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How Do Income and Bequest Taxes Affect Income Inequality? The Role of Parental Transfers

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Abstract

This paper studies how income and bequest taxes affect income inequality. We firstly explore this relationship empirically using a panel of 20 OECD countries from 1980 to 2008. The data shows that an increase of income taxation tends to strengthen income inequality, while the inequality effect of bequest taxation is ambiguous. In order to explain these findings, we develop an overlapping generation model with inter-generational transfers. Altruistic parents face the joint decision of making educational investment and leaving financial bequests. A change in tax structure will affect both asset allocation decision and wealth transmission, which in turn governs the dynamics of human capital accumulation and determines the net tax effects on equilibrium income distribution. We show these tax effects on income inequality analytically, and examine aggregate effects with numerical experiments. We find that the predictions of our model are consistent with data patterns.

JEL Classification: H21, H30, D64, I24

Tax structure, Human capital, Bequests, Income inequality, Intergenerational transfers

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1 Introduction

Explaining how income inequality is affected by tax and evolves over time is one of the central issues in the discussion of economic policy. Not only can tax have a direct effect on the level of income, but also policy makers often use tax instruments to equalize income distribution through the dynamic of intergenerational transfer. Indeed, since Becker and Tomes (1979, 1986), an extensive literature has shown that the transmission of physical and human capitals from parents to children plays a significant role in determining the evolution of income distribution (see, among others, Loury (1981); Mulligan (1997); Zilcha (2003); Alonso-Carrera, Caballé, and Raurich (2012)). Hence, the tax effect on parental decision would further shape the inequality through this intergenerational link. For instance, an increase in the taxes applied to bequests can induce parents to shift part of their intended bequests to *inter-vivos* giving (Bernheim, Lemke, and Scholz (2004); Joulfaian (2005)). In addition, if the *inter-vivos* transfer is tied to the expenses of schooling (Haider and McGarry (2012)), the relevant tax change will affect the educational attainment of the children and, in turn, change the earning ability and income level.

In this paper, we focus on the transmission of physical capital (through bequests) and human capital (through educational investment) from parents to children, and we investigate how explicitly modeling the effect of income and bequest taxes on intergenerational transfers would explain the empirical results. In doing so, evidence regarding income and bequest taxations on income inequality has also been estimated to serve as our empirical background. The data indicates that, among twenty OECD countries, income taxation has a positive and significant effect on the Gini coefficient of income, while the relationship between bequest taxation and the Gini coefficient is not clear.[1] Thus, when considering the role of intergenerational transfers in our model economy, we study the following questions:

- How do income and bequest taxes affect parents' educational investment and bequest transfers? How do the changes of these taxes further shape the income distribution and inequality of the economy?
- Can the empirical findings regarding tax effects on income inequality be explained by our theoretical model? How well is our model's predication?

To answer these questions, we develop an overlapping-generations model with heterogeneous agents, where each generation is altruistically linked to their descendants.[2] As mentioned above, an important feature of our model is that it allows parents to allocate their transfers between two forms: human capital investment and physical bequest. The former determines the children's earning ability in the labor market during their adulthood, and the latter is a simple form of wealth transfer, which includes all types of gifts and bequests. We assume the realized human capital returns of the descendants is uncertain and would depend on an idiosyncratic shock in their life cycle, while

[1] Both income and bequest taxations are measured by their percentages of GDP. This will be explained below.

[2] A similar framework is commonly used to study the issues related to intergenerational transfers. See, for example, Hendricks (2001) and Nishiyama (2002).

the available physical bequest of the older generation is a risk-free asset saved from their working period. Furthermore, we assume that an upper bound for human capital investment exists, which could be considered as the tertiary education. Building upon this setup, we show that an individual's wealth consists of both risky (human capital) and risk-free (physical bequest) assets.[3] An altruistic parent's decision to allocate intergenerational transfers resembles a "portfolio-choice," and the optimal decision on human capital investment and bequest transfers would follow a simple rule that is linearly related to total after-tax wealth. In addition, the distribution of income is itself endogenous and depends on the distribution of human capital. Therefore, the income inequality is jointly determined by the parents' decision rule on children's human capital investment and aggregate wealth distribution, where both are directly and indirectly affected by the structure of taxes. We calibrate the model to match relevant features of the U.S. economy, and analyze the net effects of income and bequest taxes on equilibrium income distribution.

The mechanism behind our model economy delivers the following insight: Taxes affect income equality through two different impacts on the distribution of human capital. Since there exists an upper bound for educational investment, the distribution of human capital depends on the proportion of parents whose wealth values are higher than a threshold level. The lower the wealth threshold, the larger the proportion of people who share the same level of human capital and the smaller the human capital inequality (measured by the Gini coefficient). This is the first dimension where the relative changes of income and bequest taxes can affect income inequality.[4] Secondly, taxes also influence the degree of wealth dispersion given the *risky* nature of human capital investment. When the wealth bundle includes a large proportion of this risky asset, the human capital dispersion increases and so as income inequality. Let us look at the case regarding how do income tax affect income inequality. As the income tax rate goes up, it reduces parents' incentive to make educational investment so that the wealth threshold increases. In addition, the expected wealth decreases due to the lower returns on human capital and lower investment level. As a result, the proportion of population whose wealth values are higher than the threshold level decreases, and this leads to the increase of income inequality. On the other hand, since the proportion of risky asset (human capital) in the parents' wealth bundle decreases, the stationary wealth distribution becomes less disperse. This reduces the income inequality. The net income tax effect on income inequality is then determined by combining these different forces. For the same reason, bequest taxes have opposite effects.

As a test of our calibrated model, we compare its predictions with our main empirical findings from OECD countries: the estimated parameters of income and bequest taxations on the Gini

[3]Empirical evidence suggests that, due to the risk of employment and the fluctuations of wages, the human capital investment is an rather risky asset (see Palacios-Huerta (2003) and Singh (2010)). Palacios-Huerta (2003) points out that the risk of human capital return in the U.S. is about 1/3 to 2/3 of that of U.S. equity return. The ratio depends on the demographic characteristics of workers, such as education level, yeas of working experience, race, etc. In addition, using a life-cycle model Singh (2010) studies the risky nature of human capital investment and discusses its quantitative implication in a macroeconomic framework.

[4]In other words, a tax change will affect individual's policy function on wealth and human capital, and the steady state wealth distribution. In a simplified version of our model, we show that the human capital investment is positively related to human capital risk premium. See the discussion in subsection 5.3.

coefficient in the regression model. With respect to income taxation, we find that the result of our numerical experiment is both qualitatively consistent and quantitatively close to the empirical findings from the OECD countries. With respect to bequest taxation, we find the result from our calibrated model is sensitive to both the extent of tax change and transfer schemes. We explain why this could happen and discuss the effect of bequest tax change under different scenarios. Although the effect of bequest tax on income inequality may be sensitive, we also demonstrate that the increase of bequest tax would unambiguously increase the human capital accumulation in our experimental cases.

The reminder of this paper is organized as follows. In Section 2 we provide the empirical evidence of the relationship between tax structure and inequality. In section 3 we present the model economy and the equilibrium conditions. In Section 4 we describe the model specification and parameter choices in the calibration. In Section 5 we analyze the agents' policy function based on numerical simulations. We discuss the intuition and driving forces. In Section 5 we explain the underlying channel of tax effects on income inequality. The exposition is based on both the analytical solution of individual's decision problem and the numerical exercises from the calibrated model. In Section 6 we performs the numerical experiment and examines the prediction of our model. Section 7 concludes the paper and some model extension such as incorporating progressive tax is also discussed.

2 Empirical Background

This section describes the data and presents empirical method. Some observations regarding income inequality and tax structure are documented to provide the motivation for the theoretical model presented in Section 3.

2.1 Data and methodology

Data description

Our data set covers 20 OECD countries between 1980 and 2008 from 4 data sources. Income inequality data come from Castellacci and Natera (2011), human capital data come from Barro and Lee (2012), democracy index data are the Unified Democracy Scores developed by Pemstein, Meserve, and Melton (2010), and the remaining data regarding economic production, population, and taxation are from the OECD Statistics Database (See Appendix A).

In our empirical analysis, the set of conditioning variables includes log of GDP per capita and its squared term, population growth rate, fertility rate, democracy index, and human capital[5]. As for taxation variables, we aggregate OECD's functional classifications of tax data into five categories, as described in Table 1. The variable of total tax revenue is the sum of income taxation, bequest taxation, social security taxation, consumption taxation, and other revenue.

There are six periods in our panel data: 1980 to 1984, 1985 to 1989, 1990 to 1994, 1995 to

[5]The conditioning variables are those found in the usual Barro-type regression (e.g. Barro (2000)).

1999, 2000 to 2004, and 2005 to 2008. We follow the standard practice of taking averages of annual observations to remove the effects of the business cycle, except for human capital and initial GDP per capita variables. The detailed discussion of data handling and descriptive statistics are reported in Table A1 and A2. We apply static panel econometric techniques to these data.

Table 1: Definition of Tax Variables

Variable	Functional Classification
Income taxation	Individual taxation on income, profit and capital gains
Bequest taxation	Estate, inheritance and gift taxation
Social security taxation	Social security contribution
Consumption taxation	Taxation on goods and services
Other revenues	Corporation taxation on income, profit, and capital gain; Property taxation excluding estate, inheritance and gift taxation; Taxation on payroll and workforce; Other tax revenues

Note: Functional classification refers to the classifications given in their OECD data source.

Empirical Approach

We are interested in the effects of the income and bequest taxations on income inequality, using a cross-country panel data set. To illustrate the relationship between income inequality and different types of tax revenues, we estimate the following regression

$$G_{it} = \alpha + \sum_{j=1}^{k} \beta_j Y_{jit} + \gamma_R R_{it} + \sum_{p=1}^{m} \gamma_p X_{pit} + u_{it},$$

where G is the Gini coefficient, a measure of the income distribution, Y_j is the conditioning (non-fiscal) variable, R is total tax revenue, and X_p is revenue from each tax instrument.[6] The index i refers to country and t refers to the year. The above equation includes all tax instruments, so that $\sum_{p=1}^{m} X_{pit} = R_{it}$. Therefore, one element of X must be omitted in the estimation in order to avoid perfect collinearity. This implies that the equation actually being estimated is:

$$G_{it} = \alpha + \sum_{j=1}^{k} \beta_j Y_{jit} + (\gamma_R + \gamma_m) R_{it} + \sum_{p=1}^{m-1} (\gamma_p - \gamma_m) X_{pit} + u_{it}. \quad (2.1)$$

It follows that the correct interpretation of the coefficient on each kind of taxation (X_p) is as the effect of a unit change in a particular taxation offset by a unit change in the omitted category (i.e. $\gamma_p - \gamma_m$). It also implies that if the omitted tax variable satisfies that $\gamma_m = 0$ (i.e. suggested by

[6]Kneller, Bleaney, and Gemmell (1999) estimated how fiscal policy affect growth rate in the similar fashion. We include fiscal variables on the right hand side of the equation in the same way as they did.

theory), the coefficients of remaining taxation variables can have a straightforward interpretation in the context of regression analysis.

Our regression equation follows the form of Eq. (2.1) above. We initially considered the following four panel data estimators for each regression: one-way (country) fixed effects by OLS, one-way (country) random effects by GLS, two-way (country and year) fixed effects by OLS, and two-way (country and year) random effects by GLS. Since the Hausman test rejects the null hypothesis that error terms are not correlated with individual effects at 0.01 significance level, we focus on the results from the fixed effects models. In addition, in the following subsection we report and discuss the results of two-way fixed effects model, which allows both time-specific and country-specific effects, because it receives greatest support (higher adjusted R^2) in the analysis.

2.2 Empirical findings

The results of the panel regression are reported in Table 2. For each column of the regression, we use consumption taxation as the omitted tax category. As suggested by Kneller, Bleaney, and Gemmell (1999) and Bleaney, Gemmell, and Kneller (2001), consumption taxation (taxation on goods and services) is a non-distortionary taxation so that it can be chosen as a neutral (γ_m equals zero as in equation (1)) omitted category.Therefore, we assume that the coefficient of consumption taxation is zero, and will interpret the empirical result accordingly.

Table 2 focuses on the role of tax structure. In particular, we are interested in how income and bequest taxations affect income inequality. Column (1) is a basic regression with tax variables only. It shows that country with higher income or bequest taxation, as in the percentage of GDP, also has a more unequal income distribution. Specifically, the effect of income taxation on Gini coefficient is significant at 1% level, while the effect of bequest taxation is not significant. An increase in one percentage point of income tax revenue over GDP raises Gini coefficient by 0.01. Other revenue also has a positive (much smaller and statistical insignificant) effect. On the other hand, the signs of both social security tax and total tax revenue are negative, which means that the increase of both social security and total tax revenue would reduce income inequality, but the effects are not significant.

Table 2: Panel Regression for Income Inequality

Estimation method: two-way fixed effects Dependent variable: Gini coefficient				
	(1)	(2)	(3)	(4)
	OLS	OLS	OLS	IV
Income taxation	0.010**	0.008*	0.008*	0.010^{+}
	(0.003)	(0.003)	(0.003)	(0.006)
Bequest taxation	0.004	-0.007	-0.003	-0.001
	(0.034)	(0.032)	(0.032)	(0.062)

Social security taxation	-0.001	-0.001	–	0.002
	(0.004)	(0.004)		(0.005)
Other revenues	0.001	0.002	–	0.002
	(0.003)	(0.003)		(0.004)
Social security + Other revenues	–	–	0.001	–
			(0.003)	
Total tax revenues	-0.003	-0.002	-0.002	-0.005
	(0.002)	(0.003)	(0.003)	(0.004)
Log (Initial p.c. GDP)	–	0.831	0.705	0.619
		(0.753)	(0.741)	(1.013)
Log (Initial p.c. GDP) squared	–	-0.043	-0.036	-0.034
		(0.041)	(0.041)	(0.055)
Human Capital	–	-0.0002	-0.001	-0.007
		(0.004)	(0.004)	(0.007)
Democracy index	–	-0.017	-0.018	-0.023
		(0.014)	(0.013)	(0.010)
Population growth rate	–	-0.015**	-0.014**	-0.014**
		(0.002)	(0.002)	(0.002)
Fertility rate	–	0.038**	0.038**	0.033^{+}
		(0.010)	(0.010)	(0.019)
Adjusted R^2	0.833	0.853	0.854	0.219
No. of observations	119	119	119	99

Note: Asterisks and plus signs report the level of significance (**: 1%, *: 5%, $^{+}$: 10%) and robust standard errors are in parentheses. Country and time intercepts are included in the regression, but not reported.

The variable of "total tax revenues" is the summation of taxation on income, bequest, social security, consumption, and other revenues. For each column of the regression, we use consumption taxation as the omitted tax category. Human capital and GDP per capita is measured at the beginning of each period.

Column (2) reports regression results with a set of conditioning variables, which are found to be significant in many recent studies of income inequality.[7] Regarding tax variables in this regression specification, the coefficient on the income taxation becomes slightly less positive, but it remains significant at 5% level. The sign of bequest taxation, however, changes from positive to negative. Other tax variables have very similar coefficients with slightly different standard error. As for the conditioning variables, the positive sign of log of the per capita GDP and the negative sign of its square term seem in support of the Kuznets (1955) hypothesis that income inequality and per capita GDP can be described by a inverted-U curve. The results also show that human capital

[7] See, for example, Barro (2000), Gregorio and Lee (2002), and Muinelo-Gallo and Roca-Sagalés (2013).

variable, as measured by the average years of schooling for secondary and higher education, is negatively correlated with Gini coefficient. Although this effect is not significant, it is generally consistent with existing literature that higher educational attainment would lead to more equal income distribution (Gregorio and Lee (2002)). The negative sign of democracy index confirms that if more people share the power to redistribute economic resource through political channel, the income distribution will be more equal. In addition, there is a positive and significant relationship between fertility rate and Gini coefficient, while higher population growth rate would significantly reduce income inequality.

The coefficient of fiscal variables may be sensitive to the aggregation or disaggregation of the functional classification in the data source. To examine the robustness of the result in Column (2), we reclassify the variables included in the fiscal matrix.[8] Since our tax variables has already been divided into some sub-categories and our main focus is income and bequest taxations, we simply do the reclassification by aggregating social security taxation and other revenues. The result is reported in Column (3), and the coefficient of both conditioning and tax variables (except for the aggregating one) are very similar to the that in Column (2).

The estimation of Column (2) assumes that all of the right-hand side variables are exogenously determined, and this assumption requires further examination. We address this concern by instrumental variables (IV) estimation. However, the selection of instruments in this sort of regression is always an issue. We therefore follow Fölster and Henrekson (1999) and Kneller, Bleaney, and Gemmell (1999), and estimate the first difference, two-stage least square regression using instruments for income and bequest taxations. As instruments, we use the lagged levels of all tax variables, country fixed effects, the levels and first differences of the population growth and initial GDP variables, and first difference of other conditioning variables. The results are showed in Column (4).

Comparing the IV result with those in Column (2), coefficient signs are generally unchanged, except for social security contribution, but the robust standard errors are somewhat larger (the value of adjusted R^2 become lower accordingly). In particular, the interpretation of income taxation is not affected, and the coefficient is still significant under 10% level. The sign of bequest taxation is still negative and insignificant. Therefore, the endogeneity problem should be rendered less severe when running the regression in Column (2).

In sum, a general feature of our regression results is that, holding other things equal, the increase of income taxation tend to increase income inequality. However, the inequality effect of bequest taxation is not clear. Not only that the coefficient of bequest taxation is not significant, but the sign of bequest may depend on regression model specification as well (as shown in Column (1) and (2)). Given this empirical finding, we would like to further explore the role of income and bequest tax in a theoretical framework. Specifically, we would like to know how income and bequest taxes affect the decision of intergenerational transfer, and what is the relationship between tax instrument and

[8] As argued by Kneller, Bleaney, and Gemmell (1999), the effect of fiscal variables may depend on the classification of the data. In their work, they reclassify fiscal variables by disaggregating and reallocating tax and expenditure variables into different categories because each of their original fiscal variables in concerns consists of a larger set of functional classifications.

income inequality. Can the theoretical model well explain the above empirical finding from OECD data? We answer these questions by building up an overlapping generation model in the following sections.

3 The Model

In this section, we construct a life cycle-overlapping generation model to investigate effects of taxes on inequality. The main difference between our model and the other models in the existing literature is that parents' portfolio choices of intergenerational transfers among human capital investment and financial transfers is considered.

The economy is populated by overlapping generations of people. Each generation is altruistically linked towards their descendants. Parents have incentive to leave bequest and invest in their children's human capital. New-born individuals are identical in ability and learning, but they are heterogeneous in terms of both the resources received from their parents and idiosyncratic risks of human capital returns. The physical capital market is simplified in the current framework in the sense that the economy is assumed to be a small open economy. Interest rate is determined in the world capital market and capital is perfectly mobile. Goods and labor markets are competitive. Moreover, since the analysis focuses on the steady state equilibrium, the subscript t of each variable is omitted. We also assume there is no population growth and the number of population in each generation is normalized to 1.

Individuals

Each generation contains a continuum of heterogenous agents, indexed by $j \in (0, 1)$. An agent's life consists of three periods: the child ($t = 1$), the worker ($t = 2$) and the retiree ($t = 3$). In the childhood period, agents are attached to their parents and receive educational services e based on parents' decisions. The education services are used to produce human capital h, based on a linear production function

$$h = a^h(e + e_0), \tag{3.1}$$

where a^h represents the productivity of educational services. Let $h_0 = a^h e_0$ represents the endowment of human capital. We interpret h_0 as the compulsory education, which doesn't depend on parent's decision. We assume that there exists a upper bound $\overline{e}$ for the educational investment. The upper bound represents the highest human capital level could be achieved through schooling.

An agent in the working period receives labor income w^h by renting their human capital h in a competitive labor market with rental rate w. Human capital will be fully depreciated at the end of the period. The labor income also depends on each agent's labor supply l. We assume the labor supply l is exogenously given, characterized as *i.i.d.* with bounded support $[\underline{l}, \overline{l}]$. The uncertainty of idiosyncratic labor supply shocks, l, generate the risk on human capital returns. Each agent also receives the bequest (and all other types of financial transfers) b from their parents. Government excises a proportion tax on labor income and bequest, with rate τ_w and τ_b respectively, together with a lump sum tax/transfer T. Individual's wealth I is given by,

$$I = w^h + (1-\tau_b)b + T, \tag{3.2}$$

where labor income $w^h \equiv (1-\tau_w)whl$.

Each agent gives birth of one child in the working period. An agent allocates its wealth I in the following ways: consuming by himself c_2, investing in the child's human capital $e^{'}$, and saving for the retired period s. Hereafter the prime is used to indicate the variables associated with next generation. An agent in the working period (the worker) solves the following problem:

$$V_2(I) = \max_{\{c_2,e',s\}} u(c_2) + \beta E\left\{u(c_3) + \beta_c B(I')\right\}, \tag{3.3}$$

$$s.t.\ I \geq c_2 + s + e', \tag{3.4}$$

$$s \geq 0, \tag{3.5}$$

$$e' \leq \bar{e}. \tag{3.6}$$

Utility functions $u(.)$ and $B(.)$ are assumed to be in the forms of constant risk relative aversion (CRRA), and.

$$u(c) = \frac{c^{1-\gamma_c}}{1-\gamma_c},$$

$$B(I') = \frac{I'^{1-\gamma_B}}{1-\gamma_B},$$

with coefficient of relative risk aversion γ_c and γ_B, respectively. The worker's value function V_2 contains the utility derived from discounted per period consumption $u(c_2)$ with discount factor β and the expected value from offspring $B(I^{'})$ with altruistic parameter β_c. We assume parents evaluate offsprings' value based on the living standard. $B(I^{'})$ is determined by the child's future lifetime disposable income $I^{'}$.[9] Since there is no labor income in the retired period and we also assume that the offspring will not transfer assets back to parents, the saving s must be greater or equal to 0.

In the last period, the retiree allocates his wealth between consumption c_3 and bequest b'. The retiree's financial wealth includes transfers T and financial assets $(1+r)s$, which is the gross returns on savings investing in capital markets. Suppose the idiosyncratic shock on the retiree's offspring is realized before he makes the allocation decision. A retiree's maximization problem is given by,

$$V_3(s, w^{h'}) = \max_{\{c_3,b'\}} \left\{u(c_3) + \beta_c B(I^{'})\right\}, \tag{3.7}$$

$$s.t.\ (1+r)s + T \geq c_3 + b', \tag{3.8}$$

$$I' = w^{h'} + (1-\tau_b)b' + T, \tag{3.9}$$

$$b' \geq 0. \tag{3.10}$$

[9]This bequest motive is identical to the joy-of-receiving motive described in Grossmann and Poutvaara (2009).

Production

There exists a representative infinitely lived firm in the economy. The firm hires workers equipped with human capital and rents the physical capital to produce the output Y, following the production function

$$Y = AK^{\alpha}H^{1-\alpha}. \tag{3.11}$$

where K and H represent the total capital and human capital in the economy, respectively. We assume there is no aggregate risk. The productivity A is normalized to be 1. The firm sells the output in the competitive market, and maximizes profits

$$Y - wH - rK,$$

taking the rental rates of physical and human capital (w, r) as given.

The capital rental rate is given by the world interest rate $\bar{r}$. Firm's first order conditions are

$$w = (1-\alpha)\frac{Y}{H}, \tag{3.12}$$

$$\overline{r} = \alpha\frac{Y}{K}. \tag{3.13}$$

Efficiency conditions (3.12) and (3.13) determine the rental rate of human capital w,

$$w = (1-\alpha)\left[\frac{\overline{r}}{\alpha}\right]^{\alpha/(\alpha-1)}. \tag{3.14}$$

Government

In each period the government has fixed expenditure G and provides lump sum transfers T to all the agents in the working period. These expenditures are financed by the tax revenue. We focus our attention on the labor income tax and bequest tax. In addition, to simplify the analysis, both taxes will be implemented as a flat proportional tax.[10] Let's suppose the expenditures and revenues are balanced in each period,he government's budget constraint is

$$\tau_w wH + \tau_b B = G + T, \tag{3.15}$$

where B is the aggregate bequests in the economy.

Aggregation

We assume there is no aggregate shock in the economy. Although each worker faces idiosyncratic shocks on their labor supply l^j, the expected value is normalized to be 1. Following the law of large number, the aggregation of labor supply is 1, which is shown as follows

$$\int_0^1 l^j dj = 1.$$

[10] The driving force that affects inequality in our model is the change of risk premium of human capital due to change in tax structure. The driving force would become even stronger if the tax changes to progressive tax, as we usually observed in the data.

The total effective human capital H is

$$H = \int_0^1 h^j l^j dj. \tag{3.16}$$

The total bequest B is the aggregation of the retiree j's decision on bequest b^j,

$$B = \int_0^1 b^j dj.$$

The aggregate capital supplied in the markets is the aggregation of worker's saving and the capital outflow/inflow from the world market K^w,

$$K = \int_0^1 s^j dj + K^w. \tag{3.17}$$

Stationary Competitive Equilibrium

The stationary competitive equilibrium of this model is defined as follows:

1. Invariant distributions of agents wealth and worker's human capital with probability density function $\Omega(I)$, $G(h)$.
2. A set of prices $\{w, \overline{r}\}$,
3. The government's policy rule $\{\tau_w, \tau_b, T\}$,
4. Worker's decision rules $\{c_2(I), e'(I), s(I)\}$,
5. Retiree's decision rules $\{c_3(s, w^h), b'(s, w^h)\}$,
6. Allocations of capital K and human capital H.

The stationary competitive equilibrium exists if the following conditions hold in every period: (1) given markets' prices $\{w, \overline{r}\}$ and government policy structure $\{\tau_w, \tau_b, T\}$, the decision rules $\{c_2(I), e'(I)\}, s(I)\}$ solve worker's maximization problem and $\{c_3(s, w^{h'}), b'(s, w^{h'})\}$ solve retiree's maximization problem, (2) given prices, the firm solves the profit maximization problem, (3) the markets of capital and human capital are clear, and (4) government budget constraint 3.15 is satisfied.

4 Model Specification and Parameters Selections

The theoretical model presented above is not tractable due to the heterogenous agents setting and constraints on assets allocations (*i.e.* the upper bound of human capital and the positive bequests constraint). As a first step, we will solve it numerically. Model parameters are pinned down by targeting to the moments of the U.S. economy. Given this baseline case, we will discuss the driving forces (in section 5) and conduct numerical experiments to verify if the model predictions are consistent with our empirical findings (in section 6).

Table 3: Parameters and data targets

Parameters		value	source and target
Capital share	α	0.33	Standard value in the literature
Discount factor (period)	β	$(1/\overline{r})^{25}$	Standard value in the literature
World interest rate (annually)	$\overline{r}$	1.04	Standard value in the literature
RRA of utility function	γ_c	2	Standard value in the literature
RRA of child value function	γ_B	2	Assume curvature is the same as utility function
Maximum human capital	$\overline{e}$	0.3145	Wage ratio between workers with college degree and high school is 2.578
Bequest tax	τ_b	0.032	Proportion of inheritance and gift taxation to GDP is 0.25% (U.S. 2008)
Wage tax	τ_w	0.176	Proportion of income taxation to total GDP is 11.8% (U.S. 2008)
Initial human capital	h_0	1	
Child value discount variable	β_c	3.058	Fraction of workers with education above college degree 31% (U.S. 2010)

To simplify the analysis, in the model we assumed each agent lives for three periods, which represent different stages of life. In the numerical experiments, each period is assumed contains 25 years. Parameters' values and their calibration targets are summarized in Table 3 and the details are described as follows:

Labor income: To characterize the relative size of the wealth accumulated in the working periods and the financial assets transferred in the retired period, the quantity of assets and the returns of physical and human capital need to be carefully measured. Let us suppose in the beginning of the period workers have the information of the arrival idiosyncratic shocks on human capital returns and the the future bequest transferred by parents, while the retirees have the information of their offspring's income streams. Under this assumption, agent's human capital returns are the discounted value of income streams over the entire working period:

$$w^h = \sum_{i=1}^{25} \frac{(1-\tau_w)whl_i}{(1+\overline{r}^1)^i} = (e+e_0) \sum_{i=1}^{25} \frac{(1-\tau_w)wa_h l_i}{(1+\overline{r}^1)^i}, \tag{4.1}$$

where $\overline{r}^1$ is the annually world interest rate. For each unit of goods spending on investing in human capital in the end of parents working period (*i.e.* the beginning of child working period), its effective annual rate of return $r^{1,h}$ is equal to $wa_h l$ in our model. The aggregate human capital returns for parents is $r^h = \kappa_T r^{1,h}$, where $\kappa_T \equiv \left(\overline{r}^1\right)^{25}(1-(1+\overline{r}^1)^{-25})/\overline{r}^1$.[11] w^h can be defined in terms of rate

[11]Here $\overline{r}^1$ represents the annually returns of human capital for children in the working period, while the value of r^h

of returns of human capital,

$$w^h = (e + e_0) \sum_{i=1}^{25} \frac{(1-\tau_w) r_i^{1,h}}{(1+\overline{r}^1)^i} = (e+e_0)(1-\tau_w) r^h \tag{4.2}$$

Recall that the idiosyncratic labor supply shock l' is used to model the risks of human capital returns. The distribution of w^h can be derived form the distribution of returns on human capital, $r_i^{1,h}$. We assume $r_i^{1,h}$ is *iid* according to log normal distribution

$$\log r_i^{1,h} \sim N(\mu_h, \sigma_h^2). \tag{4.3}$$

Follow the estimates from Palacios-Huerta (2003), we take the value $\mu_h = 9\%$ and $\sigma_h^2 = 7.6\%$.[12] The distribution of r^h and w^h are pinned down by equation (4.2).[13]

The initial human capital h_0 is normalized to be 1. The maximum human capital $\bar{h}$ can be pined down by the wage ratio of workers with college degree to the group with lowest education background. We use the annual median salary ratio of workers with college degree and workers to some high school degree as the proxy. The fraction of workers with education college degree is used to pin down altruistic parameter β_c .

Taxes structure: In the present study, since the focus is the effects of each type of tax relative to the other, both of them are simplified as a proportional tax in the analysis. The tax structure are characterized by two parameters (τ_w, τ_b). Each of the proportional tax rates is pinned down by the ratio of the taxation to output. That is, given the ratio of total income tax to nominal GDP, ϕ_w (11.8% in the U.S. in 2008) and the ratio of bequest tax to nominal GDP, ϕ_b (0.25% in the U.S. 2008), the tax rates τ_w and τ_b can be pinned down by

$$\phi_w = \tau_w \frac{wH}{Y} = \tau_w(1-\alpha), \tag{4.4}$$

$$\phi_b = \frac{\tau_b B}{Y}, \tag{4.5}$$

where the total output Y each year can be calculated by equation (3.12). In the baseline case, we assume the government transfer T is 0.

Other parameters: We set the risk aversion coefficient γ_c equal to 2. We also assume that the risk aversion coefficient for the child value γ_B is identical to agent's preference of consumption. As for other parameters related to economic environment, we follow the standard value in the related literature: the capital share α is 0.33 and the annual interest rate $\overline{r}$ is 4% per year. Since the focus is on the steady state equilibrium, the discount factor is assumed to be $1/\overline{r}$.

We use these parameters to construct the baseline case for the later numerical experiments.

is the information for parents making decision in the beginning of their period Therefore κ_T includes a factor $\left(\overline{r}^1\right)^{25}$.

[12]We use the group of college education, with working experiences 6-15 years. The estimates do not vary so much across education/experiences groups. Our quantitive results also remain consistent if we change the parameters values of distribution to other group.

[13]For the parameters in our general equilibrium framework, (a^h, w), wage rate can be determined by equation (3.14), given the values $(\alpha, \overline{r}^1)$; while a^h can be determined by the identity $E(wa_h l) = E(r^{1,h})$.

5 Discussion

This section explains the underlying channel of tax effects on income inequality. The exposition is based upon both the analytical solution of the agents' decision rule and the results of numerical exercises based on the parameters pinned down in the previous section.

Specifically, we proceed the discussion in several steps: first, we examine the agent's life cycle decisions. Given the distribution of human capital return r^h and risk free assets return $(1+r)$, we show that the agents' optimal decisions on consumption,human capital investments and bequests follow a simple rule linearly related to the disposable wealth. With this conclusion, next, we explore the determinants of income inequality. We show that income inequality is jointly determined by the agents' decision rule on their offspring's human capital investment and also aggregate wealth distribution. Both are directly and indirectly affected by tax. Therefore, in the last part of the discussion, we examine tax effects based on the dimension they affect. We show that both income and bequest tax may generate several different effects, while each of them influences inequality through different channels with different magnitudes. Given that, we use the results of the numerical exercises to conclude the dominant driving force.

5.1 Individual's decision

In order to capture the main idea of the agent's decision rule, we simplify the model to some extent to obtain the analytical solution. We assume that government transfer T is equal to 0, and the curvature of parent's utility function γ_c is equal to that of the offspring's value function γ_B.

Retiree's problem

A retiree chooses consumption c_3 and bequest b' to maximize the utility from consumption and child's value. The equilibrium condition is given by

$$u'(c_3) \geq \beta_c(1-\tau_b)B'(I'), \tag{5.1}$$

where the condition (5.1) holds when $b > 0$. Let w^r denote the total after-tax wealth (disposable wealth) owned by a retiree and his working-age offspring. Since the sum of consumption c_3 and next generation's wealth I' is equal to w^r, we have the following expression:

$$w^r \equiv (1+r)s + w^{h'} - \tau_b b' = c_3 + I'. \tag{5.2}$$

Parents' decisions making can be regarded as the optimal allocation of w^r among c_3 and I' which satisfies equilibrium condition (5.1). Given that utility functions follow the CRRA functional form, from equation (5.1) we obtain that the optimal levels of c_3 and I' are constant fractions of the disposable wealth w^r, and this can be expressed by

$$c_3 = \alpha^r w^r, \text{ and} \tag{5.3}$$

$$I' = (1-\alpha^r)w^r, \tag{5.4}$$

where $\alpha^r = \frac{\kappa}{1+\kappa}$ and $\kappa = (\beta_c(1-\tau_b))^{\frac{-1}{\gamma}}$.

This consumption ratio α^r only holds for the case of interior solution ($b > 0$). If the optimal consumption $\alpha^r w^r$ is less than the wealth level $(1+r)s$, parents simply consume all their wealth and leave no bequests. This case happens when a parent's wealth is relatively small, compared with his offspring's labor income w^h. In other words, the situation with $b = 0$ can be described as the offsprings' realized human capital return r^h is higher than a critical level. We can show it by substituting the optimal consumption rule (5.3) into the retiree's budget constraint (3.8), and obtain the bequest decision rule by

$$b'(s, w^h) = \begin{cases} \frac{(1-\alpha^r)(1+r)s - \alpha^r(w^h)}{1-\alpha^r\tau_b} & if\, r^h < X, \\ 0 & if\, r^h \geq X, \end{cases} \tag{5.5}$$

where $X = \frac{1}{\kappa}\left(\frac{1+r}{1-\tau_w}\right)\left(\frac{s}{e'+e_0}\right)$.

We further substitute the definition of w^h and the solved b' into w^r, and find that

$$w^r = \left(\tilde{r}^h(e' + e_0) + \tilde{r}s\right), \tag{5.6}$$

where $\tilde{r}^h \equiv (1-\tau_w)r^h/(1-\alpha^r\tau_b)$ and $\tilde{r} \equiv (1+r)(1-\tau_b)/(1-\alpha^r\tau_b)$. The optimal levels of a retiree's consumption c_3 and the offspring's wealth I' are

$$c_3 = \begin{cases} \alpha^r w^r, & if\, r^h < X \\ (1+r)s & if\, r^h \geq X \end{cases} \tag{5.7}$$

$$I' = \begin{cases} (1-\alpha^r)\, w^r & if\, r^h < X \\ (1-\tau_w)r^h((e^{'} + e_0)) & if\, r^h \geq X \end{cases}. \tag{5.8}$$

Equation (5.6), (5.7), and (5.8) imply that in the case of interior solution, the disposable wealth w^r owned by both generations can be regarded as parent's gross returns of the investments in human capital $e' + e_0$ and the risk-free assets s in the working period, with the adjusted returns $(\tilde{r}^h, \tilde{r})$ by tax τ_b and allocation rule α^r. It is worth noting that τ_b increases the returns on the risky assets while decreasing the returns on the risk free assets.[14] In other words, the increase of τ_b does not only reduce the disposable wealth, but it also generates distortion on the agents' asset allocation choices by changing the assets' returns. Noted that, e_0 is not in the parent's budget constraints in the working period. We can, however, interpret it as a parent's predetermined decision, and it can be incorporated into parents' maximization problem in the working period as specified below.

Worker's problem

By adding the initial level of educational investment e_0 on both sides of the parents' budget constraint in the working period, we obtain

$$c_2 + e' + e_0 + s = I + e_0. \tag{5.9}$$

[14]It can be shown that $\tilde{r}^h \geq r^h$ and $\partial\tilde{r}^h/\partial\tau_b > 0$; $\tilde{r} > 1 + r$ and $\partial\tilde{r}/\partial\tau_b > 0$.

The right hand side (RHS) of (5.9) can be regarded as a worker's lifetime wealth that incorporates his child's initial educational investment, and is denoted by $w^w \equiv I + e_0$. The left hand side (LHS) is the allocation decision made in working period, which includes consumption c_2, savings in the risk-free asset s, and investments in human capital $e' + e_0$. The sum of the later two parts is the total investments in the working period that is denoted by $\bar{s} \equiv e' + e_0 + s$. We will show that the optimal levels of c_2 and $\bar{s}$, and the allocations between two type of investments all follow some constant ratios of the w^w, which are given by

$$
\begin{aligned}
c_2 &= \alpha^w w^w, &(5.10)\\
\bar{s} &= (1-\alpha^w) w^w, &(5.11)\\
e' + e_0 &= \alpha^p \bar{s}, &(5.12)\\
s &= (1-\alpha^p)\bar{s}. &(5.13)
\end{aligned}
$$

The term α^w represents the optimal consumption ratio and α^p represents the proportion of savings that is invested in human capital. These two ratios can be obtained by solving the agent's first order conditions on s and e':

$$
\begin{aligned}
u'(c_2) &= \beta E \frac{\partial V_3(s,e')}{\partial s}, &(5.14)\\
u'(c_2) &= \beta E \frac{\partial V_3(s,e')}{\partial e'}. &(5.15)
\end{aligned}
$$

By substituting the individual's allocations rules in the retired period, V_3 can be written as

$$
\begin{aligned}
V_3(s,e') = &\int_0^X \{u(\alpha^r w^r) + \beta_c B((1-\alpha^r)w^r)\} f(r^h) dr^h \\
&+ \int_X^\infty \left\{ u((1+r)(1-\alpha^p)\bar{s}) + \beta_c B(r^h \alpha^p \bar{s}) \right\} f(r^h) dr^h. \quad (5.16)
\end{aligned}
$$

Note that this value function V_3 consists of both the interior solution (i.e. $b > 0$ if $r^h \in [0, X]$) and the corner solution (i.e. $b = 0$ if $r^h \in [X, \infty]$). In addition, using asset allocation rules (5.12) and (5.13), we can rewrite (5.6) as

$$
w^r = \bar{s}\left(\tilde{r}^h \alpha^p + \tilde{r}(1-\alpha^p)\right) = \bar{s}\tilde{r}^p, \quad (5.17)
$$

where $\tilde{r}^p$ is the return of total savings $\bar{s}$.

Substituting all the policy rules into efficiency conditions (5.14) and (5.15), we have

$$
\begin{aligned}
\left(\tfrac{\alpha^w}{1-\alpha^w}\right)^{-\gamma} &= \beta\left\{\theta\tilde{r}\int_0^X (\tilde{r}^p)^{-\gamma} f(r^h) dr^h + \int_X^\infty \{(1+r)(1-\alpha^p)\}^{-\gamma} f(r^h) dr^h \right\} &(5.18)\\
\left(\tfrac{\alpha^w}{1-\alpha^w}\right)^{-\gamma} &= \beta\left\{\theta\int_0^X \tilde{r}^h (\tilde{r}^p)^{-\gamma} f(r^h) dr^h + \int_X^\infty (1-\tau_w) r^h \beta_c \left\{(1-\tau_w) r^h \alpha^p\right\}^{-\gamma} f(r^h) dr^h \right\} &(5.19)
\end{aligned}
$$

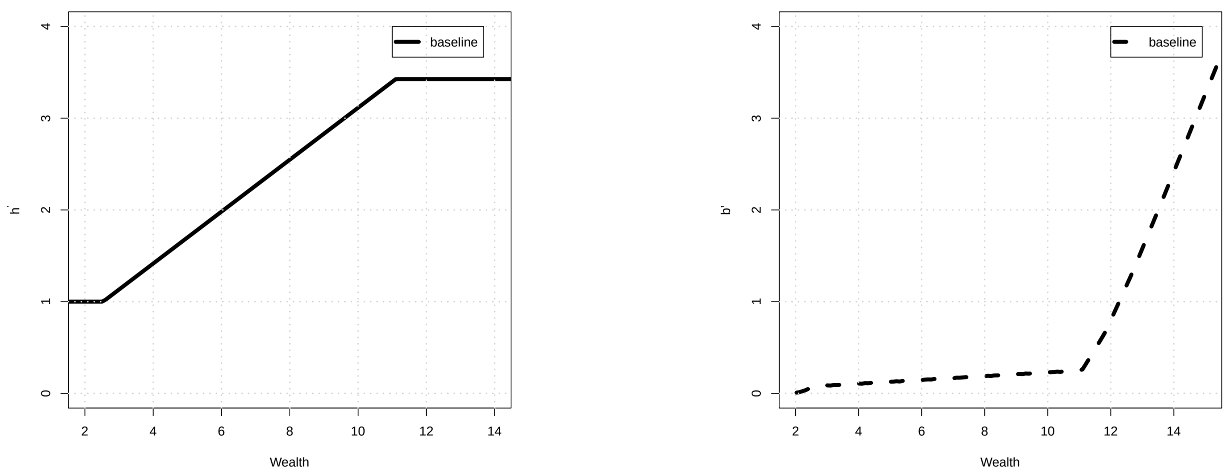
where $\theta = \{(\alpha^r)^{-\gamma} + \beta_c(1-\alpha^r)^{-\gamma}\}$. In the appendix we show explicitly that by combining the RHS of equation (5.18) and (5.19), α^p is found to be a constant and is uniquely determined. Given

α^p is a constant, the above equations imply that α^w is a constant as well. The ratio α^p and α^w are determined by consumers' preference, assets returns and tax rates.

Due to the initial level of educational investment e_0 and the constraint $\bar{e}$, the policy function of educational investment $e' = f_e(I)$ is a kinked-shaped graph:

$$e' = f_{e'}(I) = \begin{cases} 0 & if\, I \leq I_l \\ \phi_h\,(I + e_0) - e_0 & if\, I \in (I_l, I_h) \\ \overline{e} & if\, I \geq I_h \end{cases},$$

where $\phi_h = (1-\alpha^w)\alpha^p$, $I_l = ((1-\phi_h)/\phi_h)\, e_0$, and $I_h = (\bar{e}+e_0)/\phi_h - e_0$. Parent's educational expenditures e' has a linear relationship with wealth level I if $I \in (I_l, I_h)$, and the slope is determined by human capital investments ratio α^p and saving ratio $(1-\alpha^w)$. In the case of $I \geq I_h$, a wealth threshold I_h exists, above which all those (relatively) wealthy parents invest in their children's educational attainment up to the same level of $\overline{e}$. On the other hand, when parents' wealth is low $(I \leq I_l)$, expenditures fall to zero.



(a) Children's human capital

(b) Expected bequest transfer in the retired period

Figure 1: Individual's policy functions of intergenerational transfers

We summarize the above discussion of parents' transfer rules in Figure 1. These figures (and all other figures in this section) are depicted using the calibrated parameters in the baseline case. The left panel (Figure 1a) illustrates the relationship between parents' wealth I and children's human capital h'. Given that parents' educational investments e' is a kinked-shaped function of wealth I, the relationship of human capital h' and wealth I must follow the same pattern. On the right panel (Figure 1b), we demonstrate the relationship between *expected* bequest transfer b' and parents' wealth I. Notice that the bequest is shown by expected value because the bequest function incorporates the next generation's income w^h that has yet to be realized in the parents' working period. The expected bequest is also a kinked-shaped function, and this graph is kinked at $I = I_h$. This reflects the fact that for those wealthy parents ($I > I_h$) who invest in their offspring's education to the upper bound level can do no better by leaving more bequests.[15]

So far we have summarized the agent's decisions as various simple expressions. Next, we explore how taxes may affect income inequality through these decision rules.

5.2 How do taxes affect income inequality?

In the model economy, workers' labor income differs due to an idiosyncratic labor supply shock and the level of human capital h. These are the causes of income inequality, but only the latter is affected by taxes. Therefore, to study the tax effects on income inequality, we can simply explore how do taxes affect human capital distribution.

Notice that the tax effects on steady state distribution can not be directly examined by equilibrium conditions. We thus start our analysis with the convergence process of the stationary distribution. Suppose the distribution of one particular generation's wealth $\Omega(I)$ is given, the distribution of the next generation's human capital can be obtained by $f_{e'}(I)$. From the equilibrium conditions, we obtain the following results:

Proposition 1. *For any given wealth distribution $\Omega(I)$ of a generation, the Gini coefficient of the next generation's human capital, which is generated by educational expenditure policy function $f_{e'}(I)$, is increasing in the value of wealth threshold I_h, given I is bounded below and above $I \in [a, b]$ and $a > I_l$, $b > I_h$.*

Proof: see appendix

Proposition 1 is a key relationship in the model: It implies that, the larger the proportion of people who share the same level of human capital, the smaller the human capital inequality (measured by the Gini coefficient). We illustrate this relationship by using the calibrated baseline model. Figure 2 shows both the policy function of human capital investment (solid line) and the stationary wealth distribution (dashed line). The grey area represents the proportion of parents who invest in their offspring to the college level. Proposition 1 implies that the size of this area is negatively associated with human capital inequality. It can be observed in figure 2 that this area is jointly determined by parents' decisions on human capital investment (specifically speaking, the value of I_h) and steady state wealth distribution. Therefore, a tax change may affect inequality through two possible channels: firstly it changes the wealth threshold I_h by influencing parents' investment decision. Secondly, given I_h fixed, the proportion of parents whose wealth level is higher than I_h is changed by tax as well.

The above discussion focuses on the tax effects on the proportion of parents who invest in the maximal level of human capital. On the other hand, taxes also affect inequality through changing

[15] The bequest function itself is also a kinked-shaped function of the retiree's wealth, but the cause of the kinked point is different from that in 1b. All the parents, regardless of wealth level, may face the situation that children's human capital returns are relatively higher than the retiree's assets. That is, there is a positive probability that none of the parents make bequest transfers.

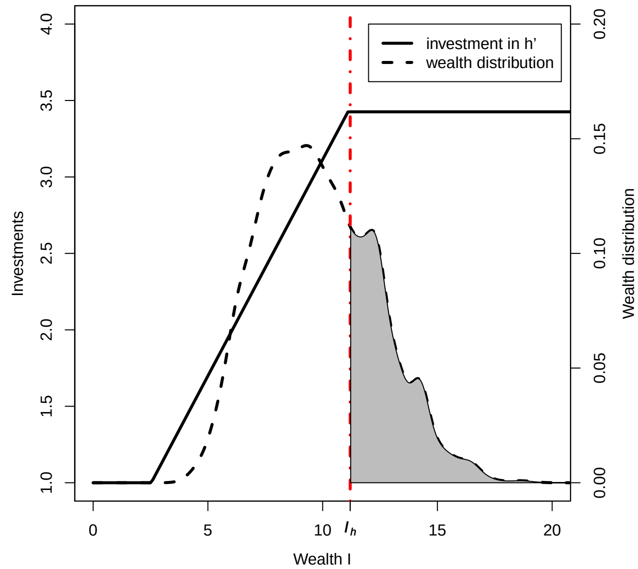


Figure 2: Human capital Investments and Wealth Distribution

the degree of human capital dispersion of those children whose educational attainment is lower than the maximal level. It means that their parents' wealth level are lower than I_h as shown in Figure 2. If the wealth distribution left to I_h is more dispersed, the degree of overall human capital dispersion is also higher. Recall that a parent's wealth I incorporates a large proportion of risky assets - the human capital w^h, which creates the volatility of the parent's wealth level. If, due to tax rate changes, the parent incorporates a lager (or smaller) proportion of risky assets in the wealth bundle, the volatility of aggregate wealth distribution increases (or decreases) as well. The degree of human capital dispersion rises and so as human capital inequality.

Given the above analysis, we next examine how taxes affect human capital inequality through their impacts on the following two dimensions: the parent's decision on human capital investment, and the aggregate wealth distribution.

5.3 Tax effects on parents' human capital investment

The earlier discussion concludes that the wealth threshold I_h is one of the determinants affecting inequality, while I_h is inversely related to the ratio of human capital to lifetime wealth (i.e. $(I_h+e_0) \propto \frac{1}{\phi_h}$). Recall that ϕ_h consists of both the proportion of savings invested in human capital α^p and saving ratio $(1-\alpha^w)$, and these ratios are determined by model parameters and tax rates, as shown in equation (5.18) and (5.19).

However, due to the constraint that bequests have to be non-negative, a parent's problem in the retired period consists of two cases: the interior solution ($b > 0$) and the corner solution ($b = 0$), which is expressed in equation (5.16). We can not derive a closed form solution from this equation and the tax effects are thus unclear.

To circumvent this difficulty, we proceed the analysis by examining two extreme cases: (1) perfect intergenerational risk sharing, and (2) no intergenerational transfer. In the first case we relax the constraint $b \geq 0$. That is, parents are allowed to manage child assets and transfer them back. In other words, parents can implement perfect risk sharing between the overlapping generations. This is the extreme situation of the interior solution.[16] In the second case, we completely shut off the channel of bequest transfers, such that parents can only consume their savings while children can only obtain utility from human capital returns. This case stands for the extreme situation of the corner solution. Since, in our model economy, what parents would face in their working period is the weighted average of these two extreme cases, we can get insights on tax effects by discussing these two cases. This is summarized in the following proposition.

Proposition 2. *Suppose $\gamma_c = \gamma_B = \gamma > 1$ and $T = 0$ (1) In the case of intergenerational risk sharing, α^p increases as τ_b increases, decreases as τ_w increases; If γ is large enough, educational investment to lifetime wealth ratio $\phi_h = \alpha^p(1-\alpha^w)$ increases as τ_b increases, and it decreases as τ_w increases; (2) In the case without intergenerational risk sharing, where $b = 0$, ϕ_h increases as τ_w increases.*

Proof: see appendix

In the case of intergenerational risk sharing, in the proof in appendix we show that agents' assets allocation decision rule α^p follows

$$\alpha^p = \frac{E\log(1-\tau_w)r^h - \log(1-\tau_b)(1+r) + \sigma_h^2/2}{\gamma\sigma_h^2}. \tag{5.20}$$

This assets allocation rule is similar to Merton's solution (1971) and the findings in the related portfolio literature (see Merton (1971); Viceira (2001)), that agents portfolio choice between risky and non-risky assets over the lifecycle is a constant ratio, determined by risk premium, volatility of returns on risky assets and coefficient of risk aversion. In our model, the risk premium of asset's return is $E\log(1-\tau_w)r^h - \log(1-\tau_b)(1+r)$. It is clear that income tax τ_w compresses the risk premium of human capital, so that it negatively affects α^p. On the other hand, although the bequest tax τ_b is not imposed on saving on risk-free assets directly, its effects on the intergenerational transfers propagate back to affect agents' decision making in the working period. By lowering the returns on risk-free assets, bequest tax expands the risk premium and can positively affect α^p.

As for tax effects on the saving rate $(1-\alpha^w)$, τ_w generates positive effects while the effects from τ_b is unclear. The main reason is that in our model setting, given the CRRA utility function, the value of coefficient of risk aversion γ is also the reciprocal of the elasticity of intertemporal substitution. If γ is greater than 1, which is commonly required in the macroeconomics and portfolio studies,

[16] According to Storesletten, Telmer, and Yaron (2007), this case may not be regarded as "perfect risk sharing." That is, since there exists the borrowing constrains $s > 0$, individuals still can not do risk sharing across more than 2 generations.

agents will save more when the returns on saving decrease. For agent's optimization problem in the working period, the return on savings $\tilde{r}^p$ shown in (5.17) is a weighted average of the adjusted assets' returns $(\tilde{r}^h, \tilde{r})$. Since an increases in income tax reduces $\tilde{r}^h$ and also discourages agents savings on the high return risky assets (human capital), the expected portfolio returns decreases. As for the effects of τ_b, if τ_b increases, on the one hand, agents invest more on the high return risky assets, which raises the expected portfolio return. On the other hand, τ_b reduces the adjusted risk free assets return $\tilde{r}$, though the channel of intergenerational transfer. Therefore, the overall effects on $\tilde{r}^p$ and saving rate are ambiguous.

In the case of no intergenerational transfer, the proportion of human capital investment to wealth is positively related to τ_w. If the human capital returns decrease, parents cannot cover the this loss though his wealth in the retired period. The compensation should be made in parents' working period by investing more human capital. Therefore, if τ_w increases, causing returns on human decline, α^p increases. The saving rate is still inversely related to returns. Saving rate $1-\alpha^w$ increases as τ_b increases, which is the same as the case of intergenerational risk sharing.

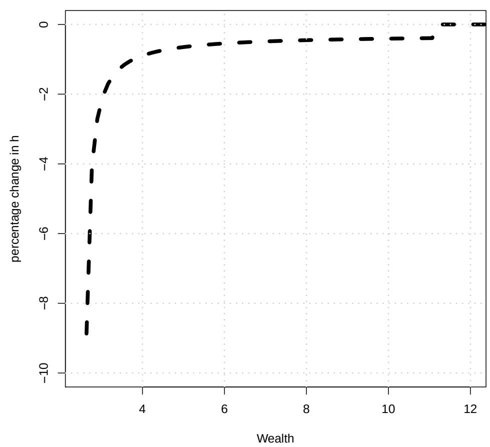
Table 4: Tax effects on human capital investment

	Perfect intergenerational risk sharing		No bequest transfer	
	portfolio choice	saving rate	portfolio choice	saving rate
Effects on h'	α^p	$1-\alpha^w$	α^p	$1-\alpha^w$
τ_w	$-$	$+$	$+$	$+$
τ_b	$+$	?	X	X

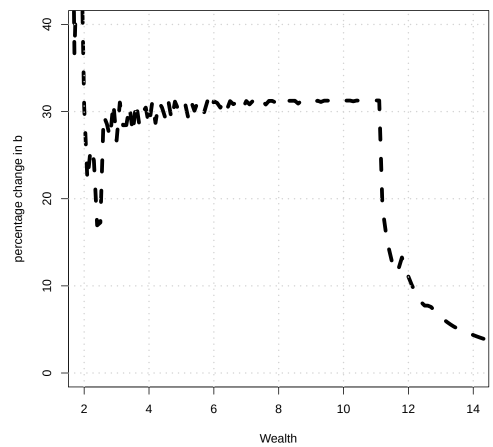
Table 4 summarizes the above discussion, in which the tax effects on human capital investment is decomposed into the effects on portfolio choice α^p and saving ratio $(1-\alpha^w)$ for both risk sharing and no transfer cases. We find that neither tax generates consistent effects on human capital investment. Thus far we can not conclude the tax effects simply based on theoretically analysis.[17] Above analysis, however, is useful to understand the underlying driving forces behind the numerical results. On the other hand, to present the numerical results could help us to distinguish which is the dominate force determines the tax effects. To this end, we conduct two numerical exercises: in the first case, τ_w is set to be 10% higher than the value in the baseline τ_w^*; in the second exercise, τ_b is set to be twice of the value in the baseline case. Except the change of tax rates, all other parameters are identical to the baseline case. We compare the results of these two cases with the baseline case. Figure 3 and the results presented in the section 5.4 are based on these two experiments.

The upper panel of figure 3 presents the effect of τ_w on human capital investment and bequest transfer for parents with different wealth level, while the bottom panel presents the counterparts of the effect of τ_b. The results are presented by showing the percentage change between the ex-

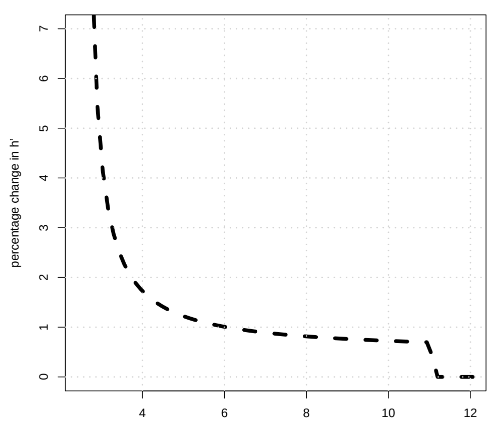
[17]However, we show that in the appendix that, as long as γ is large enough, the effects on the portfolio choice α^p in the perfect risk sharing case dominates agents' decision making.



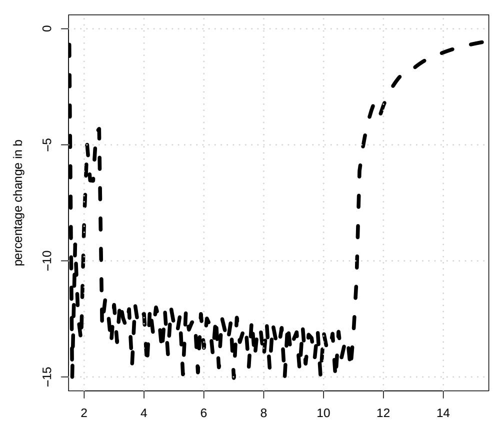
(a) Percentage change in h', $\tau_w \uparrow$



(b) Percentage change in b', $\tau_w \uparrow$



(c) Percentage change in h', $\tau_b \uparrow$



(d) Percentage change in b', $\tau_b \uparrow$

Figure 3: Change in h', b', comparing with the baseline case

perimental case and baseline case. For instance, in figure 3a, we find that a 10% increases in τ_w, causes individuals' human capital investments decrease, except for the group whose investments is bounded by investment upper bound. (*i.e.* wealth level is greater than threshold) On the other hand, an increase in bequest tax causes individuals' human capital investments increase. Comparing the numerical results with table 4, it can be easily concluded that tax effects on portfolio choice α^p is the dominate effect, since all other effects is unclear or opposite to the effects on portfolio choice. We also observed that in both case, the tax effects on bequests are all opposite to the effects on human capital investments. Such finding, taking the change of τ_w as an example, implies that individuals shift the human capital investments to savings, and then transfer to offsprings in the retired period. This supports the above arguments that the tax effects on total savings rates $1-\alpha^w$ and the effects in the case of corner solutions are relatively small. What individuals response to tax rate change is to reallocate the intergenerational transfer from one to the other.

5.4 Tax effects on wealth distribution

Here we examine how taxes affect the equilibrium distribution of wealth. For each tax, we are interested in how it affects the proportion of individuals who may invest their offsprings up to college ($i.e. I > I_h$), and how it affects the wealth dispersion for the other group of individuals ($I < I_h$).

The upper panel of Figure 4 shows tax effects of τ_w for individuals with different wealth level on offspring's wealth I' (Figure 4a) and stationary wealth distribution (Figure 4b), while the bottom panel illustrates the effects of τ_b. To highlight the effects, all plots are presented in the way of comparing with the baseline case. Since the realized human capital returns w^h is unknown, I' is a random variable as well. For the plots of I' (the left panel of 4), we show three different cases: (1) the expected value of I' (the real line) (2) the realized r^h is high (the dotted line) and (3) the realized r^h is low (the dashed line). The *high* and *low* mentioned here are defined as the realized human capital return r^h is 2.5 standard deviation away from the expected returns, *i.e.* the realized $w^{h'}$ is equal to $(Er^h + 2.5\sigma_{r_h})h'$, and $(Er^h - 2.5\sigma_{r_h})h'$. In the plots of wealth distribution (the right panel of figure 4), we demonstrate the change of the distribution. Specifically, the y axis represents the change of pdf of stationary wealth distribution, $\omega^e(I) - \omega^b(I)$, where ω^e and ω^b represents the simulated pdf of experimental case and baseline case, respectively.

When τ_w increases, as shown in Figure 4a, offsprings' expected wealth I' decreases, regardless of parent's wealth level. Earlier discussion concludes that parents reduce human capital investments in respond to the increase in τ_w. Parents do make higher bequests transfers in substitute to the human capital investment, but such shift in intergenerational transfer does cover the loss from tax payments. As a consequence, the stationary wealth distribution will shift leftward. Figure 4b confirms such impacts - the right side of the change in distribution (the dashed area in the figure) is negative, while the left side is positive. This implies that comparing with the baseline case, a rise in τ_w reduce the proportion of individuals who are willing to invest their offspring up to the college level. Therefore, income inequality rises.

An increase in τ_b generates the opposite effects to τ_w on wealth distribution, though the situation is more complicated. Offspring's expected wealth EI' increases if parent's wealth level is less than I_h. This result could happen because now parents allocate more *high-return* risky assets on their offspring's wealth bundle. On the other hand, for those wealthy parents ($I > I_h$), an increase in τ_b only discourages their bequest transfer, without any impact on human capital investments. Therefore, their offspring's expected wealth EI' decreases. The former impacts make the distribution on the left side shift rightward, while the latter impacts reduce the proportion of population on the right side of the distribution. This can be verified in the Figure 4d, in particular, the shaded area, where we find that comparing with the baseline group, the right side of distribution decreases while the distribution close to the boundary line $I = I_h$ increases. Note that, due to the combined effects, we cannot conclude the change of distribution in the shaded area, so as the the effects on income inequality.

We now discuss how tax affects the dispersion on the left side of wealth distribution. Recall that, as discussed earlier, a dispersed wealth distribution in the current generation implies a more dispersed human capital distribution of next generation.[18] While the proportion of risky assets (human capital) of each agent's wealth bundle is the main factor which determines the dispersion of wealth distribution. That is, a wealth bundle which includes larger proportion of risky assets comes with a higher volatility. If it applies to all individuals in the economy, in aggregate, there will be a widely dispersed wealth distribution. Therefore, a rise in τ_w, which reduces parents human capital investments, will further reduces aggregate wealth dispersion, while a rise in τ_b generates the opposite effects.

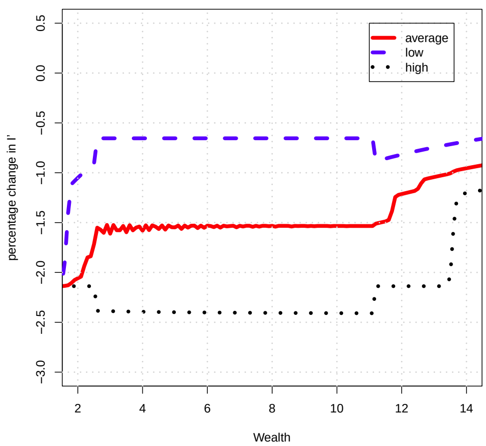
We use the case of τ_b to illustrate the discussion. In Figure 4c, consider any particular agent with wealth I ($I < I_h$): if the realized return r^h is high (the dotted line), his offspring's wealth is around 0.5% higher than the baseline case; on the other hand, when the realized return is low (the dashed line), his offspring's wealth will be 0.45% lower than baseline case. These results imply that for any agent, given the same wealth level and the same value of r^h, after τ_b increases, his offspring's wealth is more volatile. In aggregate, there will be a more widely dispersed wealth distribution. Such conclusion can be verified by Figure 4d. We find that both right and left tail of distribution over the range $I < I_h$ increases.[19]

In the case of τ_w, from Figure 4a, an effect opposite to τ_b is observed. Although in both cases I' is lower than the baseline case, we find that when r^h is high, I' is particularly lower than the baseline case, comparing with the case r^h is low. It implies in aggregate, there will be a more centered wealth distribution, which can be verified in Figure 4b.

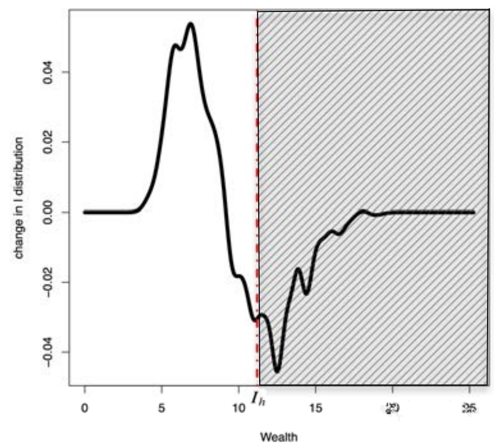
In summary, in this section we examined the tax effects on agents' asset allocation and on the aggregate wealth distribution. These two different type of impacts jointly determine the net

[18]Eventually both distributions will converge to stationary distributions. The discussion here focuses on the convergence process.

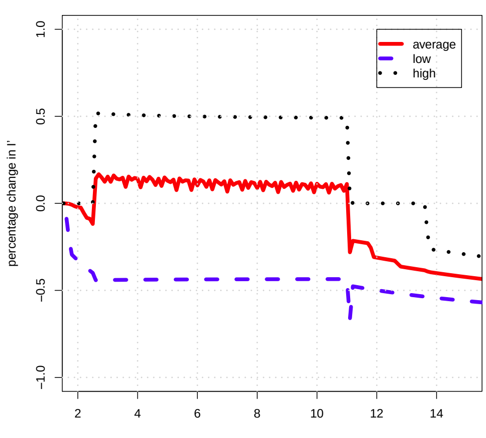
[19]To further support above argument, note that, all offspring's expected wealth in this area increases (the real line in Figure 4c). Without the effects on changing wealth volatility, it is not supposed to observe that the left tail of the distribution increases.



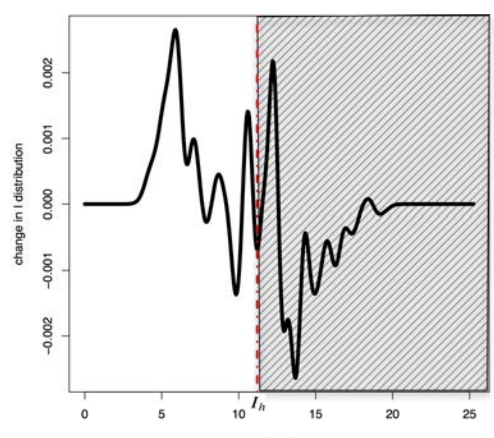
(a) Percentage change in I', $\tau_w \uparrow$



(b) Change in wealth distribution , $\tau_w \uparrow$



(c) Percentage change in I', $\tau_b \uparrow$



(d) Change in wealth distribution, $\tau_b \uparrow$

Figure 4: Change in I' and wealth distribution, comparing with baseline case

tax effects on income inequality. Table 5 summarizes all the effects for each tax. For instance, an increase in the income tax reduces parents' human capital investment so that the wealth threshold I_h increases. This decreases the proportion of population achieving the college degree, and increases income inequality. On the other hand, individuals' expected wealth decreases due to the lower returns on human capital and lower investment. As a consequence, the proportion of population whose wealth is level higher than wealth threshold decreases, which increases income inequality. Moreover, since the proportion of risky assets among agent's wealth decreases, the stationary wealth distribution becomes more centered. This reduces the income inequality. The net income tax effect is determined by considering all these forces. As for the effects of income tax, it affects the income inequality through similar channels but with opposite direction.

Table 5: Tax effects

	Human capital investments	Wealth distribution	
	I_h	EI'	Dispersion of I'
τ_w	$+$	$-$	$-$
Effects on inequality	$+$	$+$	$-$
τ_b	$-$	?	$+$
Effects on inequality	$-$	?	$+$

6 Numerical experiments

In the last section we examined how the *tax rates* affect income distribution. Here we conduct numerical experiments to see if model predictions are consistent with the empirical counterparts reported in section 2. Recall that in section 2, to examine the effects of a particular tax, in the regression we control for the total taxation and other types of taxation (measured as a percentage of GDP).[20] Therefore, to ensure the results of numerical experiments to be comparable with the our empirical findings, in each experiment we only change the taxation we are considering, while the total taxation and the alternative taxation must be kept as the same as that in the baseline case. The details of the procedure are as follows:

In the first group of experiments, we identify the effects of income taxation on income inequality. Specifically, we would like to know how much change in income Gini coefficient for a unit change in income taxation, *i.e.* $\Delta Gini_i/\Delta Tax_w$. The empirical counterparts of this is the coefficient of income taxation in the regression (2). The experiments start by raising the income tax rate τ_w by 10% (*i.e.* $\tau_w = 1.1\tau_w^*$, where τ_w^* is the income tax rate in the baseline case). By doing so, income taxation is expected to rise. However, all other endogenous variables in the equilibrium change as well. For instance, since the increase in τ_w discourages parents' investment in child's human capital, the equilibrium aggregate output and bequests are expected to decrease. To keep all the other control variables being the same as that in the baseline case, other parameters related

[20]In this section, the term *taxation* refers to the ratio of tax revenue to output. The changes in taxation refers to the change of this ratio.

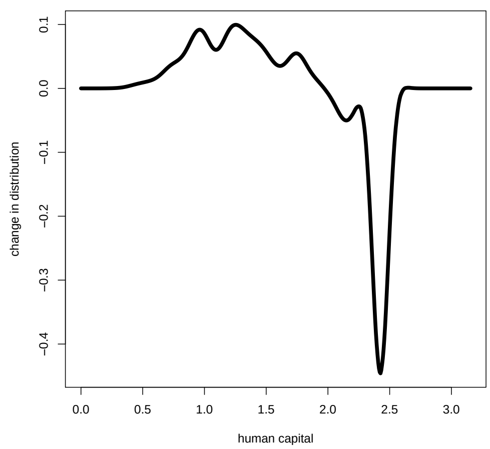
to government choices should be adjusted accordingly. We increase bequest tax rate τ_b to keep bequests to output ratio be the same as the baseline case. Government transfers T have to be adjusted for letting the total taxation remains the same as the baseline case. These adjustments ensure that in the equilibrium, income taxation is the only variable that changes the value among all the variables related to taxation structure. We then calculate the value $\Delta Gini_i/\Delta Tax_w$ and compare it with our empirical results. Besides, to check the robustness of our quantitive results, we also conduct an experiment in which τ_w is raised by 5%.

In the second group of experiments, we follow similar procedures described above to identify the effects of bequest taxation. The experiments again start by rising the bequest tax rate. For the robustness check, two cases are considered: (1) $\tau_b = 1.5\tau_b^*$ and (2) $\tau_b = 2\tau_b^*$, where τ_b^* is the bequest tax rate in the baseline case. The income taxation remains the same as the baseline case, as long as τ_w does not change, as shown in equation (4.4). However, total taxation are expected to rise due to the gain from bequest taxation. Thus the extra tax revenue give back to agents through government transfers T. How government transfers back the extra tax revenue affects the agents' wealth level. It affects the wealth distribution and thus further changes the income inequality. Therefore, for the second group of experiments, two schemes of government transfers are considered. For the first scheme (T_a), we follow the original model setting that government transfers apply to both worker and retirees. In the second scheme (T_r), we assume government transfers only apply to the retirees.

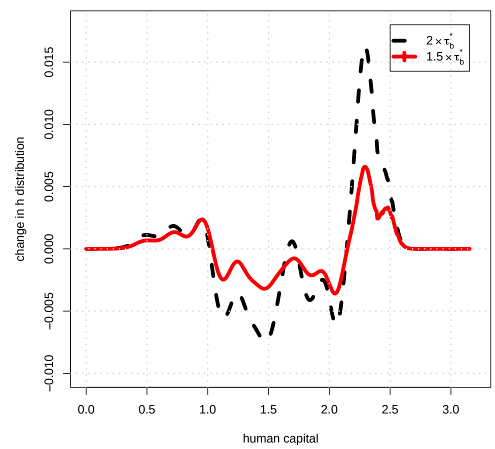
Table 6: Experiment Results

Experiment		Details	$\Delta Gini_i/\Delta tax$ Model	$\Delta Gini_i/\Delta tax$ Data	$\Delta h/h$
τ_w	(1)	$\tau_w = 1.05\tau_w^*$	0.0082	0.008	-3.2%
	(2)	$\tau_w = 1.1\tau_w^*$	0.0076		-6.5%
τ_b	(1)	$\tau_b = 1.5\tau_b^*$, T_a	0.0005	-0.007	0.05%
	(2)	$\tau_b = 1.5\tau_b^*$, T_r	0.0001		0.07%
	(3)	$\tau_b = 2\tau_b^*$, T_a	-0.0001		0.13%
	(4)	$\tau_b = 2\tau_b^*$, T_r	-0.0004		0.16%

Table 6 reports the relevant statistics for each numerical experiment and its empirical counterpart. As shown in the column labelled $\Delta Gini_i/\Delta Tax$, a rise in income taxation increases the income Gini coefficient. This result does not vary so much as the level of change in the τ_w. In addition, the model predictions are very close to our empirical findings. On the other hand, the effects of bequest taxation on income Gini coefficient are sensitive to the change of bequest rates and also the transfer scheme. However, this kind of result is not far away from our empirical finding, in which the relationship between bequest taxation and income Gini coefficient is unclear. The column labelled $\Delta h/h$ reports the percentage change of aggregate human capital. Labor income taxation reduces the aggregate human capital while bequests taxation positively affects human capital. The effects of each taxation on human capital are consistent with the discussion earlier: A change in tax rates affects the human capital risk premium. Agents respond to this by reallocating their intergenerational transfers from bequests to human capital investments, or vice versa. The finding that labor income tax discourages human capital accumulation is widely recognized in the literature (see, for instance, Trostel (1993))[21] Moreover, the finding that bequest taxation has positive effects on aggregate human capital is consistent with the findings in Grossmann and Poutvaara (2009), although the underlying channel is different.



(a) Effect of income taxation



(b) Effect of bequest taxation

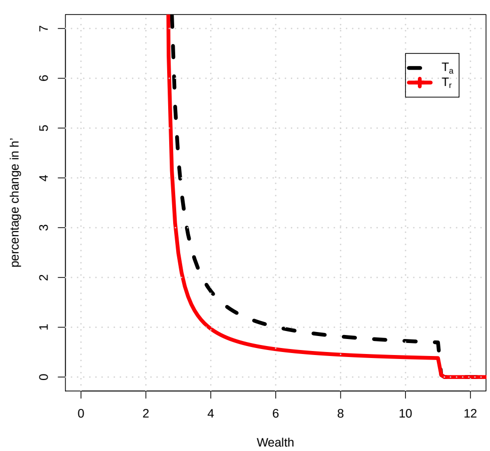
Figure 5: Change in human capital distribution

In section 5 we conclude that there exist two main factors associated with income inequality: (1) the proportion of people who attains the college education and (2) wealth dispersion. Both are affected by tax change. The former is negatively while the latter is positively related to the income inequality. We now examine the experiments results and see the relative importance of these two factors. Figure 5 depicts the difference of human capital distribution (measured by pdf) between experimental and baseline case for each tax. The effects of a rise in income tax is shown in the Figure 5a. One may clearly notice that there exists a big drop around the value 2.4. It implies that comparing with the baseline case, the proportion of individuals who receive the education up to upper bound decrease by 0.45%. This causes a rise in income inequality. We also notice that the left side of the distribution becomes more centered, which comes from a less dispersed wealth distribution. This implies the income inequality will be alleviated. Since we have aware that income taxation increases the income inequality. We know the former impact dominates.

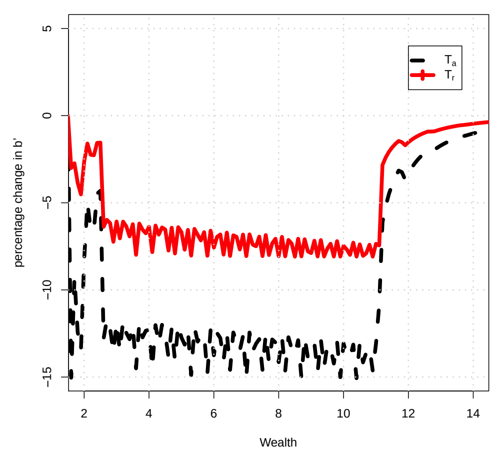
However, the tax effects on wealth dispersion is also relevant, in particular, for bequest taxation. As reported in Table 6, a rise in bequest tax increases the human capital. This implies that a higher proportion of people attains the college education, and as a consequence income inequality should

[21] In Trostel (1993), he demonstrates that a 1 percent increase in the income tax rate would cause the human capital stock to decline by 0.39 percent in the long run. Our simulation results indicate the corresponding vale is 0.65.

be reduced. This result happens in the case $\tau_b = 2\tau_b^*$. Yet, we find that in the case $\tau_b = 1.5\tau_b^*$, income inequality actually rise. We illustrate this differences by Figure 5b, which depicts the change of distribution for both cases $\tau_b = 1.5\tau_b^*$ and $\tau_b = 2\tau_b^*$, under transfer scheme T_a. Notice that now in both cases, the left and right tails of the distribution increase. The reason is that, on the one hand, the proportion of people achieving the education upper bound increases due to the rise of human capital investment. On the other hand, due to individuals' wealth bundle containing higher proportion of risky assets , the wealth and human capital distribution becomes more spread out. Comparing with these two cases, we conclude that when the change of τ_b is not high enough, the force from the increase in wealth dispersion dominates. As τ_b increases more, the force from the increases proportion of college education turns to dominate.



(a) Percentage change in h'



(b) Percentage change in b'

Figure 6: Comparing effects between T_a and T_r

We further compare the effects of these two transfer scheme. For any given tax rate, $\tau_b = 1.5\tau_b^*$ or $\tau_b = 2\tau_b^*$, the transfer scheme T_r generates a higher level of aggregate human capital, and a lower income inequality. We talk about the difference between these two scheme by Figure 6, which reports the percentage change of b' and h', comparing with the baseline case. Although the human capital risk premium is identical for both transfers scheme, we find that under the transfer scheme T_A, individuals tend to transfer more human capital in substitute to bequests. The reason is that for agents in the working period, they are now hiding a proportion of risk free assets - government transfers, which affects parents decision on the assets allocation. Specifically, due to this risk-free assets, parents are encourage to invest more risky assets on their offspring. On the other hand, in the case T_r, parents need to prepare more savings in the retired period to against the risk child may experience. As a result there are more bequests transfer. Moreover, under this situation parents' wealth is higher is the retired period, so that there will be higher possibility for bequest transfers in the retired period. (*i.e* the transfer threshold decreases). The consequence is that larger proportion of wealth will be transferred across generations. Therefore, the aggregate wealth, human capital investments, and the proportion of individuals who receive college education are higher in the case T_r. To confirm this model implications, we do an extra empirical examination. We redo the regression model (2) in section 2 for a subsample, where government expenditures on pensions are relatively higher.[22] Countries in this group are regard as the case T_r. We find that the coefficient of bequest taxation becomes -0.002 at a 5% significance level. This results is qualitatively consistent with our finding, that in the economy where agents have higher level of risk free bequest tax, bequest taxation is likely negatively correlated with income inequality.

7 Conclusion

In this study we examined the effects of income and bequest taxation on income inequality. Based on OECD countries panel data we found that income taxation are positively related to income Gini coefficient, while the relationship between bequest taxation and Gini coefficient is not clear. We constructed a theoretical overlapping generation lifecycle model to explain this findings. The main idea is that tax rates creates distortion on parents' decision on intergenerational transfers. For instance, a higher income tax induces parents to allocate less transfers toward educational investment but more on bequests. Such behavior changes the distribution of education in the economy. Besides, tax rates also affect the wealth distribution over generation and the volatility of agents' assets. All these forces contribute to the tax effects on income inequality. Based on the calibrated parameter values from U.S. data, our model prediction in the effect of income taxation is quantitively consistent with the empirical findings. We also provide some explanations for the ambiguous effect of bequest taxation.

Existing literature usually focus on the effects of progressiveness of taxation on income inequality. (see, for example Sarte (1997); Farhi and Werning (2010)) Our focus is on the effects of each taxation, *i.e.* the composition of taxation. Although we did not directly touch the issue of tax progressiveness, this can still be seriously considered in our framework. For instance, it should be not difficult to provide a progressive tax scheme on labor income in a way that the expected return on human capital remains constant but the volatility of returns are reduced. In this case, our model implies that parents will invest more on human capital, as shown in equation 5.20. The income inequality will be reduced. On the other hand, a progressive bequest tax may not only compress the human capital risk premium for the less wealthy parents, but also expand risk premium for those wealthy parents (who already invest their offspring up to college). As a results, the left tail of the human capital distribution must rise, while the right tail may remain the same. Therefore, the progressiveness of bequest tax may have negative effects on educational investment and income inequality.

[22] Specially, we rank all countries based on the pension paid as a percentage of GDP. The top 10 countries are selected.

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A Data Source and Characteristics

Data are available for 20 OECD countries from 1980 to 2008. Those countries are Australia, Austria, Belgium, Denmark, Finland, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland Turkey, UK, USA.

The income inequality data, measured by Gini coefficient, come from Castellacci and Natera (2011). They construct a new dataset (CANA) with state-of-the-art imputation method for cross-country analyses. The CANA panel dataset provides a rich and complete set of 41 indicators, including Gini coefficient, for over 80 countries in the period 1980-2008. Their data source comes from United Nations, and the calculation of Gini coefficient is based on disposable income. In the CANA panel dataset, there is no missing value for the data of Gini coefficient, which is a usual limitation when doing cross-country analyses. Our human capital data come from Barro and Lee (2012). This panel dataset on educational attainment are available at 5-year intervals for 146 countries from 1950 to 2010. The human capital variable we applied in our analysis is the average years of schooling for secondary and higher education in the population aged 25 and over. The democracy index is developed by Pemstein, Meserve, and Melton (2010). They apply a Bayesian latent variable approach and synthesize a new measure of democracy, the Unified Democracy Scores (UDS), from 10 extant scales. The scores are available for virtually every country in the world from 1946 through 2008. The data about GDP per capita, population growth rate, and fertility rate are obtained from OECD database. Table A1 shows the definition of these conditioning variables.

Table A1: Definition of Conditioning Variables

Variable	Definition or Measurement
Population growth rate	Annual growth rate of total population (%)
Fertility rate	Number of children per women
Democracy index	A subjective measure of democracy from 10 extant scales
Initial per capita GDP	Measured by 1980 US$
Human Capital	Average years of schooling for secondary and higher education (population aged 25 and over)

Note: GDP per capita and average years of schooling for secondary and higher education are measured at the beginning of each period.

These data are annual, except that human capital data are available at 5-year interval. For all the taxation variables, population growth rate, fertility rate, and democracy index, we follow the standard practice of taking 5-year averages of annual observations to remove the effects of the business cycle. There are six periods in our panel data: 1980 to 1984, 1985 to 1989, 1990 to 1994, 1995 to 1999, 2000 to 2004, and 2005 to 2008 (we take 4-year average for the last period). As for the log of per capita GDP and its squared term, and human capital variables, they are measured at the beginning of each period.

Descriptive statistics for the data are reported in Table A2. It can be seen that population grew, on average, 0.66%, per annum, with fertility rate around 1.7, and average years of schooling for secondary and higher education are 3.88. Among the tax variables, the (individual) income taxation accounts for the largest share of GDP (10.61%) and is slightly higher than the share of consumption taxation (10.51%). The third largest tax revenue comes from social security contribution (8.55%), and bequest taxation, on average, only accounts for 0.17% of the total tax revenue. As for the category of other tax revenues, which include corporate income taxation, part of property taxation, tax on payroll and workforce, and other tax revenues contribute 5.28% to GDP.

B Model solution and Proofs of Propositions

B.1 Solving α^p and α^w

We have show that in equilibrium, the following condition holds:

$$\left(\frac{\alpha^w}{1-\alpha^w}\right)^{-\gamma} = \beta\left\{\theta\tilde{r}\int_0^X (\tilde{r}^p)^{-\gamma} f(r^h)dr^h + \int_X^\infty \{(1+r)(1-\alpha^p)\}^{-\gamma} f(r^h)dr^h\right\}, \tag{B.1}$$

$$\left(\frac{\alpha^w}{1-\alpha^w}\right)^{-\gamma} = \beta\left\{\theta\int_0^X \tilde{r}^h(\tilde{r}^p)^{-\gamma} f(r^h)dr^h + \int_X^\infty (1-\tau_w)r^h\beta_c\left\{(1-\tau_w)r^h\alpha^p\right\}^{-\gamma} f(r^h)dr^h\right\}. \tag{B.2}$$

Combining the RHS of equation (B.1) and (B.2),

$$\begin{aligned}&(1+r)\left\{\theta(1-\tau_b)\textstyle\int_0^X (\bar{r}^p)^{-\gamma} f(r^h)dr^h + \int_X^\infty \{(1+r)(1-\alpha^p)\}^{-\gamma} f(r^h)dr^h\right\}\\ =\ &\theta\textstyle\int_0^X (1-\tau_w)r^h(\bar{r}^p)^{-\gamma} f(r^h)dr^h + \int_X^\infty (1-\tau_w)r^h\beta_c\left\{(1-\tau_w)r^h\alpha^p\right\}^{-\gamma} f(r^h)dr^h\end{aligned} \tag{B.3}$$

where $X = \frac{1}{K}\left(\frac{1+r}{1-\tau_w}\right)\left(\frac{1-\alpha^p}{\alpha^p}\right)$. In equation (B.3), α^p is the only endogenous variable. Reorganize equation (B.3), we obtain,

$$\begin{aligned}&\theta\left\{\textstyle\int_0^X \{(1-\tau_w)r^h - (1+r)(1-\tau_b)\}(\bar{r}^p)^{-\gamma} f(r^h)dr^h\right\}\\ =\ &\beta_c\textstyle\int_X^\infty \left\{(1+r)^{1-\gamma}(1-\alpha^p)^{-\gamma} - \left\{(1-\tau_w)r^h\right\}^{1-\gamma}(\alpha^p)^{-\gamma}\right\} f(r^h)dr^h.\end{aligned}$$

It is easy to verify that LHS is decreasing in α^p and RHS is increasing in α^p. Moreover, when $\alpha^p \to 0$, LHS is greater than RHS. A unique α^p can be determined by equation (B.3).

B.2 Proof of Proposition 1

Since the policy function of educational expenditures is a increasing linear function of parents's life time disposable wealth $w^w = I + e_0$. The human capital h is also an increasing function of w^w. The Gini coefficient is calculated by,

$$Gini_i = 1 - 2\left(\int_0^1 v(z)dz\right),$$

where z is the percentile of population ranking from the lowest h up to z. $v(z)$ represents for the proportion of human capital owned by the bottom z proportion population. For the agent, whose human capital at z's percentile position, his parents' wealth is also at $z's$ percentile due to the increasing property of policy function. Parent's wealth level can be obtained by,

$$I = \Omega^{-1}(z).$$

The $v(z)$ function can be calculated by

$$\begin{aligned}
v(z) &= \frac{y(s)}{y(b)} \\
y(s) &= \begin{cases} y_1(s) = \int_a^s mxf(x)dx & if\, s < c^* \\ y_2(s) = \int_a^{c^*} mxf(x)dx + \bar{h}(\Omega(x) - \Omega(I_h)) & if\, s \geq c^* \end{cases} \\
\text{where } s &= \Omega^{-1}(z) + e_0 \\
c^* &= I_h + e_0 \\
m &= a^h \phi_h \\
\bar{h} &= mc^*
\end{aligned}$$

$y(x)$ calculates the accumulated human capital from the lowest up to x, where x is the z's percentile-agent's parents' wealth plus e_0. Since distribution $\Omega(I)$ is assumed to be bounded by a and b, we rule out the case where parents investments is 0. In addition, there should exist positive proportion of people whose educational attains the upper bound. Give that, $y(b)$ represents the sum of total human capital.

We will show that $\left(\int_0^1 v(z)dz\right)$ decreases as I_h (or c^*) increases, so that $Gini$ increases. Let $\theta = \Omega^{-1}(I_h)$.

$$\begin{aligned}
\frac{d}{dc^*}\left(\int_0^1 v(z)dz\right) &= \frac{d}{dc^*}\left(\int_0^\theta \frac{y_1(s)}{y(b)}dz + \int_\theta^1 \frac{y_2(s)}{y(b)}dz\right) \\
&= \underbrace{\theta' \frac{y_1(c^*)}{y(b)}}_{term\, 1} + \underbrace{\int_0^\theta \frac{d}{dc^*}\left(\frac{y_1(s)}{y(b)}\right)dz}_{term\, 2} - \underbrace{\theta' \frac{y_2(c^*)}{y(b)}}_{term\, 3} + \underbrace{\int_\theta^1 \frac{d}{dc^*}\left(\frac{y_2(s)}{y(b)}\right)dz}_{term\, 4} \qquad \text{(B.4)}
\end{aligned}$$

It is easy to verify that the term 1 and term 3 on the RHS of equation (B.4) can be cancel out. We will show that the function inside the integral of both term 2 and term 4 are negative.

The function inside the term 2:

$$\frac{d}{dc^*}\left(\frac{y_1(s)}{y(b)}\right) = \left(\frac{y_1'(s)y(b) - y_1(s)y'(b)}{y(b)^2}\right) \qquad \text{(B.5)}$$

The function inside the term 4:

$$\frac{d}{dc^*}\left(\frac{y_2(s)}{y(b)}\right) = \left(\frac{y_2'(s)y(b) - y_2(s)y'(b)}{y(b)^2}\right) \qquad \text{(B.6)}$$

where

$$
\begin{aligned}
y_1'(s) &= \frac{-1}{c^*}\int_a^s mxf(x)dx = \frac{y_1(s)}{c^*} \\
y_2'(s) &= \frac{-1}{c^*}\left(y_2(s) - \bar{h}(\Omega(x) - \Omega(I_h))\right) \\
y'(b) &= \frac{1}{c^*}\left(y(b) - \bar{h}(1 - \Omega(I_h))\right)
\end{aligned}
$$

Substitute these into (B.5) and (B.6), we obtain

$$
\frac{d}{dc^*}\left(\frac{y_1(s)}{y(b)}\right) = -\frac{1}{c^*}\left(y_1(s)\bar{h}(1-\Omega(I_h))\right) < 0 \tag{B.7}
$$

$$
\frac{d}{dc^*}\left(\frac{y_2(s)}{y(b)}\right) = c^* y_2'(s)(1-\Omega(s)) < 0 \tag{B.8}
$$

Above two inequality confirms $\frac{d}{dc^*}\left(\int_0^1 v(z)dz\right) < 0$, which implies $\frac{d}{dc^*}Gini > 0$. Given the relationship between c^* and I_h by construction, that $c^* = I_h + e_0$, it implies that Gini is increasing in I_h.

B.3 Proof of Proposition 2

Case 1: Complete Intergenerational risk sharing

Portfolio Choice α^p To make the problem to be tractable and the results to be illustrative, we log linearize efficiency conditions based on the approximation technique from Viceira (2001). We take log on all assets return: define $\bar{r}_p = \log \bar{r}^p$, $\bar{r}_h = \log((1-\tau_h)r^h)$ and $\bar{r} = \log((1-\tau_b)(1+r))$ The portfolio return $\bar{r}_p$ can be approximated as

$$
\bar{r}_p = \alpha^p(\bar{r}_h - \bar{r}) + \bar{r} + \frac{1}{2}\alpha^p(1-\alpha^p)\sigma_h^2
$$

where σ_h^2 is the variance of $\bar{r}_h$. Recall that human capital return r^h follows a log normal distribution with parameters $N(\mu_h, \sigma_h^2)$. The variance of after tax (log) returns $\bar{r}_h$ is identical to the variance of $\log r^h$.

Worker's efficiency conditions are given by,

$$
\begin{aligned}
1 &= \beta(1+r)E\left(\alpha^r \tilde{r}^p \left(\frac{1-\alpha^w}{\alpha^w}\right)\right)^{-\gamma} \\
&= \beta\beta_c E(1-\tau_w)r^h\left((1-\alpha^r)\,\tilde{r}^p\left(\frac{1-\alpha^w}{\alpha^w}\right)\right)^{-\gamma}
\end{aligned}
$$

In this case all the uncertainty terms in RHS follows log normal distribution, these two equations can be log linearized as follows,

$$
0 = \log\beta - \gamma\left\{\log\left(\frac{1-\alpha^w}{\alpha^w}\right) + E\bar{r}_p + \log\frac{\alpha^r}{1-\alpha^r\tau_b}\right\} + \bar{r} + \frac{1}{2}Var(\bar{r} - \gamma(C_1 + \bar{r}_p)) \tag{B.9}
$$

$$
0 = \log\beta - \gamma\left\{\log\left(\frac{1-\alpha^w}{\alpha^w}\right) + E\bar{r}_p + \log\frac{\alpha^r}{1-\alpha^r\tau_b}\right\} + E\bar{r}_h + \frac{1}{2}Var(\bar{r}_h - \gamma(C_2 + \bar{r}_p)) \tag{B.10}
$$

Subtracting (B.9) from B.10 yields,

$$\alpha^p = \frac{E\bar{r}_h - \bar{r} + \sigma_h^2/2}{\gamma\sigma_h^2}. \tag{B.11}$$

We could examine the tax effect on the portfolio choice

$$\begin{aligned}\frac{\partial\alpha^p}{\partial\tau_w} &= \frac{1}{\gamma\sigma_h^2}\left(\frac{-1}{1-\tau_w}\right) < 0\\ \frac{\partial\alpha^p}{\partial\tau_b} &= \frac{1}{\gamma\sigma_h^2}\left(\frac{1}{1-\tau_b}\right) > 0\end{aligned}$$

That is, α^p decreases as τ_w increases; α^p increases as τ_b increases.

Saving Ratio $1-\alpha^w$ Substitute the solved α^p into equation (B.9), we can solve the log value of saving to consumption ratio

$$\Delta = \log\frac{1-\alpha^w}{\alpha^w} = \frac{1}{\gamma}\left\{\log\beta + \bar{r}_h - \gamma\log\frac{1-\alpha^r}{1-\alpha^r\tau_b} - \gamma\bar{r} + \frac{1}{2}\sigma_h^2\left[\gamma(1-\gamma)\left(\alpha^p\right)^2 - 2\alpha^p\gamma + 1\right]\right\}$$

The partial derivative of saving ratio with respect to each tax is

$$\begin{aligned}\frac{\partial(1-\alpha^w)}{\partial\tau_b} &= \frac{\partial\Delta}{\partial\tau_b}(1-\alpha^w)\alpha^w\\ \frac{\partial(1-\alpha^w)}{\partial\tau_w} &= \frac{\partial\Delta}{\partial\tau_w}(1-\alpha^w)\alpha^w\end{aligned}$$

Case 2: Without bequest transfer

In the case of without risk sharing. $b=0$. Efficiency conditions are give by

$$c_2^{-\gamma} = \beta(1+r)c_3^{-\gamma} \tag{B.12}$$

$$c_2^{-\gamma} = \beta\beta_c E(1-\tau_w)r^h I'^{-\gamma} \tag{B.13}$$

Assume parents still follow linear rules $\alpha^{w'}$ to allocating w^w into consumption c_2 and saving $\bar{s}$. Among the saving, a fraction $\alpha^{p'}$ is allocated in human capital investments and $1-\alpha^{p'}$ in risk-free assets. Substitute the decision rules into (B.12) and (B.13), we have

$$\left(\frac{\alpha^{w'}}{1-\alpha^{w'}}\right)^{-\gamma} = \beta(1+r)^{1-\gamma}(1-\alpha^{p'})^{-\gamma}, \tag{B.14}$$

$$= \beta\beta_c E\left((1-\tau_w)r^h\right)^{1-\gamma}(\alpha^{p'})^{-\gamma}. \tag{B.15}$$

(B.14) and (B.15) pin down the value of $\alpha^{w'}$ and $\alpha^{p'}$,

$$\begin{aligned}\alpha^{p'} &= \frac{\kappa^p}{1+\kappa^p},\\ \alpha^{w'} &= \frac{\kappa^w}{1+\kappa^w},\end{aligned}$$

where

$$\kappa^p = \beta_c^{\frac{1}{\gamma}} \left(\frac{1+r}{1-\tau_w}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{E\left(r^h\right)^{1-\gamma}}\right)^{\frac{-1}{\gamma}}$$

$$\kappa^w = \beta^{\frac{-1}{\gamma}} (1+r)^{\frac{\gamma-1}{\gamma}} (1+\kappa^p)^{-1}$$

It is easy to verify that $\partial \alpha^{p'}/\partial \tau_w > 0$ and $\partial \alpha^{w'}/\partial \tau_w < 0$.

$$\frac{\partial \alpha^{p'}}{\partial \tau_w} = \left(\frac{1}{1+\kappa^p}\right)^2 \left(\frac{\gamma-1}{\gamma}\right) \left(\frac{1}{1-\tau_w}\right) \kappa^p > 0,$$

$$\frac{\partial \alpha^{w'}}{\partial \tau_w} = -\left(\frac{1}{1+\kappa^w}\right)^2 \left(\frac{k^w}{1+\kappa^p}\right) \left(\frac{\partial \alpha^{p'}}{\partial \tau_w}\right) < 0.$$

Therefore, the educational investment ratio $\phi_h = \alpha^{p'}(1-\alpha^{w'})$ is increasing in τ_w.