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Dynamic Economic Analysis on Invasive Species Management: Some Policy Implications of Catchability*

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Abstract

The problems of controlling invasive species have been emerging as a global issue. In response to these threats, some governmental programs have been proposed for supporting eradication. This article challenges this view by studying the optimal strategies of controlling invasive species in a simple dynamic model. The analysis mainly focuses on deriving policy implications of catchability in a situation where a series of controlling actions incurs operational costs that derive from the fact that catchability depends on the current stock size of invasive species. We analytically demonstrate that the optimal policy sequence can drastically change, depending on the sensitivity of catchability in response to a change in the stock size, as well as on the initial stock. If the sensitivity of catchability is sufficiently high, the constant escapement policy with some interior target level is optimal. In contrast, if the sensitivity of catchability is sufficiently low, there could exist a threshold of the initial stock which differentiates the optimal policy between immediate eradication and giving-up without any control. In the intermediate range, immediate eradication, giving-up without any control, or more complex policies might be optimal. Numerical analysis is employed to present economic intuitions and insights in both analytically tractable and intractable cases.

Key Words: bioeconomic model, catchability, eradication, invasive species management, dynamic programming

JEL Classification: Q57; Q58

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1 Introduction

An international conference, '*Turning the tide of biological invasion: Eradication of invasive species*,' sponsored by the World Conservation Union (IUCN), was held in 2001. The occurrence of this event represents increasing salience of making decisions on whether or not to aim at eradication of invasive species in a globalized world and called for much attention on this issue in public. Many researchers claim in the proceedings of this conference that although eradication is the first best goal among several policy options, it cannot easily be concluded as a "desirable goal" in reality due to various reasons (see, e.g., Clout and Veitch (2002) and Simberloff (2002)).

Many factors that affect success or failure of eradication have already been well documented (see, e.g., Bomford and O'Brien (1995), Myers, Savoie, and van Randen (1998), and Simberloff (2002)). Among them is an economic factor that is concerned with the nature of operational costs for controlling invasive species. It is represented by the following anecdote: "Killing the first 99% of a target population can cost less than eliminating the last 1%." More precisely, the operational cost of removing one unit of invasive species may be escalated as the existing population decreases. It is evident in the case of killing the last 1-10% of the population. Focusing on such a escalating cost structure where the cost depends not only on removals but also on the current stock level, this paper studies the optimal decision rule of the removal of invasive species, including whether or not to eradicate, through using a simple dynamic model.[1]

To the best of our knowledge, only a few papers examine the optimal strategies of removing invasive species in an economic dynamic model in which the objective of a social planner is to minimize the long term social cost. Olson and Roy (2002) theoretically develop a discrete-time dynamic model under a stochastic invasion growth and study the optimal policy of eradication under the assumption that the cost of removal operations is independent of the stock size. This assumption is also employed in the work of Eisewerth and Johnson (2002), which develops a continuous-time

[1]This study focuses on the case in which the operational cost of removing one unit of invasive species is non-decreasing as the population decreases. In general, the cost structure is highly dependent on which method or technology is employed for removal operations, and there may be the case that an escalating cost structure does not hold. For example, the sterile insect technique is the one with which the cost does not escalate, or may even decrease as the population gets smaller (see, e.g., Ito and Kakihana (1999) and Dick, Hendrichs, and Robinson (2006)).

optimal control model but does not analyze the decision of eradication, and their analysis is mainly on the long-run equilibrium outcomes. Olson and Roy (2004) is the only previous work that incorporates the cost function that depends on current stock size into a dynamic model.[2] They examine the long run dynamic behavior of an optimally controlled invasion and show a wide variety of possible results under very general settings.

Building upon these previous works, this paper focuses more on deriving concrete policy implications by modelling explicitly an escalating cost structure in a discrete-time dynamic model. A new feature of this paper is to translate the aforementioned nature of the operational cost as the fact that catchability depends on the stock size, while no previous works on invasive species management consider such catchability. We demonstrate that the cost function, which is derived from stock-dependent catchability, possess a set of plausible features suggested by ecologists or resource managers.

In our specification of the cost function, the more rapidly catchability decreases with a reduction in the population, the more the cost for removal operations skyrockets. More precisely, such a property can be captured by the sensitivity of the operational cost of removing one unit of invasive species, that is 'the sensitivity of catchability,' in response to a decline in the population size. The building block in terms of catchability we adopt in the model is not new in the economics of fishery, and also Reed (1979) and Clark (1990) provide some justification in the context of the renewable resource management. They state that catchability increases in the stock size when the catch per unit of effort increases with population abundance. In other words, as the stock size of invasive species decreases, the removal operation per unit of effort becomes less effective.

Despite the fact that the cost function based on catchability seems to play an important role in the invasive species management, no previous works have analyzed such related issues and their policy implications. Given this state of affairs, the goal and contribution of this paper are to find policy implications in relation to catchability by answering the following question: how does the sensitivity of catchability in response to a change in invasive species stock affects the optimal

[2]As an example of related studies, Nyarko and Olson (1991) study the optimal policy within the context of a stochastic growth model with stock-dependent rewards.

decision-making?

The deterministic dynamic model is developed, although uncertainty such as measurement error or environmental variability is present in reality (see, e.g., Roughgarden and Smith (1996) and Sethi, Costello, Fisher, Hanemann, and Karp (2005)). This is due to the fact that even under deterministic settings, a wide variety of policy implications are obtained, and complex situations arise, which cannot be seen in the other fields of renewable resource management. It is our view that the analysis in the deterministic dynamic framework could provide a benchmark and be extended to the model under uncertainties for the purpose of comparison.

The results show that the optimal policy sequence can drastically change, depending not only on the initial stock of invasive species but also on the sensitivity of catchability in response to a change in invasive species stock. More importantly, the sensitivity of catchability is confirmed as a key for determining the type of the optimal policy. A series of recommended policies is derived as follows. The constant escapement policy with some interior target level is optimal if the sensitivity of catchability is sufficiently large. In contrast, if the sensitivity of catchability is sufficiently low, there could exist a threshold of the initial stock which differentiates the optimal policy between immediate eradication and giving-up without any control.[3] In the intermediate range, immediate eradication, giving-up without any control, or more complex policies might be optimal. However, there exist some cases that the optimal policy becomes complex and analytically intractable to be characterized in general. Thus, we employ numerical analysis on such cases to illustrate the optimal policy and to provide economic intuitions of our results and further insights.

The main contribution of this paper is summarized as follows. It shows the concrete conditions of catchability under which each of various optimal policies, i.e., eradication, non-eradication, or constant escapement policies, should be adopted. Contrary to Olson and Roy (2004), since our analysis is based on the model that considers catchability and its stock-dependence in a simple form, we are successful in connecting the conditions of optimal policies into catchability. We believe

[3]In some cases, there exists a threshold of the initial invasive species stock which differentiates long-run behaviors of stock dynamics by optimal programs. Such a threshold is generally called a Skiba point, which is typically discussed in relation to non-convexity (Skiba (1978)).

that this paper is the first one on the invasive species management in which the constant escapement rule with some interior target level may be optimal and the emergence of a Skiba point is assured in relation to stock-dependent catchability. In addition, a set of conditions for optimal policies are dependent only on parameters that could be estimable from raw data, and thus the results may potentially be informative for practitioners.

A set of novel results derived in this paper can be interestingly in contrast to those of the standard bioeconomic models in which profit maximization is the objective of the planner. In a class of standard harvesting models, Clark (1973) is the pioneering work that analyzes conditions for the optimal extinction of animals. Reed (1979) derives the most general results for the conditions under which the constant escapement policy is optimal. The structure of the model adopted in this paper for invasive species management is closely parallel to these previous researches. However, the objective of our model is to minimize the long-run social costs which consists of damage out of invasive species and culling costs. This distinction of the objective between this and theirs gives rise to the sharp contrast of conditions for optimal eradication as well as constant escapement policy that are partly common to the standard bioeconomic models, but some are totally opposite. We elaborate on such differences and provide some intuitive explanations.

The remainder of this paper is organized as follows. In the next section, we elaborate on the basic elements of the model with catchability. The section is followed by presenting the analysis with some important results. In the next section, the results of numerical analysis are shown. In the final section, we offer some conclusions.

2 The Model

This section first provides a basic dynamic framework of invasive species management and then discuss the cost of removal operation following the standard bioeconomic models.

2.1 Basic Elements

We consider an infinite-period dynamic model of invasive species, following Olson and Roy (2002), Eisewerth and Johnson (2002), and Olson and Roy (2004). Let $x_t \geq 0$ and $y_t \in [0, \gamma(x_t)]$ denote the stock (population) of existing invasive species and the number of invasive species removed in an operation by the society in period t, respectively, where $0 \leq \gamma(x_t) \leq x_t$ for any x_t. The value of $\gamma(x_t)$ represents the maximum feasible number of invasive species removed when the current stock is x_t.[4] Accordingly, $s_t \equiv x_t - y_t$ represents the escapement of invasive species in period t. We assume that the social cost in each period consists of the social damage from the escapement of invasive species and the cost associated with the removal operation of invasive species. The former cost in period t is given by $D(s_t)$, where D is an increasing and convex function with $0 < D'(s) < \infty$ for all $s \geq 0$. The latter cost in period t is given by $C(y_t, x_t)$, where C is increasing in y_t with $C(y_t, x_t) \geq 0$ for any y_t and x_t, as will be explained later. This implies that the removal cost in each period depends not only on the number of removed invasive species but also on the stock of existing invasive species in that period. Then, the payoff for the society in period t is given by:

$$u_t \equiv u(x_t, y_t) = -D(x_t - y_t) - C(y_t, x_t). \tag{1}$$

The society official maximizes the present value of the payoffs (minimizes the present value of the payoff losses) by choosing a sequence of invasive species removal $\{y_t\}_{t=0}^{\infty}$:

$$\max_{\{y_t\}} \sum_{t=0}^{\infty} \rho^t u(x_t, y_t)$$

subject to $x_{t+1} = F(s_t)$ and $s_t = x_t - y_t$, where $\rho \in (0, 1)$ is the discount factor, and $F(s)$ is the reproduction function of invasive species with $F(0) = 0$ and $F''(s) < 0$.[5] We assume that there exists an undisturbed level of the stock of invasive species, $\tau > 0$, with $\tau = F(\tau)$ such that $F(s) > s$

[4] The feasibility constraint on the number of invasive species removed may depend on the current stock from various reasons (see, e.g., Rondeau and Conrad (2003)).

[5] The reproduction function F is different from a conventional growth function, as 'growth' generally refers to an increase in the population. In this paper, F reflects growth plus the value of the stock in the previous time period.

and $F'(s) \geq 0$ if $s \in (0, \tau)$. Thus, we can restrict ourselves into the case in which the initial stock is smaller than τ, i.e., $x_0 < \tau$, so that the stock never exceeds τ in any period. The Bellman equation for this problem is:

$$v(x) = \max_{y \in [0, \gamma(x)]} \{u(x, y) + \rho v(F(x - y))\}, \tag{2}$$

where $v(x)$ is the value function given the current stock of existing invasive species, x.

It is assumed that a social planner can observe the stock size and then determine the number of invasive species removed at the beginning of each period t. Removal operations in each period t are implemented during a fixed time interval $[t, t + \epsilon]$ for some small $\epsilon \in (0, 1)$. The length of time needed for removal operations, ϵ, is short enough that it does not significantly affect the underlying population dynamics except removals themselves. After removal operations, the size of escapement in current period is realized, and then the invasive species stock for the next period is generated.[6] Furthermore, this study only considers a deterministic dynamics because our main aim is to provide a benchmark analysis on invasive species management.[7]

2.2 The Cost of Removal Operation

In this subsection, we derive the total cost of removal operations in each period t, $C(y_t, x_t)$. The main issue in this study is on the catchability of invasive species. To discuss this, we consider a situation in which, during a fixed time interval $[t, t + \epsilon]$ in each period t, the society involves the removal of invasive species, as in Reed (1979), Clark (1973), and Clark (1990). The crucial assumption is that the removal cost varies with the stock of the existing invasive species. The unit cost of removal operation, when the stock is at the level x, is given by $c(x)$, where $c(\cdot)$ is some

[6] In this time framework, removal cost in period t is assumed to be specified as a function of the stock at the beginning of that period and the stock (escapement) after removal operation during time interval $[t, t+\epsilon]$, and social damage in period t is assumed to be specified as a function of the stock (escapement) after removal operation.

[7] Once some uncertainty is incorporated into the model, analytical derivation of the optimal policy might be difficult due to mathematical complexity. Most authors adopt numerical approaches in such stochastic cases on harvesting models (see, e.g., Clark and Kirkwood (1986), Moxnes (2003) and Sethi, Costello, Fisher, Hanemann, and Karp (2005)). Examining how uncertainty affects the optimal policy must be an interesting topic to be addressed in the future research.

non-increasing function.

To derive the unit cost function $c(x)$, we consider a production function model of the removals, following Reed (1979). We assume that instantaneous removals $\dot{y}$ is related to the removal effort E and the stock x such that $\dot{y} = xq(x)E = p(x)E$, where $q(x)$ is the removal rate (mortality) per unit of effort, and $xq(x) \equiv p(x)$ is the number of removals per unit of effort or catch per unit of effort (CPUE). The dot on a variable represents its time derivative. The value of $q(x)$ is called the catchability coefficient in Reed (1979). We assume that $p(x)$ is increasing in x with $p(0) = 0$, i.e., the number of removals per unit of effort is increasing in the stock of invasive species, but the removal rate (mortality) per unit of effort may be increasing or decreasing in the stock.

The well-known Schaefer production function is one special case in which the removal rate per unit of effort is independent of the stock size, i.e., $q(x) = \bar{q}$. This corresponds to the case in which the number of removals per unit of effort is proportional to the stock, i.e., $p(x) = \bar{q}x$. If $p(x)$ is increasing and convex, then $q(x)$ is increasing, i.e., the removal of invasive species starts with the high level of the removal rate per unit of effort, resulting in its rapid decline. In contrast, if $p(x)$ is increasing and concave, then $q(x)$ is decreasing, i.e., the removal of invasive species starts with the low level of the removal rate per unit of effort, resulting in its rise. Figure 1 illustrates the graph of $p(x)$ in these two cases, in which x_t represents the current stock. Clark (1990) provides another justification for the cost function by introducing the concept of the concentration profile that explains the relationship between the exploited density and population size.[8]

The aggregate effort in a time increment $\Delta\bar{t}$ needed to catch an amount Δy when the stock is at the level x is inversely proportional to $p(x) = xq(x)$, i.e., $E(x) = 1/p(x)$. Assuming that the removal costs are proportional to the removal effort, the unit cost function is of the form:

$$c(x) = kE(x) = \frac{k}{xq(x)} = \frac{k}{p(x)}, \tag{3}$$

where $k > 0$ represents the constant cost per unit of effort. Since $p(x)$ increases with x, $c(x)$ is

[8]According to the classification by Clark (1990), the linear function $p(x) = x\bar{q}$ may correspond to diffusive species like tuna, the case of the convex function $xq(x)$ sedentary species like cod, and the case of the concave function $xq(x)$ aggressive species like anchoveta.

decreasing in x. Then, given the stock of invasive species at the beginning of period t, x_t, and the total number of removals during time interval $[t, t+\epsilon]$ in period t, y_t, the total cost of removal operations in each period t is described by:[9]

$$C(y_t, x_t) = \int_{x_t - y_t}^{x_t} c(z)dz. \tag{4}$$

This specification implies that the feasibility of eradication depends on the functional form of the unit cost function $c(x)$. That is, for given stock x, the eradication is feasible if $C(x, x)$ is finite, and it is infeasible if $C(x, x)$ is infinity.

Figure 1 provides some illustration of relation between the CPUE, $p(x)$, and the unit cost function, $c(x)$. We confirm the fact that if $p(x)$ is convex, the unit cost of removing one invasive species skyrockets as the existing invasive species are getting smaller. In such cases, the eradication may be infeasible since the total cost of eradication is very costly (the area below the graph of $c(x)$ is infinite or very large). Notice that the feasibility of eradication depends on the form of $c(x)$, in particular, on the characteristic of $c(x)$ around the origin. Even though $c(x)$ goes to infinity as x goes to zero, it is possible to have the case where the eradication is feasible, i.e., $C(x, x) < \infty$.

3 Analysis

This section first demonstrates the way of characterizing the solution to dynamic invasive species management problems and discusses the optimal policy with the unit cost function kept in a general form. We then introduce a specific functional form of the unit cost function, which is conventional in the field of renewable resource management, in order to capture the catchability and to attempt

[9]More precisely, the total cost of removal operations during each period can be derived as follows. Let $\bar{x}_t(\bar{t})$ and $\bar{y}_t(\bar{t})$ denote instantaneous number of invasive species (stock) and instantaneous total number of removals at time $\bar{t} \in [t, t+\epsilon]$ for some $\epsilon \in (0, 1)$ during period t, respectively. The number of escapements (stock) at time $\bar{t}$ is described by $\bar{x}_t(\bar{t}) = x_t - \bar{y}_t(\bar{t})$. The dynamics of the total number of removals follows that $d\bar{y}_t(\bar{t})/d\bar{t} = p(\bar{x}_t(\bar{t}))E(\bar{x}_t(\bar{t}))$ with $\bar{x}_t(t) = x_t$ and $\bar{x}_t(t+\epsilon) = x_t - y_t$. With the assumption that instantaneous removal cost is proportional to instantaneous removal effort, i.e., $\bar{c}(\bar{t}) = kE(\bar{x}_t(\bar{t}))$, we obtain that $d\bar{x}_t(\bar{t})/d\bar{t} = -p(\bar{x}_t(\bar{t}))\bar{c}(\bar{t})/k$. Then, given x_t and y_t, the total cost of removal operations over time interval $[t, t+\epsilon]$ during period t is given by $C(y_t, x_t) = \int_t^{t+\epsilon} \bar{c}(\bar{t})d\bar{t} = \int_{x_t-y_t}^{x_t} \frac{kdz}{p(z)} = \int_{x_t-y_t}^{x_t} c(z)dz$.

to get more concrete policy implications in relation to it. Note that all proofs of lemmas and propositions are in the appendix.

3.1 Optimal Policy

In order to characterize the officials' optimal strategy in the dynamic problem, we transform the payoff in period t during a decrease in the stock size from x_t to s_t into the form of $u(x_t, y_t) = -C(y_t, x_t) - D(s_t) = -[\Psi(s_t) - \Psi(x_t)] - D(s_t)$, where $\Psi(x) \equiv \int_x^\tau c(z)dz \leq \infty$ represents the total operational cost of removing invasive species from the undisturbed stock level, τ, to some stock level, x. Using the state equation of $x_{t+1} = F(s_t)$, we rewrite the objective function in the dynamic problem as:

$$\sum_{t=0}^{\infty} \rho^t u(x_t, y_t) = \sum_{t=0}^{\infty} \rho^t \left\{-[\Psi(s_t) - \Psi(x_t)] - D(s_t)\right\} = \Psi(x_0) + \sum_{t=0}^{\infty} \rho^t W(s_t), \tag{5}$$

where $W(s_t) \equiv -\Psi(s_t) + \rho\Psi(F(s_t)) - D(s_t)$ is discounted growth per period in the immediate escapement value of the population. In general, the shape of the graph $W(s)$ is highly dependent on the functional forms of $c(s)$, $F(s)$ and $D(s)$. However, this transformation of the objective functions simplifies the analysis of optimal policies since characterization of $W(s)$ suffices for that purpose.

To characterize $W(s)$, we differentiate $W(s)$ with respect to s as follows:

$$W'(s) = [c(s) - \rho c(F(s))F'(s)] - D'(s) = c(s)\left[1 - \frac{\rho c(F(s))F'(s)}{c(s)}\right] - D'(s). \tag{6}$$

Notice that $c(s)$ is simply the marginal increase in current cost associated with a unit removal at the escapement level s, $\rho c(F(s))F'(s)$ represents the discounted present value of the marginal increase in sustained future removal cost resulting from a unit increase in the escapement, and $D'(s)$ represents the marginal damage associated with the unit escapement. The value of $B(s) \equiv c(s) - \rho c(F(s))F'(s)$ can be regarded as the (removal-cost related) marginal benefit associated with the unit escapement in current period. If $B(s)$ is relatively large compared to $D'(s)$, i.e., $c(s)$ is relatively large compared to $\rho c(F(s))F'(s)$ and $D'(s)$, then it is more costly to remove the stock in the current period so that the policymakers should not involve the removal in the current period. In contrast, if $B(s)$ is relatively small compared to $D'(s)$, or if $B(s)$ is negative, i.e., either $\rho c(F(s))F'(s)$ or $D'(s)$ is relatively large compared to $c(s)$, then it is more costly to remove the stock in the future so that the policymakers should involve the removal in the current period.

Evaluating $W(s)$ around $s = 0$ and $s = \tau$, we examine whether or not the eradication policy and the giving-up policy can be optimal. Here, the *eradication policy* and the *giving-up policy* are defined as follows: given the initial state of the invasive species, the eradication policy means a class of policies that control the stock of invasive species to zero sooner or later, while the giving-up policy means a class of policies that involve non-removal operation sooner or later in order that the invasive species stock increases up to undisturbed population level τ. If a policy is neither the eradication policy nor the giving-up policy, then it is called the *interior escapement policy*, i.e., a policy that involves some partial control of the stock in the way that invasive species stock does not reach zero or undisturbed population level, τ in future periods.

Noticing that $\lim_{s\to 0} c(s) \leq \infty$ in the specification of our model, we deduce the following preliminary results:

Lemma 1 *$\lim_{s\to 0} W'(s) < 0$ if $\lim_{s\to 0} B(s) < D'(0)$, and $\lim_{s\to 0} W'(s) > 0$ if $\lim_{s\to 0} B(s) > D'(0)$. $W'(\tau) < 0$ if $B(\tau) < D'(\tau)$, and $W'(\tau) > 0$ if $B(\tau) > D'(\tau)$.*

The first part of this lemma demonstrates that $W(s)$ never attains even its local maximum at $s = 0$ if $\lim_{s\to 0} B(s) > D'(0)$. The second part states that $W(s)$ never attains even its local maximum at $s = \tau$ if $B(\tau) < D'(\tau)$. It should be noted that $\lim_{s\to 0} c(s)$ can be either finite or infinite. If $\lim_{s\to 0} c(s)$ is finite so that $\lim_{s\to 0} B(s) = 0 < D'(0)$, then it must always hold that $\lim_{s\to 0} W'(s) < 0$. On the other hand, if $\lim_{s\to 0} c(s)$ is infinite, then equation (6) implies that the sign of $\lim_{s\to 0} W'(s)$ depends on that of $1 - \rho \lim_{s\to 0}[c(F(s))F'(s)/c(s)]$, independent of the functional form of $D'(0)$. We assume that $\lim_{s\to 0} c(s) = \infty$ since our interest in this paper is on the property of escalating cost structures as the stock decreases close to zero. Here an important point to be noted is that this condition of $\lim_{s\to0} c(s) = \infty$ does not imply that eradication is infeasible. In other words, even when $\lim_{s\to0} c(s) = \infty$, we may have $C(x, x) < \infty$. In this case, the result in the first part depends not on $D(s)$ but on $c(x)$ and $F(x)$. Concerning the evaluation of $W'(\tau)$, $B(\tau) \gtreqless D'(\tau)$ implies $c(\tau) - \rho c(F(\tau))F'(\tau) \gtreqless D'(\tau)$. By Lemma 1, we directly deduce the following results related to the possibility of the eradication and the giving-up policies:

Proposition 1 *Suppose that* $\lim_{s\to0} c(s) = \infty$. *Then, the eradication policy cannot be optimal for any initial stock level if* $1 > \rho \lim_{s\to0}[c(F(s))F'(s)/c(s)]$. *On the other hand, the giving-up policy cannot be optimal for any initial stock level if* $c(\tau) - \rho c(F(\tau))F'(\tau) < D'(\tau)$. *Furthermore, the interior escapement policy is optimal for any initial stock level if* $1 > \rho \lim_{s\to0}[c(F(s))F'(s)/c(s)]$ *and* $c(\tau) - \rho c(F(\tau))F'(\tau) < D'(\tau)$.

This result demonstrates the possibility that an interior escapement policy is optimal, i.e., the optimal policy calls for some control of the stock through partial removals in some future periods, or neither the eradication policy nor the giving-up policy can be optimal for any initial stock level.

Notice that around $s = 0$, the sign of $\lim_{s\to0} W'(s)$ depends not on the damage function $D(s)$ but on the evaluation of $\lim_{s\to0} B(s) = \lim_{s\to0}[c(s) - \rho c(F(s))F'(s)]$. In other words, this condition suggests the possibility that there may exist a situation where eradication policy is never optimal no matter how large the social damage is, even though eradication is feasible. If the marginal increase in current cost associated with the unit removal around $s = 0$, $\lim_{s\to0} c(s)$, is larger than the discounted present value of the marginal increase in sustained removal cost resulting from a unit increase in the escapement around $s = 0$, $\lim_{s\to0} \rho c(F(s))F'(s)$, i.e., $\lim_{s\to0}[c(s) - \rho c(F(s))F'(s)] > 0$, then the removal cannot be justified so that the eradication policy cannot be optimal.

In contrast, if $D'(s)$ dominates $B(s)$ around $s = \tau$, the marginal damage associated with the unit escapement is larger than the corresponding marginal benefit. This shows that the escapement around $s = \tau$ cannot be justified, and thus the giving-up policy cannot be optimal. This result confirms our intuition that some control must be implemented whenever significant social damage out of invasive species is present.[10]

[10]The limiting case is when there is no social damage derived from invasive species. In this case, it is obvious that

Thus far, we have derived the sufficient conditions under which eradication or giving-up policy cannot be optimal, or equivalently interior escapement policy is optimal for any initial stock level. However, we have not shown any sufficient condition for some important class of policies, i.e., the optimality of constant escapement policy (defined below). Following the works of harvesting models in the context of conventional resource economics, we will show such conditions on the invasive species management. Recall that the feasibility constraints on the number of removals, $y_t \in [0, \gamma(x_t)]$, implies that $s_t \in [x_t - \gamma(x_t), x_t]$. Given the state $x_t \in [0, \tau]$, a policy $\{s_t\}_{t=0}^{\infty}$ is called the *constant escapement policy* with target $s^* \in [0, \tau]$ if

$$s_t = \begin{cases} x_t & \text{if } x_t \leq s^* \\ s^* & \text{if } s^* \leq x_t \leq s^* + \gamma(x_t) \\ x_t - \gamma(x_t) & \text{if } x_t \geq s^* + \gamma(x_t). \end{cases}$$

In terms of y_t, this is equivalent to the condition that $y_t = 0$ if $x_t \leq s^*$, $y_t = x_t - s^*$ if $s^* \leq x_t \leq s^* + \gamma(x_t)$, and $y_t = \gamma(x_t)$ if $x_t \geq s^* + \gamma(x_t)$.

Three types of the constant escapement policy exist. Given the state or the stock level x_t, when the target level is interior such that $s^* \in (0, \tau)$, the constant escapement policy is called the *interior constant escapement policy.* As special cases, given the state x_t, the constant escapement policy with target $s^* = 0$ is called the *constant eradication policy* in which all existing invasive species will be removed as soon as possible; and the constant escapement policy with target $s^* = \tau$ is called the *constant giving-up policy* in which no removal operation is implemented all over the periods. Note that the difference between the eradication policy in the previous arguments and the 'constant' eradication policy is that given the state, the eradication policy is to control the stock of invasive species to zero in some future period, while the constant eradication policy is to eradicate the species as soon as possible. Similarly, the difference between the giving-up policy and the 'constant' giving-up policy is that given the state, the giving-up policy is to involve non-removal

the optimal policy is giving-up.

operations at some future periods in order that the stock increases up to undisturbed population level τ, while the constant giving-up policy is not to control the invasive species at all forever from the current period. It should also be noted that the constant eradication (giving-up) policy implies the eradication (giving-up) policy.

To explore the possibility that constant escapement rule is optimal, we can directly deduce the following result from Spence and Starrett (1975), Reed (1979), and Clark (1990).

Proposition 2 *Suppose that $W(s)$ is quasi-concave in $s \in [0, \tau]$. Then, there exists a unique value $\bar{s} \equiv \arg\max_s W(s) \in [0, \tau]$ such that the constant escapement policy with target $\bar{s}$ is optimal for any stock level $x \in [0, \tau]$.*

In order to maximize the present value in equation (5) under the condition of quasi-concavity of W, it is sufficient to choose $s_t = \bar{s}$ for any period t if this sequence of escapements is feasible. Thus, if the current stock level is large enough such that $x_0 \geq \bar{s} + \gamma(x_0)$, the optimal policy is to remove the maximum feasible number of invasive species. If the current stock level is such that $\bar{s} \leq x_0 \leq \bar{s} + \gamma(x_0)$, the optimal policy is to cut the stock down to the target escapement $\bar{s}$. Furthermore, if the current stock level is small enough that $x_0 < \bar{s}$, the optimal policy is not to cut any stock until the stock recovers to $x_l > \bar{s}$, after which sustained cutting with the target escapement $\bar{s}$ should be employed.[11] Indeed, this result states that the optimal strategy results in the most rapid approach to the targeted escapement $\bar{s}$ with corresponding removals $\bar{y} = F(\bar{s}) - \bar{s}$. Furthermore, the optimal policy is the constant eradication policy if $\bar{s} = 0$, and it is the constant giving-up policy if $\bar{s} = \tau$. For simplicity, in the rest of the paper we assume that there is no feasibility constraint of removals such that $\gamma(x) = x$.

The condition that $W(s)$ is quasi-concave is that $W(s)$ is either monotone or unimodal. The unimodal case is that for some unique value $\bar{s} \in [0, \tau]$, the marginal benefit associated with the unit escapement dominates the marginal damage ($W'(s) = B(s) - D'(s) > 0$) if $s < \bar{s}$, and the marginal damage dominates the marginal benefit ($W'(s) = B(s) - D'(s) < 0$) if $s > \bar{s}$. The monotone $W(s)$ is

[11] It is assumed that the maximum number of removed invasive species in the next period is larger than the growth of invasive species from the current period to the next period, i.e., $F(x) - x < \gamma(F(x)) = \gamma(s)$. This condition guarantees that it is feasible to reduce the stock of invasive species by removal operations.

just the special case of the unimodal case: $W(s)$ is monotone decreasing in s when $\bar{s} = 0$, and $W(s)$ is monotone increasing in s when $\bar{s} = \tau$. However, as shown in a later part, the quasi-concavity of $W(s)$ holds only under certain conditions. In fact, there are various possible cases in which the quasi-concavity does not hold, and the optimal policy can be out of a class of constant escapement rules. In order to characterize $W(s)$ more carefully and to derive concrete policy implications in relation to stock-dependent catchability, we assume a specific form of the unit cost function and attempt to derive conditions not only for quasi-concavity but also for non-quasiconcavity in $W(s)$.

3.2 Catchability

For the better understanding of the role of the catchability in our bioeconomic model, from now on, this paper assumes that the removal rate per unit of effort (catchability coefficient) is represented by $q(x) = x^{\theta-1}$ and the CPUE is represented by $p(x) = xq(x) = x^{\theta}$, where $\theta > 0$. This specification of catchability is adopted by many previous researches on bioeconomic models (see, e.g., Reed (1979), Clark (1990), Moxnes (2003) and others).[12] If $\theta \in (0, 1)$, i.e., $p(x)$ is increasing and concave, then the removal rate per unit of effort is decreasing in the stock size, while the CPUE is increasing in the stock size. In contrast, if $\theta > 1$, i.e., $p(x)$ is increasing and convex, then both the removal rate per unit of effort and the CPUE are increasing in the stock size. Moreover, if $\theta = 1$, the removal rate per unit of effort is constant ($q(x) = 1$), the CPUE is linear ($p(x) = x$), and the unit cost function is represented by $c(x) = k/x$, as in the Schaefer function. In a limiting case where $\theta = 0$, the CPUE is independent of the stock ($p(x) = 1$), and the unit removal cost is constant at k.

One possible economic interpretation on the parameter θ may be that the value of θ is the constant sensitivity (elasticity) of the unit cost of removal operations in response to a change in the stock of invasive species: $\theta = -xc'(x)/c(x) > 0$. The unit cost of removal operations becomes more sensitive (elastic) to a change in the stock of invasive species as θ becomes larger, and it becomes

[12] In general it can be specified as $q(x) = bx^{\theta-1}$ for empirical purposes or adjustment of measurement unit where b is some parameter to be estimable. In this paper, we simply normalize it as $b = 1$ so as to fix attention on the sensitivity of catchability and confirm that the normalization does not impact any qualitative feature of the results. In addition, the detailed explanation for justifying this functional form of catchability is found in Section 7 of Clark (1990).

less sensitive (elastic) as θ becomes closer to zero. Thus, the parameter θ can be considered a reasonable index representing the *sensitivity of catchability* in response to a change in the stock. In addition, it may be considered that the removal technology forms the sensitivity of catchability and is different from species to species (see, e.g., Clark (1990)).

Similar to the previous subsection, in order to obtain various policy implications of catchability, we examine the optimal policy under the assumption of $c(x) = k/[xq(x)] = kx^{-\theta}$. Focusing on the sensitivity of catchability θ, we rewrite $W(s)$ by $W(s;\theta) \equiv -\Psi(s;\theta) + \rho\Psi(F(s);\theta) - D(s)$. Using equation (4), we obtain:

$$W(s_t;\theta) = \begin{cases} \frac{k}{1-\theta}[s_t^{1-\theta} - \rho F(s_t)^{1-\theta}] - D(s_t) + N(\theta) & \text{if } \theta \neq 1 \\ k[\ln s_t - \rho \ln F(s_t)] - D(s_t) + N(\theta) & \text{if } \theta = 1, \end{cases} \tag{7}$$

for some constant $N(\theta)$. To understand the impact of a change in the escapement on $W(s_t;\theta)$, we differentiate equation (7) and obtain the net marginal benefit associated with the unit escapement:

$$\frac{\partial W(s_t;\theta)}{\partial s_t} = B(s_t) - D'(s_t) = \frac{k}{[F(s_t)]^\theta}\left[\left(\frac{F(s_t)}{s_t}\right)^\theta - \rho F'(s_t)\right] - D'(s_t), \tag{8}$$

where $\lim_{s\to 0}[F(s)/s] = F'(0) > 1$, $F(\tau) = \tau$, and $\rho F'(\tau) < 1$. In general, the shape of the graph $W(s;\theta)$ is highly dependent on the sensitivity of catchability, θ, and the functional forms of $F(s)$ and $D(s)$.

Based on the results of Lemma 1 and Proposition 1, we first connect the sensitivity of catchability into the discussion of whether or not the eradication policy and the giving-up policy can be optimal through characterizing $W(s;\theta)$ around $s = 0$ and $s = \tau$. Let

$$\hat{\theta} \equiv 1 + \frac{\ln \rho}{\ln F'(0)} < 1 \quad \text{and} \quad \bar{\theta} \equiv \frac{1}{\ln \tau} \ln\left(\frac{k[1-\rho F'(\tau)]}{D'(\tau)}\right). \tag{9}$$

From equations (7) and (8), evaluating $W(s;\theta)$ around $s = 0$ and $s = \tau$ yields the following preliminary results corresponding to Lemma 1:

Lemma 2 *$\lim_{s\to 0} \frac{\partial W(s;\theta)}{\partial s} < 0$ if $\theta < \hat{\theta}$ and $\lim_{s\to 0} \frac{\partial W(s;\theta)}{\partial s} > 0$ if $\theta > \hat{\theta}$. $\frac{\partial W(\tau;\theta)}{\partial s} > 0$ if $\theta < \bar{\theta}$ and $\frac{\partial W(\tau;\theta)}{\partial s} < 0$ if $\theta > \bar{\theta}$.*

The first part shows that $W(s;\theta)$ never attains even its local maximum at $s = 0$ if $\theta > \hat{\theta}$, and the second part states that $W(s;\theta)$ never attains even its local maximum at $s = \tau$ if $\theta > \bar{\theta}$. Since $c(s)$ satisfies $\lim_{s\to 0} c(s) = \infty$, the critical value $\hat{\theta}$ depends on the form of $F'(0)$, irrespective of $D'(0)$. On the other hand, the critical value $\bar{\theta}$ is dependent on the forms of $F'(\tau)$ and $D'(\tau)$. In particular, if the marginal damage associated with the unit escapement, $D'(\tau)$, is larger, then the critical value $\bar{\theta}$ becomes smaller so that the giving-up policy is less appropriate for relatively high θ. Notice that whether $\hat{\theta}$ is larger than $\bar{\theta}$ is in general ambiguous, and it partly depends on the value of $D'(\tau)$. Furthermore, the conditions in this lemma partially share the results of Clark (1973) in the discussion of whether or not the extinction is optimal in his harvesting model. Then, by Lemma 2, we directly deduce the following results related to the possibility of the eradication and the giving-up policies in Proposition 1:

Proposition 3 *The eradication policy cannot be optimal for any initial stock level if the sensitivity of catchability is large enough such that $\theta > \hat{\theta}$. On the other hand, the giving-up policy cannot be optimal for any initial stock level if the sensitivity of catchability is large enough such that $\theta > \bar{\theta}$. Furthermore, the interior escapement policy is optimal for any initial stock level if $\theta > \max\{\hat{\theta}, \bar{\theta}\}$.*

If the sensitivity of catchability is sufficiently large such that $\theta > \max\{\hat{\theta}, \bar{\theta}\}$, then some interior escapement policy is optimal, or neither the eradication policy nor the giving-up policy can be optimal for any initial stock level.

A larger sensitivity of catchability implies that the unit cost of removal is increasing more rapidly as the stock level decreases closely to the eradication. Thus, when the stock level is so small, the marginal benefit associated with the unit escapement, $B(s)$, is very large, and in other words, the cost of the removal of all existing invasive species to the eradication dominates its benefit. As a result, the eradication policy cannot be supported as optimal for any initial stock level.

It should be noted that only when technology with sufficiently low sensitivity of catchability with $\theta < \bar{\theta}$ is available, the eradication policy may be justified. More importantly, even when eradication is feasible in the sense that the removal cost of eradication is finite, i.e., $C(x, x) = \int_0^x c(z)dz < \infty$ or $\theta < 1$, the eradication cannot be supported as optimal if the sensitivity of catchability is such that $\theta \in (\hat{\theta}, 1)$.

Concerning the giving-up policy, a larger sensitivity of catchability implies that the marginal benefit associated with the unit escapement, $B(s)$, dominates the marginal damage, $D'(s)$, around $s = \tau$. The unit cost of removal is declining more rapidly as the stock level increases to the undisturbed level $\tau = F(\tau)$. Thus, when the stock level is close enough to the undisturbed level, the cost of the removal of some invasive species becomes relatively small compared to an increase in social damage associated with a rise in the stock under no control of invasive species. As a result, the giving-up policy cannot be supported as optimal for any initial stock level.

Recall from Proposition 2 that for given θ, if $W(s;\theta)$ is quasi-concave in s, there exists a unique value $\bar{s}(\theta) \equiv \arg\max_s W(s;\theta) \in [0, \tau]$ such that the constant escapement policy with target $\bar{s}(\theta)$ is optimal for any stock level $x \in [0, \tau]$. However, there may be various possible cases in which the quasi-concavity of $W(s;\theta)$ does not hold, and the optimal policy can be complicated. For example, if W is inverse unimodal (i.e., for some $\bar{s} \in (0, \tau)$, $\frac{\partial W(s;\theta)}{\partial s} < 0$ for all $s \in [0, \bar{s}]$ and $\frac{\partial W(s;\theta)}{\partial s} > 0$ for all $s \in [\bar{s}, \tau]$), then the optimal policy could be either constant eradication or constant giving-up policy, depending on the initial stock x_0. In this case, there could be a threshold of the initial stock level separating different long-run behaviors. The logic of this threshold could be consistent with a Skiba point that is first proposed by the pioneering work of Skiba (1978).[13]

In order to make our discussion clarified, we should identify when W satisfies the quasi-concavity. However, given the functional form of $c(x)$, it is generally difficult to fully derive the condition that W is quasi-concave. Admitting such a difficulty, the following subsections discuss the relation among the catchability and the optimal policy through examining two cases: the first is a case where W is strictly concave; and the second is a case where W is strictly convex. In the first case, W always satisfies the quasi-concavity that is the condition in Proposition 2 (we call this case 'quasi-concavity

[13]See e.g., Maler (2000) for an application of the Skiba point in the context of resource economics. A similar property is also found in Majumdar and Mitra (1983) which studies the problem of optimal intertemporal allocation in a model with a non-convex technology.

case'). The second case provides the possibility that W is not quasi-concave. In fact, W is inverse unimodal unless it is monotone.

3.3 Quasi-Concavity Case

We now attempt to connect the catchability into the model and to characterize when a class of constant escapement policy is optimal, independently of the initial stock level. Differentiating equation (8) with respect to s yields:

$$\frac{\partial^2 W(s;\theta)}{\partial s^2} = \frac{k}{(F(s))^{\theta+1}} \left[-\theta \left(\frac{F(s)}{s} \right)^{\theta+1} - \rho F''(s)F(s) + \theta\rho(F'(s))^2 \right] - D''(s). \tag{10}$$

Concerning the sufficient condition of strict concavity of W, we deduce the following preliminary result:

Lemma 3 *There exists some value $\theta^* \geq 0$ such that for all $\theta > \theta^*$, W is strictly concave in $s \in [0,\tau]$, i.e., $\frac{\partial^2 W(s;\theta)}{\partial s^2} < 0$ for all $s \in [0,\tau]$.*

This result implies that W is strictly concave and hence quasi-concave in s if the sensitivity of catchability is sufficiently high. In this case, W must satisfy the condition in Proposition 2 so that the constant escapement policy with target $\bar{s}(\theta) \equiv \arg\max_s W(s;\theta) \in [0,\tau]$ is optimal for any stock level. The concavity of W can be attained in a situation where the marginal benefit associated with the unit escapement, $B(s)$, is decreasing in s, or the convexity property of damage function $D(s)$ is relatively large for any s. Notice that $\theta^* \geq \min\{\hat{\theta}, \bar{\theta}\}$ must hold since $\theta^* < \min\{\hat{\theta}, \bar{\theta}\}$ does not allow W to be strictly concave and quasi-concave in s by Lemma 2. Then, by Propositions 2 and 3 and Lemma 3, we obtain the following results:

Proposition 4 *Suppose that the sensitivity of catchability is sufficiently high such that $\theta > \theta^*(\geq \min\{\hat{\theta}, \bar{\theta}\})$. Then, (1) the interior constant escapement policy with some level $s^* \in (0,\tau)$ is optimal for any stock level if $\theta > \max\{\hat{\theta}, \bar{\theta}\}$; (2) the constant giving-up policy is optimal for any stock level if $\hat{\theta} < \theta < \bar{\theta}$; and (3) the constant eradication policy is optimal for any stock level if $\bar{\theta} < \theta < \hat{\theta}$.*

Notice that the optimal policy is independent of the initial stock level. Under strict concavity of W, W must be either unimodal or monotone. There are three cases in the analysis related to the sensitivity of catchability to confirm the intuitions in this proposition.

Case A-I ($\theta > \max\{\hat{\theta}, \bar{\theta}\}$ and $\theta > \theta^*$): By Proposition 3, if $\theta > \max\{\hat{\theta}, \bar{\theta}\}$, then the interior escapement policy is optimal, or neither the eradication policy nor the giving-up policy can be optimal for any initial stock level. Since the condition of $\theta > \theta^*$ implies the strict concavity of W, it must hold that W is unimodal in s and attains its maximum at some interior escapement level $s^* \in (0, \tau)$. As a result, the constant escapement policy with s^* is optimal for any stock level. Case A-I is simply likely to occur when the marginal change in social damage is sufficiently large so that $\bar{\theta}$ is sufficiently small (see equation (9)), and when the sensitivity of catchability are sufficiently large.

Case A-II ($\hat{\theta} < \theta^* < \theta < \bar{\theta}$): If $\hat{\theta} < \theta < \bar{\theta}$, then the eradication policy cannot be optimal and the giving-up policy can be optimal for any initial stock level, by Proposition 3. The condition of $\theta > \theta^*$ requires that W must be increasing in s and attain its maximum at $s = \tau$, which yields that the constant giving-up policy is optimal for any initial stock level. Case A-II could occur when the marginal change in social damage is not sufficiently large such that $\bar{\theta} > \hat{\theta}$ holds (see equations (9)). Given these conditions, Case A-II emerges if the sensitivity of catchability takes some intermediate value between $\hat{\theta}$ and $\bar{\theta}$.

Case A-III ($\bar{\theta} < \theta^* < \theta < \hat{\theta}$): If $\bar{\theta} < \theta < \hat{\theta}$, then the giving-up policy cannot be optimal and the eradication policy can be optimal for any initial stock level, by Proposition 3. The condition of $\theta > \theta^*$ requires that W must be decreasing in s and attain its maximum at $s = 0$, which yields that the constant eradication policy is optimal for any initial stock level. Case A-III could occur when the marginal change in social damage is sufficiently large such that $\bar{\theta} < \hat{\theta}$ holds (see equations (9)). Given these conditions, Case A-III emerges if the sensitivity of catchability takes some intermediate value between $\bar{\theta}$ and $\hat{\theta}$.

The most important message in Proposition 4 is that whatever the relation of $\theta^*, \bar{\theta}$ and $\hat{\theta}$ is, the interior constant escapement policy is optimal when the sensitivity of catchability is sufficiently large. That is Case A-I. The other two cases of Case A-II and A-III are shown to illustrate the situations where constant eradication or constant giving-up policy may be optimal when the sensitivity of catchability takes some intermediate values.

3.4 Possibility of Non-Quasiconcavity Case

The aim of this subsection is to show the possibility that W is not quasi-concave. In particular, as will be explained, if W is inverse unimodal, the optimal policy is not in a class of the constant escapement policy that is independent of the initial stock. To discuss that, we focus on a case where W is strictly convex, in contrast to the previous case where W is strictly concave. Notice that even when W is strictly convex, it can be quasi-concave if it is monotone.

Concerning the condition that W is strictly convex in s, we first deduce the following preliminary results:

Lemma 4 *Suppose that ρk is sufficiently large such that* $\sup_{s\in[0,\tau]}[\rho k F''(s) + D''(s)] < 0$. *Then, there exists some value $\theta^{**} > 0$ such that for all $\theta \in [0, \theta^{**})$, W is strictly convex in $s \in [0, \tau]$, i.e., $\frac{\partial^2 W(s;\theta)}{\partial s^2} > 0$ for all $s \in [0, \tau]$.*

Since $\frac{\partial W(s;\theta)}{\partial s} = B(s) - D'(s)$, the convexity of W can be attained in a situation where the marginal benefit associated with the unit escapement, $B(s)$, is increasing in s, and its property dominates the convexity of damage function $D(s)$ for any s. The assumption of $\sup_{s\in[0,\tau]}[\rho k F''(s) + D''(s)] < 0$ requires that the concavity property of the reproduction function F is relatively large compared to the convexity property of damage function D, or that the discount factor ρ or the cost per unit of effort k is relatively large. Given this assumption, W is strictly convex and hence may not be quasi-concave if the sensitivity of catchability is sufficiently small. Notice that $\theta^{**} \leq \max\{\hat{\theta}, \bar{\theta}\}$ must hold since $\theta^{**} > \max\{\hat{\theta}, \bar{\theta}\}$ does not allow W to be strictly convex in s by Lemma 2. Then, by Propositions 2 and 3 and Lemma 4, we obtain the following results:

Proposition 5 *Suppose that ρk is sufficiently large such that* $\sup_{s\in[0,\tau]}[\rho k F''(s) + D''(s)] < 0$, *and that the sensitivity of catchability is sufficiently low such that $\theta < \theta^{**}(\leq \max\{\hat{\theta}, \bar{\theta}\})$. Then,* (1) *if*

$\theta < \min\{\hat{\theta}, \bar{\theta}\}$, there could exist a unique stock level $\bar{x}_0 \in (0, \tau)$ such that the constant eradication policy is optimal if the initial stock level is sufficiently small such that $x_0 < \bar{x}_0$, and the constant giving-up policy is optimal if the initial stock level is sufficiently large such that $x_0 > \bar{x}_0$; (2) *the constant giving-up policy is optimal for any stock level if $\hat{\theta} < \theta < \bar{\theta}$; and* (3) *the constant eradication policy is optimal for any stock level if $\bar{\theta} < \theta < \hat{\theta}$.*

Under the strict convexity of W, the graph of W must be either inverse unimodal or monotone. As before, there are three cases in the analysis related to the sensitivity of catchability to confirm the intuitions in this proposition.

Case B-I ($\theta < \min\{\hat{\theta}, \bar{\theta}\}$ and $\theta < \theta^{**}$): By Proposition 3, if $\theta < \min\{\hat{\theta}, \bar{\theta}\}$, then either the eradication policy or the giving-up policy may be optimal depending on the initial stock level. Since the condition of $\theta < \theta^{**}$ implies the strict convexity of W, it must hold that W is inverse unimodal and attains its minimum at some interior escapement level. In this case, the optimal escapement policy depends on the answer to the following question: Which maximum (corner) points should we seek to reach from an arbitrary starting point $x_0 \in [0, K]$? The answer is dependent on how many periods it takes for us to reach each maximum point as well as cumulative payoffs with a discount factor during that time. Thus, the optimal policy is also highly dependent on the initial stock size, and there would be a unique threshold $\bar{x}_0$ such that if the initial stock x_0 is smaller than $\bar{x}_0$, then the constant eradication policy is optimal, otherwise the constant giving-up policy is optimal. The threshold $\bar{x}_0$, which could be regarded as a Skiba point, separates different optimal policies (see Skiba (1978)).

In general, the emergence of a Skiba point may derive from non-classical assumptions, mainly: (i) a non-convex feasible set, (ii) a non-concave maximand, and (iii) a state-dependent reward (see, e.g., Tahvonen and Salo (1996), Rondeau (2001), Dasgupta and Maler (2003), and Maler, Xepapadeas, and de Zeeuw (2003) for studies on environmental issues). Such non-classical assumptions in the model can lead to multiple basins of attraction, and which one to move to may depends on the initial state. In our model, the emergence of a Skiba point relies mainly on one of non-classical assumptions that the cost of removal depends not only on the control but also on the state, that is, state-dependent reward assumption.[14]

It must be noted that Case B-I is totally opposite to Case A-I. In terms of θ, Case A-I corresponds to a sufficiently large θ, while Case B-I corresponds to a sufficiently small θ. In terms of the graph of W, Case A-I corresponds to the unimodal shape, while Case B-I corresponds to the inverse unimodal shape, with an interior optimum in both cases. The interior constant escapement policy is optimal in Case A-I, while there could exist a Skiba point that differentiates the long run behaviors of optimally controlled stock dynamics in Case B-I.

Case B-II ($\hat{\theta} < \theta < \theta^{**} < \bar{\theta}$): If $\hat{\theta} < \theta < \bar{\theta}$, then the eradication policy cannot be optimal and the giving-up policy can be optimal for any initial stock level, by Proposition 3. The condition of $\theta < \theta^{**}$ requires that W must be increasing in s and attain its maximum at $s = \tau$, which yields that the constant giving-up policy is optimal for any stock level. Case B-II could occur when the marginal change in social damage is not so dominant that the relation of $\hat{\theta} < \bar{\theta}$ holds (see equations (9)) and when the sensitivity of catchability takes some intermediate values between $\hat{\theta}$ and $\bar{\theta}$.

Case B-III ($\bar{\theta} < \theta < \theta^{**} < \hat{\theta}$): If $\bar{\theta} < \theta < \hat{\theta}$, then the giving-up policy cannot be optimal and the eradication policy can be optimal for any initial stock level, by Proposition 3. The condition of $\theta < \theta^{**}$ ensures that W must be decreasing in s and attain its maximum at $s = 0$, which yields that the constant eradication policy is optimal for any stock level. Case B-III could occur when the marginal change in social damage is sufficiently large such that the relation of $\bar{\theta} < \hat{\theta}$ holds (see equation (9)), and when the sensitivity of catchability takes some intermediate values between $\bar{\theta}$ and $\hat{\theta}$.

The most important result in Proposition 5 is that whatever the relation of $\theta^{**}, \bar{\theta}$ and $\hat{\theta}$ is, if the sensitivity of catchability is sufficiently small, there could exist a Skiba point that leads to different long run behaviors of optimally controlled stock. That is Case B-I. The other two cases of Case B-II and B-III are shown to illustrate the situations where the constant giving-up or constant eradication policy may be optimal when the sensitivity of catchability takes some intermediate values.

Notice that Cases A-I, A-II and A-III under the strict concavity of W and Cases B-I, B-II and

[14] From the results of numerical analysis in a later section, we can confirm this fact.

B-III under the strict convexity of W show a sharp contrast related to the catchability. The former cases correspond to relatively high sensitivity of catchability, while the latter cases correspond to relatively low sensitivity of catchability. If θ is sufficiently high, then the constant escapement policy with some interior target is optimal for any initial stock level (Case A-I). If θ is sufficiently low, then the optimal policy takes either the constant eradication or the constant giving-up, depending on the initial stock level, and this case can be classified into a different class of constant escapement policy that is dependent on the initial stock level (Case B-I).

3.5 Discussions

A set of results obtained thus far has several differences from standard bioeconomic models (e.g., Reed (1979) and Clark (1990)). First, our results show that as the sensitivity of catchability increases, the optimal policy may switch from a strategy of 'giving-up' to a strategy of 'some control.' The larger sensitivity of catchability means the larger marginal benefit (cost-saving) associated with the unit escapement in the current period. In standard harvesting models, this effect should cause the optimally controlled stock to increase. However, our analysis suggests that the opposite effect occurs in the sense that the optimally controlled stock should decrease. This is due to the fact that the invasive species is a nuisance and the associated variable is opposite on the invasive species management.

Second, the conditions for the optimal eradication can be compared to those of optimal extinction derived by Clark (1973) and Clark (1990). These works show that if zero profit level population is nonzero and discount factor is sufficiently small compared to marginal stock reproduction evaluated at zero, then policies that lead to extinction is optimal. The conditions for optimal eradication in our model also depends on the discount factor as well as the marginal stock reproduction evaluated at zero (see $\hat{\theta}$ in equation (9)). However, the impact of discount factors is opposite. If the discount factor gets smaller, then $\hat{\theta}$ gets smaller so that eradication is more unlikely to be supported as optimal (Proposition 3). In the invasive species management, eradication is costly in the short-run, but it can be beneficial from the long-run perspective. Therefore, a smaller discount factor is likely to yield non-eradication policy as optimal.

Third, we also show that if the sensitivity of catchability is sufficiently large, then the interior constant escapement policy is optimal, otherwise not, as in Proposition 4. This result may be partly common to those obtained by Reed (1979), but some are totally different. Reed (1979) shows that the interior constant escapement policy is likely to be optimal especially when the sensitivity of catchability is less than one, otherwise the optimal policy may be out of constant escapement rules. In contrast to his results, our model yields the opposite situation. In our model, if the sensitivity of catchability is smaller, then the marginal cost saving associated with the unit escapement becomes smaller. In this case, the interior constant escapement policy cannot be optimal since the immediate eradication policy is more attractive. As noted previously, the impact of the change in the sensitivity of catchability works in the opposite direction to the standard bioeconomic models, and thus our results are opposite, too.

In summary, a series of propositions have characterized the optimal policy especially when the sensitivity of catchability is sufficiently large or small. An important point to be noted is that the sensitivity of catchability drastically affects the optimal policy. From the practical perspective, this result could be considered significant. When the CPUE can be estimated from stock data, which is often collected by most management agencies, policymakers can estimate the approximation of the sensitivity of catchability. Such reliable information about θ may be sufficient for rational decision of control. In addition, some sensitivity analysis can be made for various forms of social damage and reproduction functions.

There are still two crucial arguments to be further explored: (i) how representative is a class of the optimal policies, which we have analytically characterized so far, given the stylized functional forms of the damage and the reproduction functions with plausible parameters? (ii) what would happen for the optimal policy when θ is in the intermediate range under which the conditions in the previous propositions are not met so that our model is analytically intractable?

To illustrate the above statements, numerical analysis is employed in the following section. Regarding the first, we identify each of the critical values (such as θ^{*}, θ^{**}, $\hat{\theta}$ and $\bar{\theta}$) and show that in most cases, the optimal policy would fall in the classes of policies discussed in the previous propositions. Regarding the second argument, there can exist more complex cases with the property of the non-quasiconcavity of W. As an interesting case, we identify the cases where W is bimodal, and explain the intuitions behind the occurrence of such cases.

4 Numerical Analysis

This section illustrates various situations via numerical analysis where the optimal policy is analytically tractable and intractable. The aim of this section is two-fold. First, we seek to confirm our analytical results by changing only the sensitivity of catchability, holding other factors fixed. Second, we also try to demonstrate complex cases that could not be fully characterized analytically.

As we noted earlier, some cases exhibit complex optimal policy. This would be the case especially (i) when social damage out of invasive species is not dominant compared with operational costs for removals, as well as (ii) when the sensitivity of catchability takes some intermediate values. That is, when the convexity of the social damage $D(s)$ is obviously significant, the quasi-concavity of $W(s)$ is more likely to be supported, and thus the class of constant escapement rules is optimal, independently of the initial stock level, otherwise the optimal decision rule is very elusive and may be dependent on the initial stock. In addition, Saphores and Shogren (2005) note that:

'*Not every invasive species causes damages to such a degree as to warrant immediate attention.*'

This statement implies that social damage caused by invasive species is not obviously significant in every case, which corresponds to the situation where $W(s)$ is NOT quasi-concave. When $W(s)$ is not quasi-concave and a Skiba point exists, there are an important question to be answered, say, how does a Skiba point move when some key parameter changes. Such a qualitative feature of the optimal policies under non-quasiconcavity is difficult to characterize and can only be approached by numerical analysis.

For the sake of computation, we make the following two specific assumptions in terms of functional forms. First, the social damage derived from invasive species is given by the linear quadratic form:

$$D(s) = a_1 s + \frac{a_2 s^2}{2},$$

where $s \in [0, \tau]$ denotes the number of escapements with $a_1 > 0$ and $a_2 > 0$. The parameter a_2 represents the degree of strict convexity of social damage. Second, the reproduction of invasive species follows the conventional logistic curve:

$$F(s) = rs\left(1 - \frac{s}{K}\right) + s,$$

where $r > 0$ is the intrinsic growth rate and $K > 0$ is the carrying capacity.[15] The two functional forms satisfy the assumptions specified in the previous sections and are also employed by some other authors in the settings of invasive species management (see, e.g., Olson and Roy (2004) and Eisewerth and Johnson (2002)). It should also be noted that there are other candidates that have been commonly used, especially reproduction functions of invasive species, such as the Ricker or depensation type (see, e.g., Quinn II and Deriso (1999)). However, it can be confirmed that as far as the basic assumptions are met, the qualitative results of the numerical analysis presented in this section could hold irrespective of the functional forms.

The value function iteration algorithms introduced in Judd (1998) are adopted to approximate the value function $v(x)$ as well as the optimal policy function $y^*(x)$, which are characterized by the Bellman equation (2).[16] This algorithm first involves the discretization of the state space, and then iterates on the Bellman equation with an initial guess for the value function. It is shown that by the contraction theorem, the Bellman equation does fix a unique value function, $v(x)$, and the iterative process converges to the true value function. Accordingly, a particular optimal policy $y^*(x)$ and the optimal escapement rule, that is, $s^*(x) = x - y^*(x)$, are obtained.

[15] It should be noted that the logistic curve may exhibit a chaotic behavior depending on the set of parameters r and K (see, e.g., Conrad (1999)). In this section, however, the values of r and K that give rise to a stable equilibrium $\tau > 0$ with $F(\tau) = \tau = K$ is chosen for the purpose of illustration. More general discussions with respect to dynamic behaviors of difference equations can be found in May (1974) and Elaydi (2005).

[16] Matlab code is written for numerical solutions.

For our baseline, we choose $a_1 = 1$ and $a_2 = 1$ for social damage function, $D(s)$; $\rho = 0.95$ for a social discount rate; $r = 0.3$ and $K = 10$ for the reproduction of invasive species, $F(s)$; and $k = 250$ for the cost function associated with removal operations. In addition, the feasibility constraint on the stock of invasive species removed is set as $\gamma(x) = x$, as in the previous sections. Given these values and assumptions, the two critical parameter values regarding the sensitivity of catchability in Lemma 2 are computed as follows:

$$\hat{\theta} \approx 0.800; \quad \bar{\theta} \approx 0.882.$$

To examine the impact of a change in the sensitivity of catchability on the optimal policy, we try the following four cases: $\theta_1 = 0.2$, $\theta_2 = 0.6$, $\theta_3 = 0.85$ and $\theta_4 = 1.1$, each of which is denoted by Cases 1, 2, 3 and 4, respectively. For each case, we draw the two graphs in Figures 2 to 5: one represents $W(s)$ and the other is the optimal escapement policy. As we discussed in the analysis, the property of $W(s)$ fully characterizes the optimal escapement rule, and these two graphs suffice to elaborate on the intuitions of our results.

We first focus on the cases in which $\theta_i < \hat{\theta} \approx 0.80$ and $\theta_i < \bar{\theta} \approx 0.88$. They correspond to Case B-I with $\theta_i < \min\{\hat{\theta}, \bar{\theta}\}$ in Proposition 5 and to Cases 1 ($\theta_1 = 0.2$) and 2 ($\theta_2 = 0.6$), which are illustrated in Figures 2 and 3, respectively. In both cases, the parameter set is chosen in the way that the convexity of social damage is not so significant that $W(s)$ is strictly convex. In addition, the fact that $\theta_1 < \theta_2 < \min\{\hat{\theta}, \bar{\theta}\}$ requires that $W(s)$ is inverse unimodal so that the shapes of $W(s)$ are characterized by an interior local minimum at $s_{\min} \in (0, K)$ and two local (corner) maximums at $s = 0$ and $s = K$. Thus, the constant escapement policy cannot be optimal, and either the constant eradication or the constant giving-up policy is optimal, depending on the initial stock level, as shown in Proposition 5.[17] This situation is corresponding to the one in which a Skiba point appears and differentiates the long-run behaviors of dynamics.

In general, as the sensitivity of catchability θ_i increases up to $\hat{\theta}$, the interior local minimum, $s_{\min}$,

[17] If a set of parameter values is chosen in the way that social damage dominates operational costs for eradication, it is easy to see that quasi-concavity holds. It is simply achieved by choosing a larger values of a_1 or a_2 in the social damage function.

gradually gets closer to zero and finally disappears. This is numerically illustrated by comparing the local minimums of $W(s)$ in Figures 2 and 3. The local minimum in Case 1 is around $s_{\min} = 4.2$, while the one in Case 2 is around $s_{\min} = 2$. The logic behind this could be that when θ becomes lower, the operational cost for eradication and the future benefit of escaping one unit of stock in the current period get lower. This implies that the constant eradication policy is more attractive than escapement as θ becomes lower. Therefore, the regions of the initial stock size that justify the constant eradication policy is larger with low θ than with high θ. In other words, a Skiba point gradually approaches to zero when the sensitivity of catchability, θ, gets closer to $\hat{\theta}$.

Next we turn to the cases in which $\theta_i > \hat{\theta} \approx 0.80$, which corresponds to Case 3 ($\theta_3 = 0.85$) and Case 4 ($\theta_4 = 1.1$). In general, when θ_i is larger than $\hat{\theta}$, an interior local minimum, $s_{\min}$, of $W(s)$ disappears as stated in the previous paragraph. Accordingly, $W'(0)$ becomes positive around $s = 0$, and thus the eradication policy should not be adopted as stated in Proposition 3. Moreover, it should be noted that $W(s)$ now gets strictly concave by increasing only the sensitivity of catchability, θ_i, as illustrated in Figures 4 and 5. Notice also that the strict concavity of W could also arise when the convexity of social damage is made so dominant.

Case 3 corresponds to Case A-II with $\hat{\theta} < \theta_3 < \bar{\theta}$ in Proposition 4 and describes a situation where $W(s)$ is monotone increasing and thus the constant giving-up policy is optimal for any initial stock (Figure 4). By inspection of equation (9), the critical value $\bar{\theta}$ becomes lower when $D'(\tau)$ is larger. As $D'(\tau)$ is sufficiently large, $\bar{\theta}$ can be lower than $\hat{\theta}$. In this case, if $\bar{\theta} < \theta < \hat{\theta}$, then $W(s)$ is monotone decreasing, and thus the constant eradication policy is optimal for any initial stock.

Case 4 describes the situation in which θ_4 is larger than both $\hat{\theta}$ and $\bar{\theta}$. It corresponds to Case A-I with $\theta_4 > \max\{\hat{\theta}, \bar{\theta}\}$ in Proposition 4. In this case, $W(s)$ is unimodal with its interior maximum, and thus the constant escapement rule with some interior target level $\bar{s} \in (0, \tau)$ is optimal, as illustrated in Figure 5.

More Complex Case Throughout the numerical analysis so far, it has been confirmed that $W(s)$ is strictly convex and inverse unimodal for a sufficiently small θ (Cases 1 and 2) and is strictly concave for a sufficiently large θ (Cases 3 and 4). However, the convexity and the concavity of $W(s)$ is not always guaranteed for all s. This subsection presents more complex situations than the previous ones, where the optimal policy can be very tricky especially when θ takes some intermediate range such that the conditions in Propositions 4 and 5 are not satisfied.

To create such situations, we change some parameters as $a_2 = 1.6$ and $\theta = 0.75$, holding the other parameters unchanged. We denote this case as Case 5, which is corresponding to the case of intermediate range θ. Observing the shape of $W(s)$ in Figure 6, it is noted that $W(s)$ is neither convex nor concave, and there exist an interior local maximum and an interior local minimum. As in Cases 1 and 2, Case 5 also exhibits a local maximum at the boundary of $s = 0$, but the critical difference is that Case 5 now has the two interior extremum points in $W(s)$. This situation seems to arise when an increasing rate of social damage, a_2, is moderately large.[18] Moreover, the relation between $\hat{\theta}$ and $\bar{\theta}$ becomes converse with Cases 1 and 2 so that $\bar{\theta} \approx 0.69 < \theta < \hat{\theta} \approx 0.80$. Notice that $\bar{\theta}$ is changed by a change in the parameters, while $\hat{\theta}$ is the same as in the previous examples since $\hat{\theta}$ does not depend on any parameter we changed. Proposition 3 says that the giving-up policy is never optimal in this case. Indeed, Figure 6 shows that immediate eradication is optimal for a relatively small stock, and the constant escapement policy with some interior target level is optimal for a relatively large stock. That is, the optimal policy is dependent on initial stock level, as in Cases 1 and 2. This is the other case in which a Skiba point emerges.

One interesting angle is that the changed parameter value, a_2, represents the convexity of the social damage function $D(s)$. Holding the sensitivity of catchability in some intermediate range, if a_2 becomes closer to unity or smaller, a similar situation as in Cases 1 and 2 arises due to the fact that relatively low convexity of $D(s)$ implies that W is strictly convex and inverse unimodal. Accordingly, either the constant eradication or the constant giving-up policy is optimal, depending on the initial stock. On the other hand, if a_2 gradually becomes larger and the convexity of $D(s)$ becomes intensified, a complex situation as in Case 5 arises. Accordingly, the optimal policy would be either the constant eradication or the constant escapement rule with some interior target level,

[18] Once a_2 becomes sufficiently large, strict convexity of $D(s)$ gets dominant so that the strict concavity of W is guaranteed. This will be discussed later on.

depending on the initial stock. These two cases are all of the situations where a Skiba point emerges as far as we examined. Finally, if a_2 becomes sufficiently large, then the constant eradication policy is optimal for any initial stock since the sufficiently large convexity of D causes W to be monotone decreasing.

Graphically, for a relatively small value of a_2, only an interior minimum point of W appears (i.e., W is inverse unimodal). As the value of a_2 rises, an interior maximum point shows up (i.e., W has two interior optimum), and then the graph of W becomes flattened with the optimum points shifting to the left. As the value of a_2 rises further, the interior optimum points ends up disappearing, and the graph of W becomes decreasing (i.e., $W(s)$ is monotone decreasing).

5 Conclusion

This paper has addressed an important concern on the invasive species management, including the arguments such as whether or not to aim at eradication and whether or not to control the stock. While there are many reasons that influence the resulting outcomes, our focus is on deriving policy implications of stock-dependent catchability. Once such feature of catchability is incorporated into a dynamic model, some novel results are obtained from such a non-classical model.

We have shown that the sensitivity of catchability is crucial in the optimal decision rule. If the sensitivity of catchability is sufficiently high, any eradication policy cannot be optimal and the constant escapement policy with some interior target level is optimal. This case spans the situation where even though eradication policy is a feasible choice, it is never optimal irrespective of the degree of social damage from the escapement. In contrast, if the sensitivity of catchability is sufficiently low, there could exist a threshold of the initial stock which differentiates the optimal policy between immediate eradication and giving-up without any control. In the intermediate range, immediate eradication, giving-up without any control, or more complex policies could be optimal. It was also argued that to some extent, the optimal policy is sensitive to the relation between the reproduction function and the degree of social damage. Furthermore, we discussed the conditions under which the optimal decisions can be tricky through numerical analysis.

What can we say more about realistic policy recommendation out of these results? First, if there are many options in technologies and methods for removal operations and also if it can be considered that the sensitivity of catchability is endogenized by the particular choice of these options, the social planner should employ the technology with sufficiently low sensitivity of catchability when the goal of a society is eradication. If there is no such technology, any attempt of eradication would be sub-optimal and should be postponed, and the interior constant escapement or giving-up rule may be recommended until the technology with low sensitivity of catchability is developed and becomes available.

Second, the social planner should prioritize identifying the current status of the invasive species stock as well as estimating how catch per unit of effort changes with stock size under available technologies. It is because the optimal decision rule could be highly dependent on these factors. Although the informational requirements necessitate some periods of experimentation, it may be worthwhile to compare (i) the long-run social cost of the optimal decision given accurate estimates with experimentation costs and (ii) the one of repeating policy failures given no elaborate estimates for important parameters.

Contrary to the stylized framework of renewable resource management, the general objective of controlling invasive species is to minimize the long-run social cost, and the first-best goal could sometimes be set as eradication in many instances. These differences give the practice and analysis of invasive species management extra difficulties, and thus many attempts of eradication policy end up being halted. Past literature appears to suggest that there are other factors as the reason for failures, which we did not incorporate into the present model. In particular, we ignore the impact of multiple uncertainty associated with removal operations such as measurement, process and implementation errors. Such errors have been acknowledged as environmental variability and managerial uncertainty (see Roughgarden and Smith (1996) and Sethi, Costello, Fisher, Hanemann, and Karp (2005)). These caveats notwithstanding, we are hopeful that our model is both a benchmark for comparison and a first step towards developing more sophisticated models that can analyze policy issues in a highly uncertain and spatial environment.

6 Appendix

In this appendix, we show the proofs of lemmas and propositions.

Proof of Lemma 1 The desired results can be directly derived from equation (6). □

Proof of Proposition 1 Since $\lim_{s\to 0} c(s) = \infty$ and $D'(0) > 0$, the sign of $\lim_{s\to 0} W'(s)$ depends on that of $\lim_{s\to 0}[1 - \rho c(F(s))F'(s)/c(s)]$, assuming that $\lim_{s\to 0}[1 - \rho c(F(s))F'(s)/c(s)]$ exists and its value is not equal to zero. Thus, it must hold that $\lim_{s\to 0} W'(s) = -\infty$ if $1 < \rho \lim_{s\to 0}[c(F(s))F'(s)/c(s)]$, and $\lim_{s\to 0} W'(s) = \infty$ if $1 > \rho \lim_{s\to 0}[c(F(s))F'(s)/c(s)]$. The first result can be derived from the fact that $W(s)$ never attains its maximum at $s = 0$ if $1 > \rho \lim_{s\to 0}[c(F(s))F'(s)/c(s)]$. The second result comes from the fact that $W(s)$ never attains its maximum at $s = \tau$ if $\rho c(F(\tau))F'(\tau) > c(\tau) - D'(\tau)$. Then, we can directly deduce the last result.
□

Proof of Proposition 2 If $W(s)$ is quasi-concave in $s \in [0, \tau]$, then $W(s)$ must be either monotone or unimodal. Then, the desired result is obtained. □

Proof of Lemma 2 Since $\lim_{s\to 0} c(s) = \infty$, $\lim_{s\to 0}[(F(s)/s)^\theta - \rho F'(s)] = (F'(0))^\theta[1 - \rho(F'(0))^{1-\theta}]$, and $0 < D'(0) < \infty$, the sign of $\lim_{s\to 0} \frac{\partial W(s;\theta)}{\partial s}$ depends on that of $1 - \rho(F'(0))^{1-\theta}$, assuming that its value is not equal to zero. Thus, it must hold that $\lim_{s\to 0} \frac{\partial W(s;\theta)}{\partial s} = -\infty$ if $\theta < 1 + (\ln \rho)/(\ln F'(0))$, and $\lim_{s\to 0} \frac{\partial W(s;\theta)}{\partial s} = \infty$ if $\theta > 1 + (\ln \rho)/(\ln F'(0))$. Concerning $\frac{\partial W(\tau;\theta)}{\partial s}$, the desired results can be directly derived from equation (8) with $\tau = F(\tau)$. □

Proof of Proposition 3 The first result is derived from the fact that $W(s;\theta)$ never attains its maximum at $s = 0$ if $\theta > \hat{\theta}$. The second result comes from the fact that $W(s)$ never attains its maximum at $s = \tau$ if $\theta > \bar{\theta}$. Then, we can directly deduce the last result. □

Proof of Lemma 3 We need to show that there exists $\theta^* > 0$ such that for all $\theta > \theta^*$ and for all $s \in [0, \tau]$, $W''(s;\theta) < 0$. To do that, let

$$L(\theta, s) = \theta \left\{ \rho(F'(s))^2 - \left(\frac{F(s)}{s}\right)^{\theta+1} \right\} \quad \text{and} \quad R(\theta, s) = \frac{D''(s)(F(s))^{\theta+1}}{k} + \rho F''(s)F(s).$$

It is enough to show that there exists $\theta^* > 0$ such that for all $\theta > \theta^*$ and for all $s \in [0, \tau]$, $L(\theta, s) < R(\theta, s)$. Pick any $s \in (0, \tau]$. Then, $R(\theta, s)$ is increasing in θ with $\lim_{\theta\to\infty} R(\theta, s) = \infty$. Differentiating $L(\theta, s)$ with respect to θ yields:

$$\frac{\partial L}{\partial \theta} = \rho(F'(s))^2 - \left(\frac{F(s)}{s}\right)^{\theta+1} \left\{1 + \theta \ln\left(\frac{F(s)}{s}\right)\right\}.$$

It is obvious that $\frac{\partial L}{\partial \theta}$ is decreasing in θ with $\lim_{\theta\to\infty} \frac{\partial L}{\partial \theta} = -\infty$ for given s. Thus, there exists $\hat{\theta}_s > 0$, finite, such that $\frac{\partial L}{\partial \theta} < 0$ holds for any $\theta > \hat{\theta}_s$, i.e., $L(\theta, s)$ is decreasing in θ for any $\theta > \hat{\theta}_s$. Since $R(\theta, s)$ is increasing in θ with $\lim_{\theta\to\infty} R(\theta, s) = \infty$ and $L(\theta, s)$ is decreasing in θ over $(\hat{\theta}_s, \infty)$, there exists $\tilde{\theta}_s (\geq \hat{\theta}_s > 0)$, finite, such that for all $\theta > \tilde{\theta}_s$, $L(\theta, s) < R(\theta, s)$ holds. Take $\theta^* \equiv \sup_s \tilde{\theta}_s$. Since $\tilde{\theta}_s$ is finite for all s, θ^* is also finite. Then, for all $\theta > \theta^*$ and for all s, $L(\theta, s) < R(\theta, s)$ holds. □

Proof of Proposition 4 Notice that W is strictly concave with $\theta^* \geq \min\{\hat{\theta}, \bar{\theta}\}$. There are three cases, depending on the values of θ, $\hat{\theta}$ and $\bar{\theta}$. Suppose first that $\theta > \max\{\hat{\theta}, \bar{\theta}\}$. By Lemma 2, W is unimodal with its maximum $s^*(\theta) \in (0, \tau)$. Then, the constant escapement policy with some interior level s^* is optimal. Suppose next that $\theta \in (\hat{\theta}, \bar{\theta})$. Then, W is monotone increasing in s over $[0, \tau]$, which implies that the constant giving-up policy is optimal. Finally, suppose that $\theta \in (\bar{\theta}, \hat{\theta})$, W is monotone decreasing in s over $[0, \tau]$, which implies that the constant eradication policy is optimal. □

Proof of Lemma 4 Evaluating equation (10) at $\theta = 0$ yields:

$$\frac{\partial^2 W(s;0)}{\partial s^2} = -[\rho k F''(s) + D''(s)] > -\sup_t [\rho k F''(t) + D''(t)] = \inf_t \frac{\partial^2 W(t;0)}{\partial t^2} > 0,$$

for all s, by the assumption of $\sup_{s\in[0,\tau]} [\rho k F''(s) + D''(s)] < 0$. Since $\frac{\partial^2 W(s;\theta)}{\partial s^2}$ is continuous in

θ, $\sup_t \frac{\partial^2 W(t;\theta)}{\partial t^2}$ is also continuous in θ. Thus, there exists $\theta^{**} > 0$ such that for all $\theta < \theta^{**}$, $\inf_t \frac{\partial^2 W(t;\theta)}{\partial t^2} > 0$, which implies that $\frac{\partial^2 W(s;\theta)}{\partial s^2} > 0$ for all $s \in [0, \tau]$. □

Proof of Proposition 5 Notice that W is strictly convex with $\theta^* \leq \max\{\hat{\theta}, \bar{\theta}\}$. There are three cases, depending on the values of θ, $\hat{\theta}$ and $\bar{\theta}$. Suppose first that $\theta < \min\{\hat{\theta}, \bar{\theta}\}$. By Lemma 2, W is inverse unimodal with its minimum $s^*(\theta) \in (0, \tau)$. Then, the desired result is obtained. Suppose next that $\theta \in (\hat{\theta}, \bar{\theta})$. Then, W is monotone increasing in s over $[0, \tau]$, which implies that the constant giving-up policy is optimal. Finally, suppose that $\theta \in (\bar{\theta}, \hat{\theta})$, W is monotone decreasing in s over $[0, \tau]$, which implies that the constant eradication policy is optimal. □

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Figure 1: Relation between CPUE and removal costs

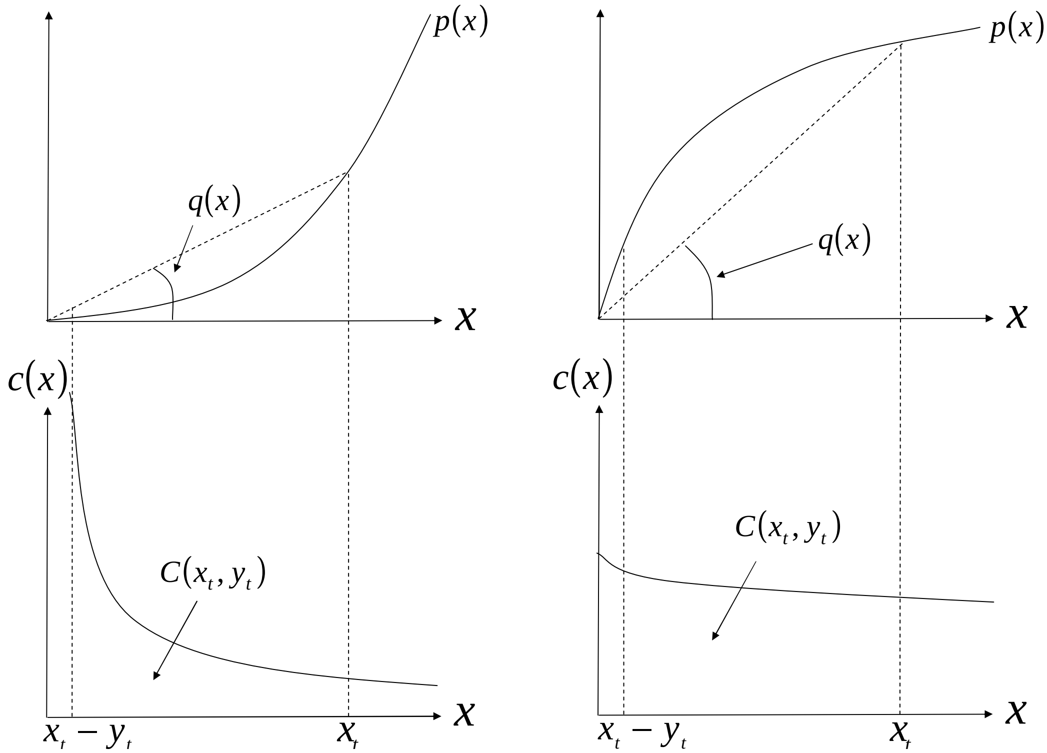
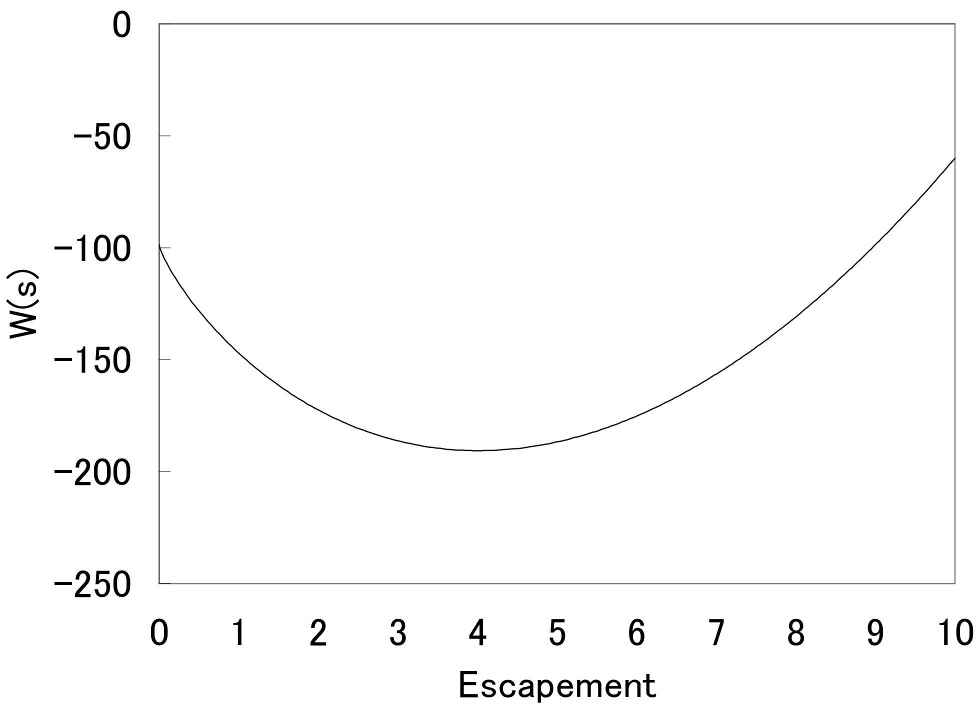


Figure 2: $W(s)$ and optimal escapement for $\theta = 0.2$



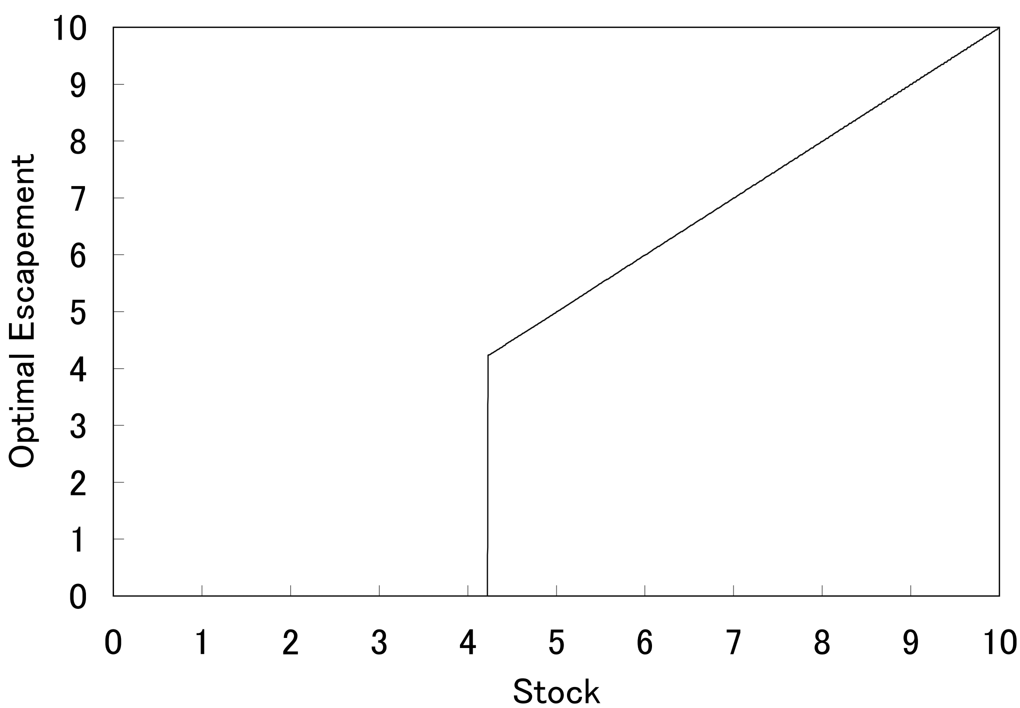
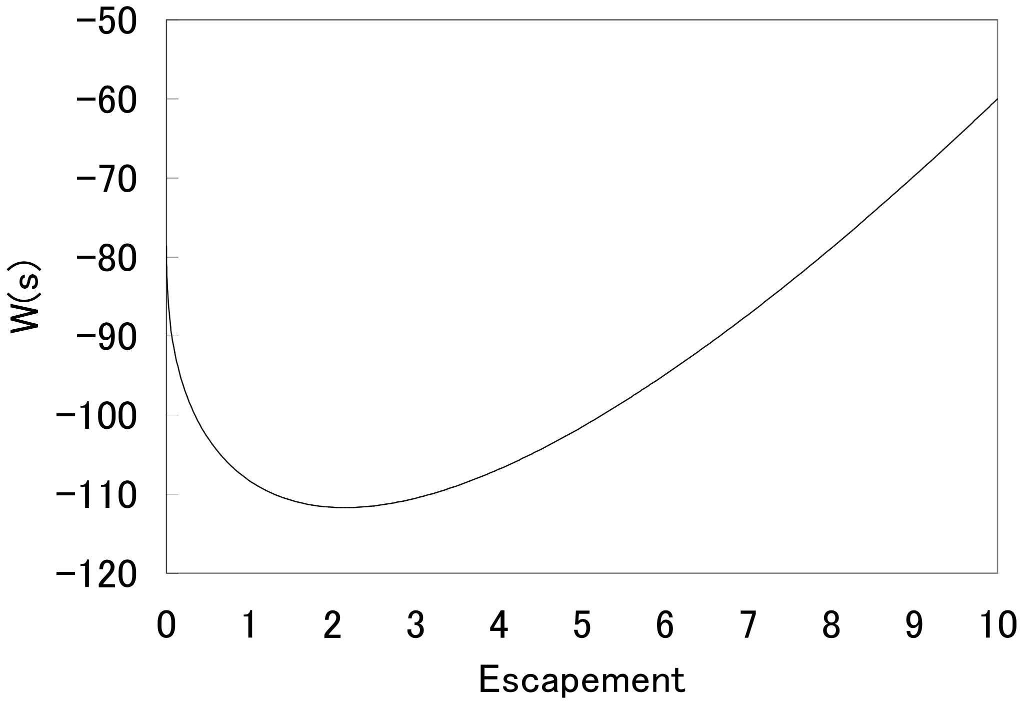


Figure 3: $W(s)$ and optimal escapement for $\theta = 0.6$



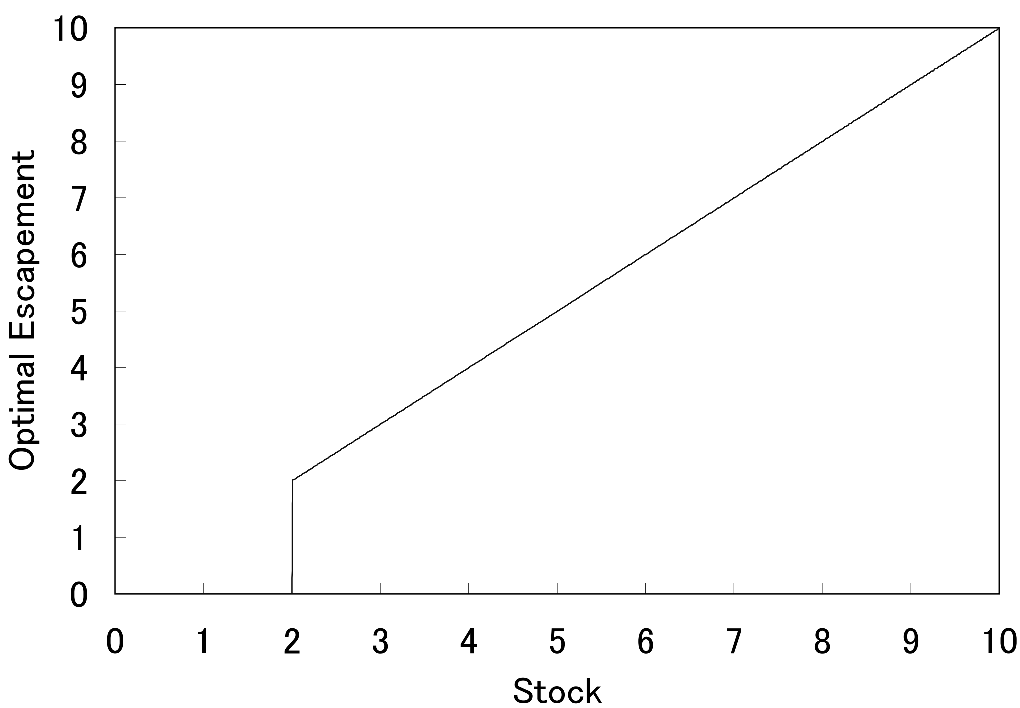
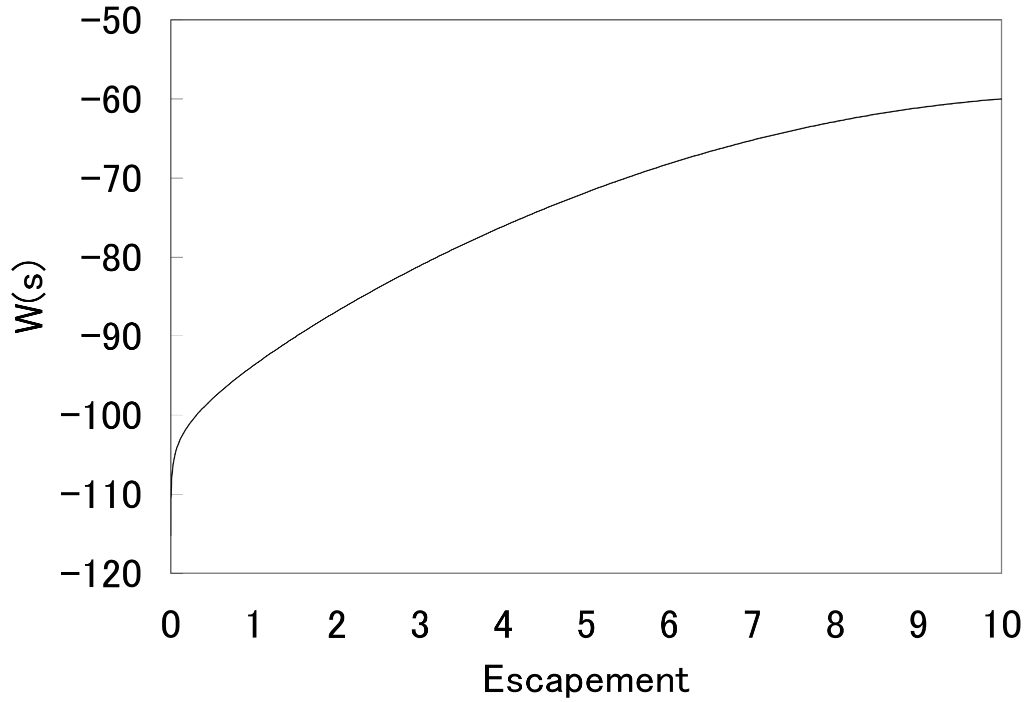


Figure 4: $W(s)$ and Optimal escapement policy for $\theta = 0.85$



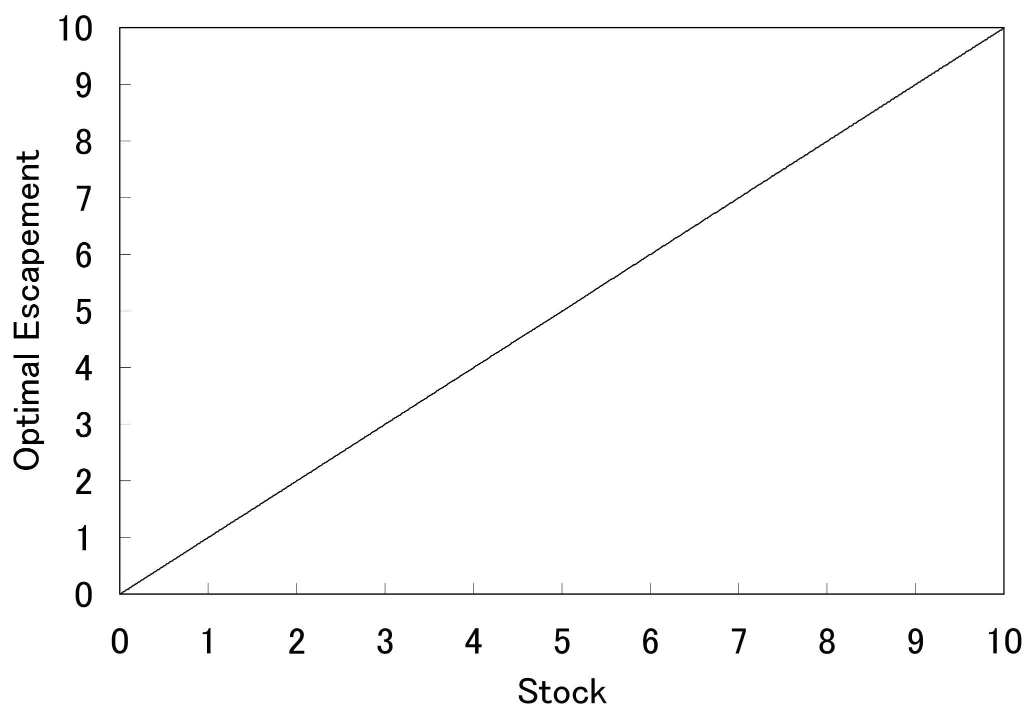


Figure 5: $W(s)$ and Optimal escapement policy for $\theta = 1.1$

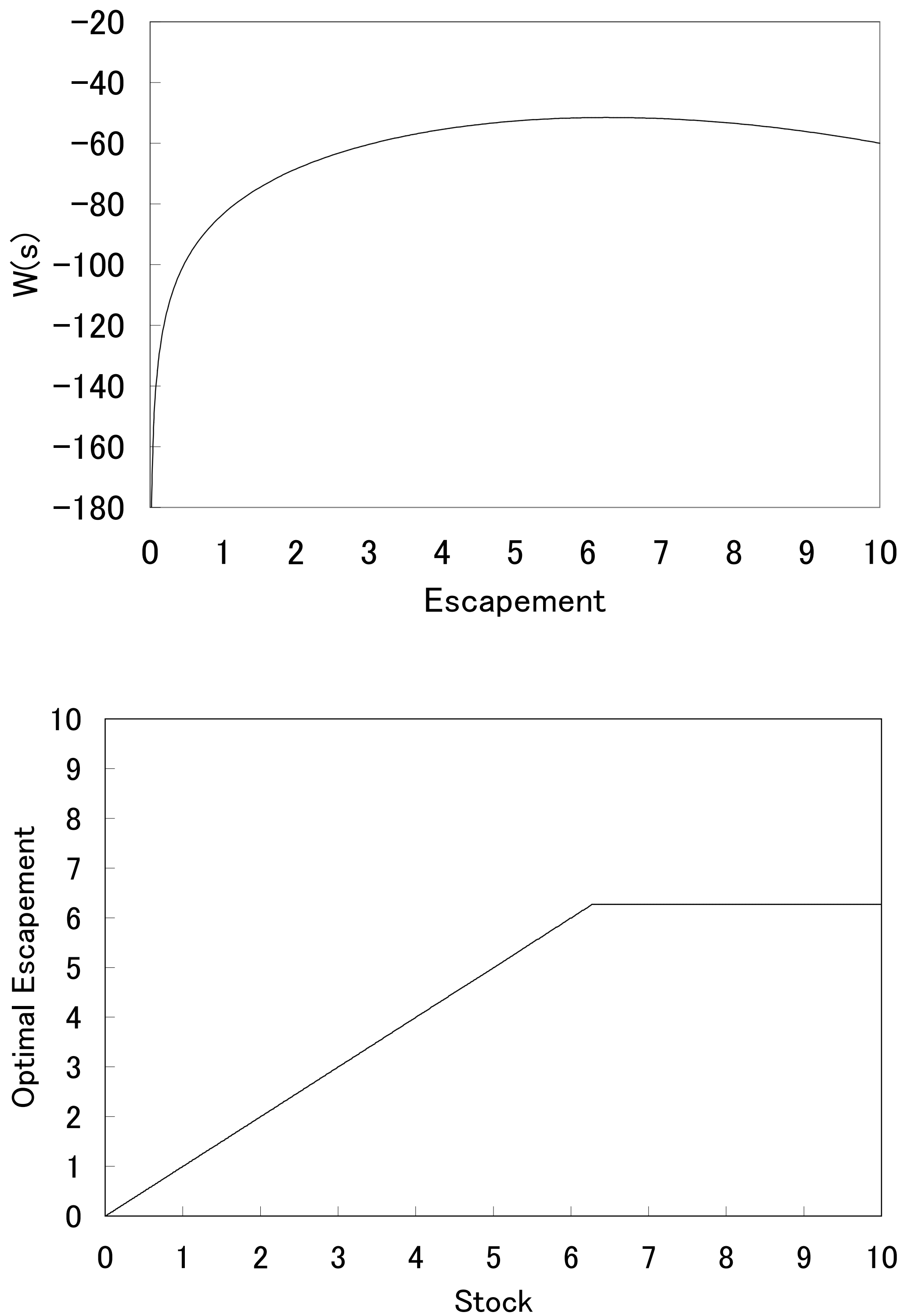


Figure 6: $W(s)$ and Optimal escapement policy for a complex example