国際大学
INTERNATIONAL UNIVERSITY OF JAPAN

Economics & Management Series *EMS-2014-01*

Timing of Motherhood and Economic Growth

Ching-Yang Lin
International University of Japan

February 2014

IUJ Research Institute
International University of Japan

Timing of Motherhood and Economic Growth

Ching-Yang Lin

February 16, 2014

Abstract

The postponement of motherhood has drawn much attention in recent research. Yet, it is the U shape pattern of the mean age of the mother at first birth that observed in the data in 1950-2005. This phenomena can not be fully explained by the explanatory variables of the delay of motherhood documented in the empirical and theoretical literature. This paper develops a theoretical model to jointly consider females' decisions on motherhood timing, as well as human capital accumulation, working, and resources spent on child rearing. The calibration results successfully generate the U shape pattern of age at maternity along the growth path. Our explanations differ from the existing human capital story, by taking into account that child rearing affects mother's decisions on human capital investment both before and after motherhood. The model predicts that the delay in motherhood timing will continue.

Keywords: Fertility, Human Capital, Time Allocation and Labor Supply

1 Introduction

In the past thirty years, the mean age of motherhood at first birth rose around three to five years among many developed countries.[1] This delay has received a large amount of attention and been studied both in empirical research based on micro level data (Heckman and Walker (1990); Walker (1995); Miller (2005)) and theoretical models (Happel, Hill, and Low (1984); Cigno and Ermisch (1989); Cigno (1991); Blackburn, Bloom, and Neumark (1993); Hotz, Klerman, and Willis (1997)). Several economic forces and variables have been considered to explain such phenomena. However, examining the historical data since 1945 as shown in Figure 1, we could find that the current trend of delayed motherhood is actually the reverse of the trend in the years following WWII: the pattern actually appears to be a U shape.[23] The purpose of this paper is to provide an explanation of this stylized fact. We do so by developing a theoretical model based on the Ben-Porath (Ben-Porath (1967)) framework to jointly consider females' decisions on motherhood timing, as well as human capital accumulation, working, and resources spent on child rearing in response to economic growth.

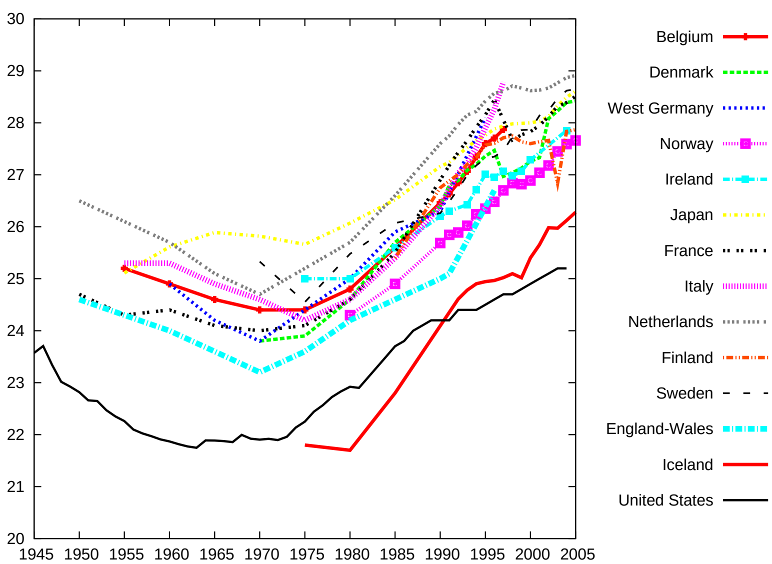

Figure 1: Mean age of motherhood at first birth (Source: Gustafsson (2001), NCHS, Eurostat)

[1]For example, Demark: 5.048 years, Belgium: 3.2 years, See Figure 1 in detail.

[2]This stylized fact was first proposed in Bosveld (1996) based on observation of European countries; The same pattern can be also observed in the U.S. data, the difference being that the low of the U shape occurs at 1960.

[3]Cohort data also confirms this pattern. See Bosveld (1996), Figure 5.2: Proportions of women with at least one child at age 25. We observed that in most European countries, there is a lower proportion of women with at least one child in the 1940 cohort, in comparison with the 1945 cohort. The proportion decreases after the 1945 cohort. Therefore the 1945 cohort constitutes the bottom of the U shape curve. US cohort data also confirms the same pattern. SeeBloom (1982) Table 1.

There is a huge literature attempting to explore fertility rates and demographic transition since they are believed to be related to economic growth, labor force participation and public policies. In contrast, the influence of childbirth timing is less studied. However, as pointed out in the literature related to the "tempo of fertility", the age at first birth is an important factor affecting the period fertility rate (Morgan (1996); Hopflinger (1984); Cigno and Ermisch (1989)), and is the main explanatory variable for the decrease in fertility in European countries (Gustafsson (2001)). Once we are concerned with economic issues related to demography, the forces involved in the timing of birth are worth taking into account.

Instead of examining the whole historical pattern, the existing literature related to motherhood timing focuses on answering the question, why do mothers wait? However, these answers are difficult to provide a consistent explanation for why mothers advance the timing during 1950 to 1975. For instance, in the existing framework, the females' human capital and wage rate at marriage are the main explanatory variables for motherhood timing.[4] However, the empirical evidence suggests that marriage age has a positive correlation with the mother's age at first birth,[5] which implies that marriage age and birth timing are both governed by the same forces in such a way that one does not provide a satisfying explanation of the other. On the other hand, human capital at marriage age, including years of schooling and work experience, is identified to postpone the maternal age. Given the fact that both the years of schooling and female labor force participation increase along the growth path, it is not clear why the U shape pattern would appear. Therefore, it seems convincing that there exists a variable that explains all these stylized facts. We consider the underlying reason for these as the growth of TFP.

Given that, in this paper the U shape pattern is explained by examining each cohort's decision over the time horizon with an exogenously given sequence of TFP. A representative agent for each cohort confronts a particular TFP level and its decision making is considered within a life cycle model. We construct the life cycle model by extending Ben-Porath's framework, in which the agent chooses to accumulate human capital and work. Each agent's years of schooling, human capital over the life cycle and age-earnings profile are endogenously

[4]The existing economic literature (see Cigno and Ermisch (1989); Hotz, Klerman, and Willis (1997); Walker (1995)) on the "tempo of fertility" in a life cycle framework shares some common assumptions: (1) A mother's human capital is accumulated through learning by doing, and the initial human capital at marriage is given. (2) The value of children is generated by child service, which is assumed to be proportional to the number of children. (3) Child care requires a mother's time and market goods, which are fixed parameters with respect to each child.(4) A mother decides her birth rate each period and the completed fertility is determined by the accumulated birth rate.

[5]In data covering the last thirty years the mean age of first marriage has a positive relationship with the mean age of first birth in Northern and Western European countries. U.S. data alos shows that the mean age of first marriage forms a U shape curve with the bottom in accordance with the bottom of mean age of the first birth.

determined for the given TFP. The basic model is extended to incorporate child rearing decisions. A mother allocates time and markets goods in rearing (investing in) a child, which determines child quality and its future value. A mother's life cycle problem is to find the best "space" to rear a child, in which she optimizes her discounted utility from consumption and future value of her child.

Two novel aspects of our model are that we consider the mother's decisions on human capital accumulation *before* motherhood and the substitutability of market goods for a mother's time in child quality formation. In the existing research, human capital accumulation is modeled as "learning by doing". The opportunity cost of rearing a child includes forgone earnings during motherhood, and forgone earnings after motherhood due to lost experiences (Cigno and Ermisch (1989); Hotz, Klerman, and Willis (1997)).[6] Specifically, the time spent on children only has an impact on a mother's human capital through loss of job experience. Thus, it is implicitly assumed that a female's decisions regarding human capital accumulation before motherhood are the same as a female who does not even have a child. This is not consistent with the empirical finding that timing of motherhood is positively correlated with education attainment (Bloom (1982, 1984); Blackburn, Bloom, and Neumark (1993)). By relaxing this assumption, our results demonstrate that child rearing not only lowers a mother's human capital accumulation, including years of schooling and on-the-job training before motherhood, but also the human capital investment after child rearing. In addition, depending on the age at childbirth and the TFP level, impacts to human capital accumulation over the life cycle are different, and the implied opportunity cost of child rearing is also different. On the other hand, by considering substitutable inputs for child quality formation, a mother's optimal child rearing time depends on the level of human capital over the maternal periods, which are also associated with the childbirth timing and the TFP level. These considerations jointly determine each cohorts decision and explain the U shape patterns as follows: the early cohort prefers to have a child at a late age, because the shadow price of rearing a child is lower; a mother can rear a child with the "best quality" without sacrificing her utility so much. As TFP grows, the force determining early cohort's decision becomes weaker. A female chooses to have child at early age because the forgone human capital acquisition is lower; this decision makes mothers attain higher earning profiles over the life cycle. As TFP continuously grows, females' decision on childbirth timing postpones because the required years of schooling and intensity on the training prolongs; the optimal maternal

[6]Blackburn, Bloom, and Neumark (1993) consider a mother's joint decision on human capital accumulation and first childbirth timing before child bearing. A mother can choose either to invest or not to invest in human capital before childbearing. This decision is determined by the variance of women's preferences toward early childbearing. Their model implies late childbearers tend to invest more in human capital than early childbearers.

periods should avoid occupying a mother's "golden time for her human capital formation"

The remainder of this paper is organized as follows. In the next section I present our model; I start from incorporating a worker training subsidy into the standard Ben-Porath framework. Based on this extended Ben-Porath model I consider a female's life cycle by incorporating child birth and child raising. In section 3 I discuss our calibration methods. In section 4 I present the results and discuss driving forces associated with the timing of motherhood. This section closes by explaining the U shape pattern. In section 5, I examine the forces which are implied in this model, but could be crucial to mothers timing decision. In section 5 I provide a conclusion and directions for future work.

2 The Model

To explain the historical pattern of females' motherhood timing decisions, I examine the decisions of each cohort, in a non-stochastic, partial equilibrium framework. Vintage human capital is assumed in our economy, that is, each cohort faces a particular TFP, w, which is interpreted as the return on human capital. Each cohort only differs in w, which is exogenously given, but except this all the other economic variables are identical. In the following exposition, I start with examining a Ben-Porath life cycle framework incorporating subsidy program for job training in the discrete time version. Next, I incorporate the child rearing decision to consider a female's problem.

2.1 A Ben-Porath life cycle framework with subsidy program for job training

A representative agent of a particular cohort maximizes the present value of discounted utility, with discount factor β, from consumption over the life cycle, given the endowment on human capital h_I, and the return on human capital the agent confronts w. We assume that agent's decisions start at age I, and ends in age T, which is the life expectancy. In each period there is a fixed amount of time, which is normalized to be 1, and the agent allocates a fraction of time n_i to produce human capital and another fraction v_i to work. The decisions on human capital investment and working proceed until the retirement period R. Agent can borrow and lend without constraint, at interest rate r. After retiring, the agent is not allowed to work and produce human capital, she/he uses the accumulated wealth to finance the consumption over periods before the end of life.

Human capital is produced by existing human capital h_i, time n_i and market goods x_i with decreasing returns to scale ($\gamma_1+\gamma_2 < 1$) and accumulates over time with the depreciation rate δ. The goods used to produce human capital are identical to the goods used for consumption, whose price is normalized to be 1. The human capital production function and its law of motion of are given by,

$$
\begin{aligned}
F(n_i, x_i) &= z_a(h_i n_i)^{\gamma_1} x_i^{\gamma_2}, &(1)\\
h_{i+1} &= F(n_i, x_i) + (1-\delta)h_i. &(2)
\end{aligned}
$$

If the time allocated in working v_i is greater than 0, it is defined as agent's working periods. Agent earns wh_iv_i by renting human capital and time to the firm, and a subsidy on human capital investment η_i. Given h_I and w and subsidy program $\{\eta_i\}_{i=I}^{R}$, agent's maximization problem is,

$$
J(h_I, w) = \max \sum_{i=I}^{T} \beta^{i-I} u(c_i), \tag{3}
$$

subject to (1), life time budget constraint (4) and time constraint (5),

$$
\begin{aligned}
\sum_{i=I}^{T} \frac{c_i}{(1+r)^{i-I}} + \sum_{i=I}^{R} \frac{x_i}{(1+r)^{i-I}} &= \sum_{i=I}^{R} \frac{wh_iv_i}{(1+r)^{i-I}} + \sum_{i=I}^{R} \frac{\eta_i}{(1+r)^{i-I}}, &(4)\\
n_i + v_i &= 1 &(5)
\end{aligned}
$$

by choosing the sequences of allocations, $\{c_i\}_{i=I}^{T}$, $\{x_i\}_{i=I}^{R}$,$\{n_i\}_{i=I}^{R}$, $\{v_i\}_{i=I}^{R}$.

In contrast to the standard Ben-Porath framework, we assume that during an agent's working periods, there is a subsidy program paying for worker's human capital investment, which could be provided by either government or the firm. The interpretation of this program is that agents' human capital accumulated during the working periods is usually firm specific, and directly (on-the-job training, off-the-job training) or indirectly (experiences) provided by the firm. On the other hand, firms may obtain a transfer on the expenditures incurred on training through tax cut. Even though some of the training cost may be reflected on the workers earning, in the model without subsidy, a worker actually pays both time and market goods for his/her human capital accumulation. Therefore, this assumption makes it clear that some proportion of the training will be firm provided. We define firms' subsidy scheme η_i as follows:

1. The payout for worker η_i is based on the workers' observable characteristic, the cohort specific return on human capital w , age i, and human capital h_i. It is paid as a lump-sum.

2. Assume η_i is efficient in that it exactly compensates for optimal investment in human capital, given agent's maximization problem, and this payment only applies for the full

time worker,

$$\eta_i = \begin{cases} x^*(i,w) & \text{if } v_i \geqslant 0.5 \\ 0 & \text{if } v_i < 0.5 \end{cases} \tag{6}$$

This subsidy program implies that firms have the knowledge of agent's maximization problem in which η_i takes as given. Given that, firms know agents' policy function of expenditure of market goods on human capital accumulations, and the policy function of labor supply. It also implies that firm can distinguish if an agent deviates to the optimal decision, based on the observable characteristic, (i, w, h_i).

2.2 The female's problem: extending a standard model to incorporate child birth and child raising

We now incorporate child birth decision into the standard model described above to implement a female's problem. The timeline of a female's decision making over the life cycle is illustrated in Figure 2. A female's life cycle decision starts at age S, the age that the woman starts to consider child birth. As in the standard model, a female works and accumulates human capital until age R, and the life ends in age T. Between age I and age S, a female's decisions are identical to those implied by the standard model. *i.e.* as if decision making did not consider future child birth. Over the female's life cycle, there exists a time interval $[\underline{U}, \overline{U}]$, in which she can choose a timing B to gives birth to a child. We assume that a female can only have one child in her life, and marriage comes with the child birth decision. Once having a child, the female must rear the child for I periods. During periods of child rearing, the mother has three uses for her time. In addition to work time v_i and human capital accumulation time n_i, the mother allocates time to child rearing l_i. Before child birth, and once the child reaches age I, $l_i = 0$ as in the standard life cycle model. The period in which child rearing is completed, a mother receives a payoff J based on the child's future value, which has been defined in agent's maximization problem in Section 2.1. *The female*'s problem is no longer just to maximized the present discounted value of utility lifetime consumption, U, but to maximize the sum of this discounted value U and the discounted child value J, the latter with discount factor β_c.

$$V(h_S, w) \equiv \max \left\{ \sum_{i=S}^{T} \beta^{i-S} u(c_i) + \beta_c^{B+I-S} J(h'_I, w'(B)) \right\} \tag{7}$$

The future value of a child $J(h'_I, w'(B))$ is determined by TFP $w'(B)$, the child's gen-

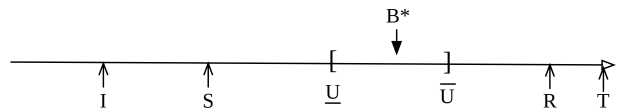

Figure 2: timeline of female's life cycle

eration faces, and the child's initial human capital h'_I at age I. When children start their life at age I, they will make their own life cycle decisions described in (3), based on $w'(B)$ and h'_I. These two variables are determined by a mother's decisions. The TFP level a child confronts $w'(B)$, is related to a mother's decision on child birth timing, B. Given that the TFP continuously grows, a child birth at later of mother's age will confront a higher TFP than that the early child confronts. A child's initial human capital h'_I is formed by a mother's production of child rearing goods, Q_i, during the child rearing periods. It is produced by mother's time l_i and market goods z_i, as shown in (8); and been accumulated following (9).

$$Q_i = G(l_i, z_i) = z_c l_i^{\alpha_1} z_i^{\alpha_2}, \tag{8}$$

$$h'_I = \min\{Q_B, Q_{B+1}, ..., Q_{I+B-1}\}. \tag{9}$$

As with agent's human capital, Q_i is produced with decreasing returns to scale ($\alpha_1 + \alpha_2 < 1$). The functional form is a special case of the child human capital production function in Cunha and Heckman (2007). We impose the following two assumptions; the first assumption is that, child rearing, Q_i is equally important in producing h'_I over all periods, *i.e.* the elasticity of substitution across periods is 0, so the child's human capital formation is assumed to be the Leontief form as seen in (9). The second assumption is that, a mother's human capital is not involved into child's human capital production. These two assumptions are made for the following reasons: under the Ben-Porath framework, an agent's life cycle human capital varies a lot over the age 20 to 30, so as the earning profile. The first assumption prevents a mother from allocating resources in certain periods with lower opportunity cost; on the other hand, if child human capital relates to mother's human capital, then so as the child's future value. The optimal timing decision would be determined by the timing at which mother's human capital is higher. Empirical evidence based on micro data demonstrate child's future performance is affected by mother's education level. However, it is not clear this effect holds within a female's life cycle.

The representative agent's (mother's) maximization problem is defined in (7), by the choices of $\{c_i\}_{i=S}^T$, $\{h_{i+1}\}_{i=S}^T$, $\{x_i\}_{i=S}^T$, $\{n_i\}_{i=S}^T$, $\{v_i\}_{i=S}^T$,B, h'_I, $\{l_i\}_{i=B}^{B+I-S}$, $\{z_i\}_{i=B}^{B+I-S}$ subject to

$$\sum_{i=S}^{T}\frac{c_i}{(1+r)^{i-S}}+\sum_{i=S}^{R}\frac{x_i}{(1+r)^{i-S}}+\sum_{i=B}^{B+I-S}\frac{z_i}{(1+r)^{i-S}}=A_S+\sum_{i=S}^{R}\frac{wh_iv_i+\eta_i}{(1+r)^{i-S}}, \tag{10}$$

$$B\in\left[\underline{U},\overline{U}\right], \tag{11}$$

$$n_i+l_i+v_i=1,\forall i\in[S,R], \tag{12}$$

$$l_i=0,\forall i\in[S,B-1]\cup[B+I,R], \tag{13}$$

where A_S is the asset holding at age S and $h_S, A_S, w, w'(B)$ and $\{\eta_i\}_{i=1}^{R}$ are given.

2.3 Efficiency Conditions

The equilibrium conditions regarding a mother's human capital accumulation are,

$$u'(c_i)=u'(c_{i+1})=u'(c^*)\ \forall i\in[S,T], \tag{14}$$

$$wh_i\leqslant\frac{\gamma_1}{\gamma_2}\frac{x_i}{n_i}, \tag{15}$$

$$\frac{u'(c_i)}{F_{x_i}}=\frac{1}{(1+r)}\left\{\left[w(1-l_{i+1}-n_{i+1})\right]u'(c_{i+1})+\left[F_{h_{i+1}}+(1-\delta)\right]\frac{u'(c_{i+1})}{F_{x_{i+1}}}\right\}, \tag{16}$$

and $l_i=0,\forall i\in[S,B-1]\cup[B+I,T]$.

Equilibrium condition (14) is the Euler equation of determining intertemporal consumption. With the assumption $\beta(1+r)=1$, (14) implies lifetime consumption is constant. Equation (15) shows that the intratemporal marginal rate of substitution between market goods and time spent on human capital accumulation is greater than or equal to the marginal gain from labor supply. Strict inequality can hold, which means the value of investment is greater than the value of working. In the beginning of the life cycle, h_i is low. However, the time spent on investment n_i is bounded above by 1. In this case, the agent specializes her time in human capital accumulation *i.e.* $v_i=0$. We refer to these accumulation periods as years of schooling. As human capital accumulates over time, n_i will eventually become less than 1. The periods in which $v_i>0$ we refer to as working periods. This feature of the model, that agents specialize in producing human capital in the beginning of the life cycle, comes from the assumption that the human capital production function exhibits decreasing returns to scale.

The LHS of (16) is the marginal cost of investment in one unit of human capital in terms of utility, and the RHS is the marginal gain. The marginal gain of investment on human capital comes from the wage return, $[w(1-l_{i+1}-n_{i+1})]$ and returns from a higher level of human capital in next period, $\left[F_{h_{i+1}} + (1-\delta)\right] \frac{u'(c_{i+1})}{F_{x_{i+1}}}$. Note that both of the RHS terms are measured in terms of utility. Equation (16) implies that if a mother spends time in child rearing the next period, the marginal return of human capital investment becomes smaller. Therefore child birth in the future will lower human capital investment for all periods before maternity.

Equilibrium conditions regarding the allocation of resources and the quantity of child rearing goods are given by,

$$
\begin{aligned}
h'_I =& \overline{Q} = G(l_i, z_i), && (17)\\
wh_i \leqslant \frac{\alpha_1}{\alpha_2}\frac{z_i}{l_i} =& \frac{\gamma_1}{\gamma_2}\frac{x_i}{n_i}, && (18)\\
\beta_c^{B+I-S} J_{h'_I} =& \frac{u'(c^*)}{(1+r)^{B-S}}\left[\frac{1}{G_{z_B}} + \frac{1}{G_{z_{B+1}}(1+r)} + ... + \frac{1}{G_{z_{B+I}}(1+r)^{I-S}}\right], && (19)\\
\sum_{i=S}^{T}\frac{c^*}{(1+r)^{i-S}} =& \sum_{i=S}^{R}\frac{wh_i + \eta_i - (wh_i n_i + x_i)}{(1+r)^{i-S}} - \sum_{i=B}^{B+I-S}\frac{wh_i l_i + z_i}{(1+r)^{i-S}}, && (20)\\
\text{and } B^* =& \text{argmax } V(\cdot|B) \;\; \forall B \in \left[S, \overline{U}\right] && (21)
\end{aligned}
$$

Equation (17) comes from the Leontief form of the child quality equation (9). In every period child specific investment is equally important, so that mothers will provide an identical quantity of child rearing goods. Equation (18) shows that the intratemporal marginal rate of substitution between market goods and time associated with human capital accumulation equals that associated with child rearing. Equation (19) is the condition which determines optimal investment in child rearing. The LHS is the present value of marginal gain from raising a child with initial human capital, h', and the RHS is the discounted marginal cost in terms of utility. In order to raise h', a mother needs to spend $1/G_{Z_B}$ units of market goods for all I rearing periods. (20) is the lifetime budget constraint. Equation(21) is the optimal condition for B^*, the optimal age for child birth. B^* is a value in $[\underline{U}, \overline{U}]$, at which (14) through (20) are satisfied and $V(h_S, w)$ is maximized.

3 Calibration

We use standard functional forms. The utility function of a mother is assumed to be CRRA

$$u(c_i) = \frac{c_i^{1-\sigma}}{1-\sigma}. \tag{22}$$

There are total six sets of parameters: They are parameters regarding to

1. economic environment: σ and r

2. human capital production: z_a, γ_1, γ_2 and δ,

3. the economic growth pattern: $\{w_t\}$, $t \in [1950, 2000]$

4. agent endowment and constraints: h_I, S, h_S, A_S, $\underline{U}$, $\overline{U}$, I, T

5. child value function $J(h', w')$ and β_c

6. child rearing: z_c, α_1 and α_2.

Table (1) summarizes the parameters of the calibration. These values are sequentially determined by following strategy:

1. For the economic environment parameters, we follow the values widely used in the previous literature. The risk aversion coefficient σ is 0.5. It is reported in recent studies that this value should around 0.5. (Cox and Oaxaca (1996): 0.67;Goeree, Holt, and Palfrey (2002): 0.52; Chen and Plott (1998): 0.48.) Following Walker (1995), we set the value of discount rate, r, to be 0.03.

2. For the second set parameters, we set the value z_a, γ_1, γ_2, δ to be 0.361, 0.60, 0.33 and 0.018 respectively, following (Manuelli and Seshadri (2006, 2009)).[7]

3. The economic growth path from year 1950 to 2000 is characterized by the sequence, $\{w_t\}_{t=1}^{t=T}$. Assuming that an agent's human capital at age 6, h_I, is the same over all generations.[8] These $T + 1$ parameters will be determined together by the moments below:

   (a) In year 2000 (the end year), school life expectancy is 17.333.[9]

   (b) In year 1985, school life expectancy is 13.333.

   (c) 2% trend in GDP growth rate.

[7]InManuelli and Seshadri (2006, 2009), the value of γ_1 , γ_2 are chosen to fit the age earning profile. Moreover, the sum of γ_1 and γ_2 , 0.93, is close to the value reported in the empirical literature. The value we choose here keep the sum of γ_1 and γ_2 to be the same, but slightly different from the parameters they use, that$\gamma_1 = 0.63$, $\gamma_2 = 0.30$. Our consideration is that if γ_1/γ_2 ratio is high, cohorts' years of schooling would grow faster, and the peak of late cohorts' earnings profile would be postpones, which lead the difficulty to find a benchmark age to pin down the TFP sequences.

[8]This assumption also implicitly restrict the child initial human capital generated by mother should be h_I.

[9]School life expectancy in year 2000 : 17.333
(source: UNESCO. See http://devdata.worldbank.org/edstats/cd5.asp in detail). It is the average of the following countries : Belgium, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Netherlands, Norway, Sweden, United Kingdom, and United States.

We use the earning of each cohort at age 36 as an index of the output each year. The earning of cohort t at age i is defined as follows:

$$\begin{aligned} e_i(i,t) &= w_t h_i v_i - x_i + \eta_{i,} \\ \text{where } \eta_i &= \begin{cases} x & \text{if } v_i \geqslant 0.5 \\ 0 & \text{if } v_i < 0.5 \end{cases} \\ v_i &= (1 - n_i - l_i). \end{aligned} \tag{23}$$

The 2% growth rate implies,

$$\frac{e(i,t+1)}{e(i,t)} = 1.02, \ \forall t \in [1, T-1]$$

Last, conditions (a) and (b) pin down the beginning and ending point of of the $\{w_t\}$ sequences.

4. We choose the age start to considering future maternal decision S to be 16. The fecund age is restricted between 18 and 30. ($i.e$ $\underline{U} = 16$, and $\overline{U} = 30$) and the child rearing periods I to be 6 years. The age at retirement T to be 64. We set up the value of initial human capital h_I to be 7.0 for all agents, including mother and child. Based on our model, h_I is an endogenous variable depends on mother's investment on child. Our calibration strategy is set up an initial value first, and it will be verified later by mother's optimal decision. Two parameters related to a mother's endowment, h_S and A_S, are endogenous determined and different across cohort. In our model setting, mother's decision-making starts from age S. Before S, the mother's life maximization problem will not consider the child rearing decision. Noted that, this setting implies even if we assume the human capital at age 6 to be the same across cohorts, the h_S is different given different w. Given all the parameters determined in the above steps, h_S, A_S for each cohort can be computed.

5. In our model setting, the agent's life value is determined by the state variables TFP that the agent confronts, w, and initial human capital, h_I. To facilitate solving mother's marginal decision on child human capital investment, (19), we approximate child value as the functional form,

$$J(h', w') \approx a_0 + a_1 (w'h')^{a_2}. \tag{24}$$

We use numerical methods to approximate these three values by following steps: First, given N_h sets of initial human capital h_I and N_w sets of TFP values, solve the agent's life cycle model without child, defined in Section (2.1), compute each optimal lifetime value, $J(h', w')$. Using these total $N_w \times N_h$ sets of results, run a non-linear regression as form (24) to estimate a_0, a_1 and a_2. We choose N_w to be 91 and N_h to be 40.[10] As for the value of β_c, we will pick a number close to 1. However, we impose a constraint on the value β_c. Following the existing literature, we assume mothers prefer to have the child earlier, given child quality is indifferent to the birth timing and mother's input. To characterize this assumption, we define a function Γ,

$$\Gamma(B|\overline{h}, \beta_c, w(B)) = \beta_c^{B-S} J(\overline{h}, w(B)).$$

$\Gamma(B)$ has to be a decreasing function in B, given β_c, $\overline{h}$, the approximated functional form of J, (24), and TFP sequences $\{w_t\}_{t=1950}^{2005}$. That is, β_c has to be greater than the growth rate of w.

6. The last set of parameters z_c, α_1 and α_2 are determined by the following three moments: time spent on child, the goods expenditure on child to household expenditure, and the initial human capital, h_I, we set up. Sayer, Bianchi, and Robinson (2004) reports 1.58 to 2.2 hours per day for the survey in 1998.[11] Rothe, Cassetty, and Boehnen (2001) report different value of the ratio of expenditure on child to household based on different estimations: it is around 25% - 33% of family expenditure per child.

parameters	z_c	α_1	α_2	h_I	a_0	a_1	a_2	β_c
value	7.85	0.070	0.053	7.00	3.3037	31.7413	0.4100	$\frac{1}{1.0059}$

Table 1: Parameters

4 Results and Discussion

The baseline results are presented in Figure 3. It plots the mean age of motherhood and year of schooling from 1950 to 2010, based on the calibration results of cohort's optimal decisions. Each point is calculated by taking the average of ages of all mothers who decide to have the childbirth at that certain year. For instance, in year 1950, there could be two or more

[10] We tried other functional form to approximate the child value function, ex: $J = a_0 + a_1 w^{a_2} h_I^{a_3}$. The parameters value are quite different, but it does not affect the results and conclusion we present in next section.

[11] Bryant and Zick (1996) use different data sets reports that the average hour working female spend on child per day is 1.13 hour.

cohorts which have their first childbirth. We average over these cohorts' ages, and report this value as the "mean age of mother at fist childbirth" for that year. The mean value of some years are missing, for instance 1987-1992, due to the discrete time model structure. We report two different values of the mean age of first child birth based on different upper bounds for motherhood $\overline{U}$: the square dots are the case when $\overline{U}$ is 32; and the diamond dots represent the case when $\overline{U}$ is 30. The mean years of schooling is depicted by the star dots. Each value comes with a curve to describe the trend.[12] From Figure 3, it is observed that our results successfully capture the U shape pattern for both upper bounds, and match the corresponding years of schooling fairly well.

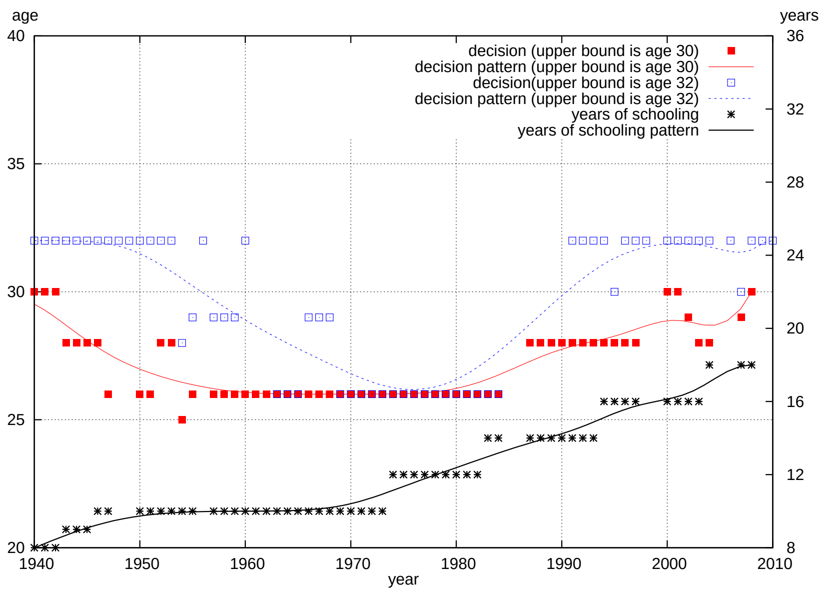

Figure 3: Mean age of motherhood (calibration results)

For both cases the results capture the pattern but do not perfectly match the data, as reported in Figure 1. When $\overline{U}$ is 30, the beginning of the trend starts at age 26 and decreases to the low during 1970s and grows up to 29. Compared to the data, motherhood age is relatively higher during the periods before 1975. When $\overline{U}$ becomes 32, the pattern does not change, but both the beginning and the ending of the pattern increase up to 32, and the low still stays at 26. We present this case by relaxing the upper bound $\overline{U}$ to see if the trend possibly increases as TFP grows. Our model prediction confirms this.

[12] A Bézier curve is used to capture the trend of optimal decisions.

4.1 Results discussion

We present the cohorts decision in Table 2. The cohorts starts from the mother over time horizon who have child at 1950, and ends in the cohort who has her first birth at 2000.[13]

Our calibration target includes the ratio of child expenditure to family expenditure, child rearing time and child initial human capital, which is set to be 7.0. All these values are reported in Table (2). Note that, some of the targets does not match precisely. The column labeled $(\frac{z}{c^*})$ is the ratio of expenditure on a child to mother's consumption, which implies child expenditure to the household expenditure is 35% - 36.7% for cohorts between 1970 to 2000. There is a difference between data suggested, that this ratio is around 33%. As for the time spent on a child $(\bar{l}_i)$, empirical evidence suggests mothers spend around 1.58-2.2 hours per day based on the survey in 1998. The ratio of child rearing time to work would be around 0.1975-0.275, which is close to our result 0.18.[14] The column labeled Q is the child rearing goods produced by a mother, it is close to 7.0, the value of initial variable we set.[15]

The column labelled $(\overline{z})$ and $(\bar{l})$ show the average time spent and expenditures of market goods in rearing a child. As TFP grows, a mother tends to spend more market goods to substitute for time input in rearing her child. The reason is clear in our model: The salary forgone for one unit of time in child rearing is wh_i. A high w induces a high sequence of human capital, $\{h_i\}_{i=B}^{T}$, over the life cycle. A mother who confronts a high TFP would like to use market goods to replace the time input. This finding contradicts what recent literature (Sayer, Bianchi, and Robinson (2004); Sandberg and Hofferth (2001); Bryant and Zick (1996)) suggests, that the historical trends in time spent caring for children is increasing. However, during 1950-1970, the total fertility rate in the developed countries is higher than that in the last 20 years. The total resources that a mother is willing to spend on children should be higher than the value that literature suggests. Given the restriction in our model that mother can only have one child, she may spend less time on each child.

4.2 Discussion of the driving forces: explicit cost and shadow price of child rearing

Although the equilibrium quantities are endogenously determined, the following discussion tries to isolate the driving forces implied by our model. Optimal timing is determined by

[13] Note that the cohort decision present here is not based on the expected child birth timing, but based on chronological order.

[14] The ratio reported from Sayer, Bianchi, and Robinson (2004) is the average value in year 1998. We use the decision of cohort 1995, who deliver the child in year 1998, to calculate this ratio $(\frac{0.167}{0.833})$.

[15] Even though the value is slightly larger than our target, however, if we restrict mother's investment to be a constant $\overline{Q}$=7, our results of the optimal timing decision still close to the one reported in Figure 3.

Table 2: Baseline Results; The cohort of a particular year represent a female at age 23 at that year.

cohort	B^*	years of schooling	$\frac{\bar{z}}{c^*}$	$\bar{z}$	$\bar{l}$	Q*
1950-1960	27.09	11	80.0%	1.44	0.41	7.52
1960-1970	25.91	12	68.3%	1.41	0.31	7.37
1970-1980	26	14	58.7%	1.39	0.23	7.22
1980-1990	27.55	15.6	54.5%	1.48	0.20	7.15
1990-2000	29	17.2	58.3%	1.79	0.18	7.19

maximizing both mother's lifetime utility, U, and the child value J, in which a trade-off exists. The discussion starts with driving forces regarding to the each parts of the shadow prices of child rearing associated with timing decision.

4.2.1 Expenditure of child rearing and the timing of motherhood

The total cost associated with rearing a child includes two parts: the expenditure for market goods and the shadow price to child rearing time. Although the total cost depends on the child investment decision, Q, the expenditure also differs with the age of motherhood. Suppose that a mother needs to spend P in period B, Then, if she chooses to postpone the child birth timing to $B+t$, she can obtain the interest return in the market $P(1+r)^t$. Thus, the decision of birth timing affects lifetime wealth, and the mother's lifetime consumption and child investment.

This income effect is closely related to the finding that the fixed expenditure on children delays the age at maternity, which is documented in the literature (Happel, Hill, and Low (1984); Hotz, Klerman, and Willis (1997); Cigno and Ermisch (1989)). Although our model does not explicitly incorporate such a fixed cost, this income effect still affects optimal timing decisions.

4.2.2 Forgone human capital accumulation and the timing of motherhood

Existing theoretical research models the human capital based on learning by doing, and the initial human capital at marriage is exogenously given. (Cigno and Ermisch (1989); Blackburn, Bloom, and Neumark (1993); Hotz, Klerman, and Willis (1997)) These frameworks consider forgone human capital accumulation based on the loss of working experience during motherhood. (*i.e.* for the periods over the life cycle except motherhood, decisions on human capital investment won't be affected.) In contrast, our model captures a females' decisions on working and education during the early life cycle in response to *future* maternal decisions. The equilibrium results demonstrate that time spent on a child has great impact on a mother's human capital investment over the life cycle. Moreover, it affects the earning profile. We conclude that there exists two opposite forces: one force advances while the other postpones the timing of child birth.

The force advancing the timing of child birth comes from the fact that a mother who has child later has a relatively lower human capital pattern during the end of her life cycle. It implies lower earnings during these periods, and leads to a lower consumption level over the life cycle. This force is explained formally as follows: First, suppose the equality of (15) holds, substituting condition (14) and (15) into (16), we have

$$\frac{1}{F_{x_i}} = \frac{1}{(1+r)}\left\{w(1-l_{i+1}) + \frac{(1-\delta)}{F_{x_{i+1}}}\right\}, \tag{25}$$

The RHS of (25) is the marginal return on human capital investment, which comes from two sources: the gain from wage return in the next period (the first term on the RHS) and the gain from the future returns on human capital (the second term). (25) implies that if a mother will spend time in rearing a child at age $i+1$, the marginal return from investment in human capital decreases (the first term in the RHS). In equilibrium, the value of $1/F_{x_i}$ becomes lower. At period $i-1$, the marginal return from human capital investment also decreases because the return on human capital in the next period, the second term in RHS, becomes lower. As a result, human capital production over all periods before motherhood decreases. Therefore accumulated human capital is lower as birth timing increases.

In addition, the effects stated above become weaker the further away a woman is from childbirth. Recursively substituting the RHS of (25), we obtain

$$\frac{1}{F_{x_i}} = \sum_{j=1}^{R-i}(\frac{1-\delta}{1+r})^{j-1}\frac{w}{(1+r)} - \sum_{j=B-i}^{B+I-1-i}(\frac{1-\delta}{1+r})^{j-1}\frac{wl_{i+j}}{(1+r)}. \tag{26}$$

As the birth timing B is further away from a mother's age i, the effect of child rearing time l_i (the second term in (26)) becomes smaller due to the discount term $\frac{1-\delta}{1+r}$. This effect can happen either early or late in a woman's life. A woman who does not have a child until age 32 will behave very much like a woman who will not have children, *until* she is close to motherhood. Similarly, a woman who has a child at age 18 will behave like a woman who does not have children *after* motherhood.

Moreover, the effect of child rearing on foregone human capital accumulation becomes weaker as TFP increases. We can see this by firstly considering the case in which there is no decision of child birth. In this case, the second term of RHS in (26) disappears. When the TFP (w) is high, an agent accumulates more human capital each period over the life cycle, since the marginal return is high. Thus, a later cohort has a higher human capital pattern over the life cycle since she faces a higher TFP. Given this conclusion, now we back to our model. That is, we consider the case in which an agent has the decision of child birth. (18) implies that given a higher h_i, child investment l_i is lower. Thus, for the later cohort, this forgone earning is less relevant (the second term in (26)), comparing with the future return (the first term of (26)).

The foregone human capital effect can be illustrated by our calibration results shown in Figure 4, 5 and 6. Theses Figures plot the human capital quantity for three different types of agent. The benchmark agent is the one does not consider child birth, as described in section 2.1, whose optimal decision on human capital is depicted in the smooth line. We also examine two different types of mothers (Young and Old) who are exogenously given the timing of motherhood. Their decision problem is solved optimally based on a given birth timing B. The Young mother is assigned to have a child at age 18, whose decisions are depicted in the line with square dots the area marked by <I> are her maternal periods. The Old mother is assigned a later age 32, whose decision is depicted in the line with triangle dots. The area marked by <II> are her maternal periods. Figure 4 plots the decisions of early cohort (with low TFP), Figure 6 plots decisions of a late cohort (with high TFP), and Figure 5 plots decisions of a cohort at 1975 (with TFP between low and high).

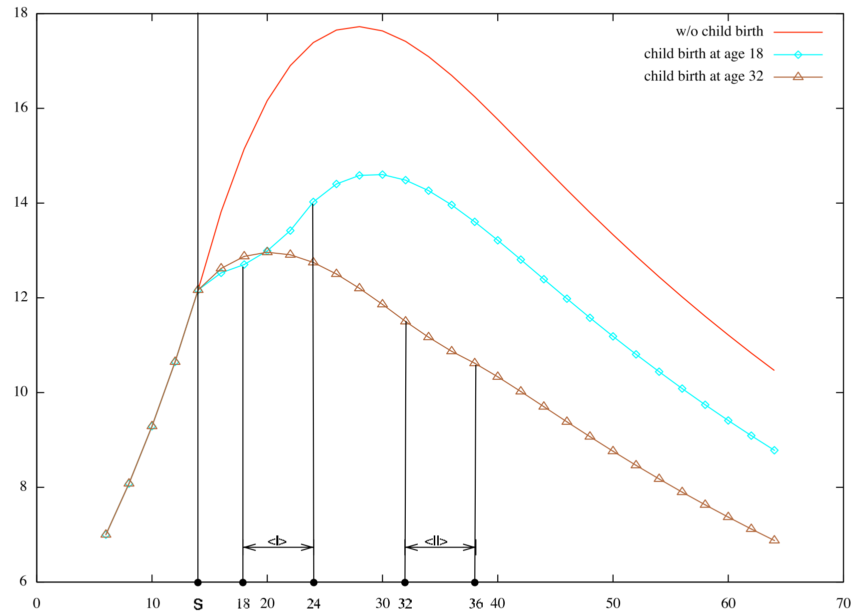

Figure 4: human capital over life cycle (TFP is low; year 1950)

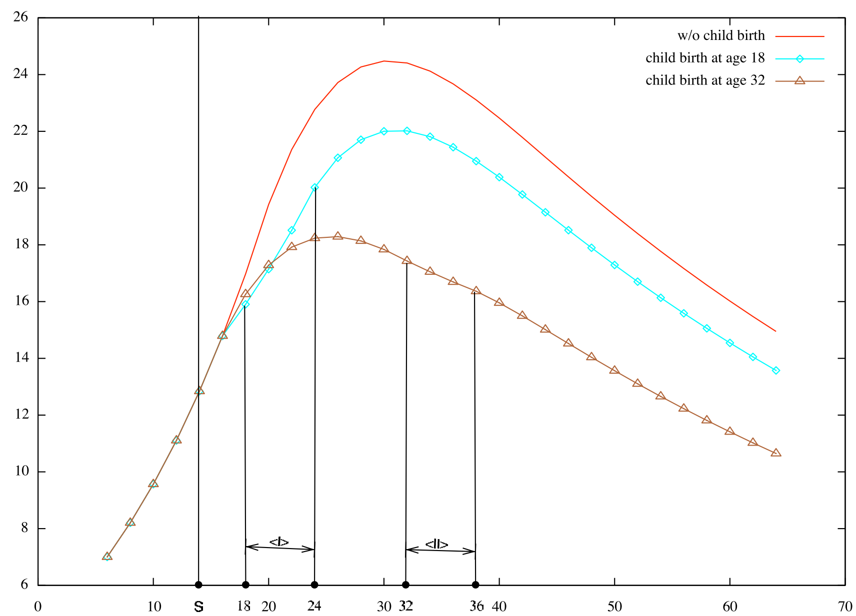

Figure 5: human capital over life cycle (TFP is in the middle range; year 1975)

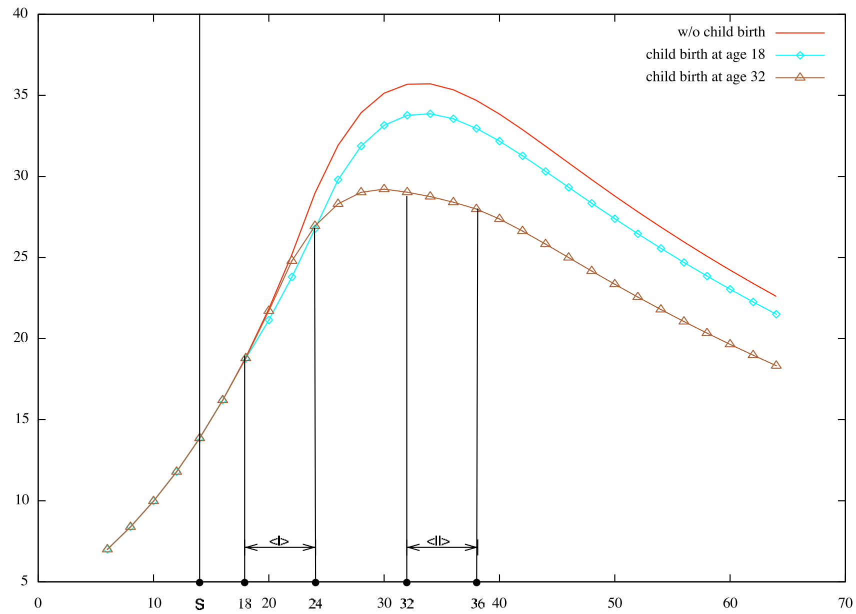

Figure 6: human capital over life cycle (TFP is high; year 2000)

A common finding from these three figures is that human capital of Young and Old mothers are lower than the benchmark agent, as concluded by equation (25). A Young mother is less affected by the time spent on a child due to a long future return. Therefore human capital foregone is relatively low. On the other hand, an Old mother has low human capital over the life cycle. This is because an Old mother's investment decisions take into in account the fact that human capital is underutilized during child rearing. We also observe that a Young mother has a relatively low h_i at the beginning (age 18), but she will catch up to the Old mother in later periods. The underlying reason for this observation is stated above, that child rearing time has a larger effect on human capital investment in periods closer to childbirth.

Another finding from Figure 4 to 6 is that the differences in human capital patterns between Young and Old mothers decreases as TFP grows. This observation verifies the discussion of (26): as TFP grows, the optimal level of l_i decreases, so the effect of child rearing on foregone human capital becomes smaller.

The different human capital profiles of Young and Old mothers have strong implications for earnings late in life. We illustrate this by using our calibration results, as shown in Figure 7 to 9. A female's earning is defined in equation (23). The notations in the Figure are the same as that used in Figure 4 to Figure 6. Observe that the Young mother has a higher earning profile in later life and this finding holds for all TFP levels, due to higher human capital late in life. Moreover, we find that the difference in earning profiles in later life between the two mothers decreases as TFP grows. This reflects the convergence of Young and Old human capital levels as TFP increases. These observations demonstrate the shadow price of time spent on a child in terms of forgone human capital. We conclude that the amount of human capital foregone is lower if mothers choose early child birth.

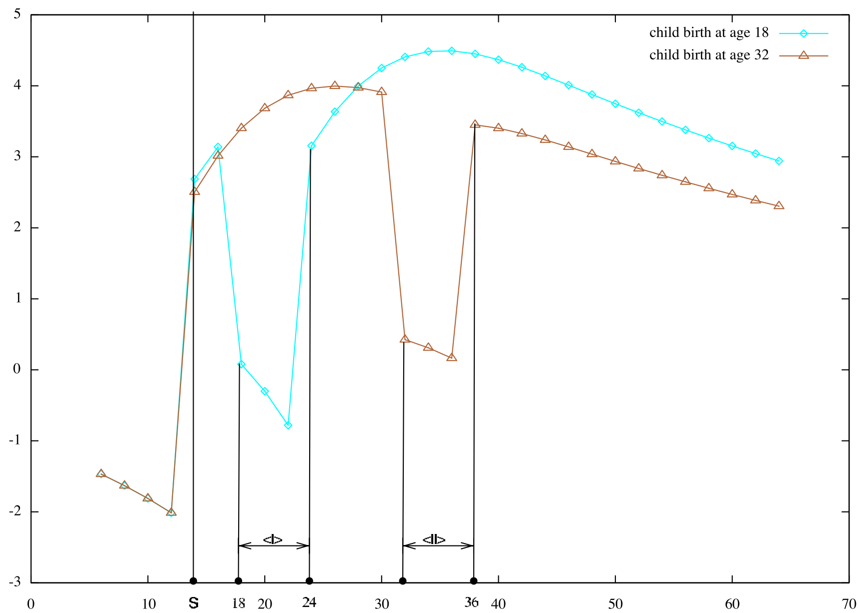

Figure 7: Earning profile over human capital over life cycle (TFP is low; year 1950)

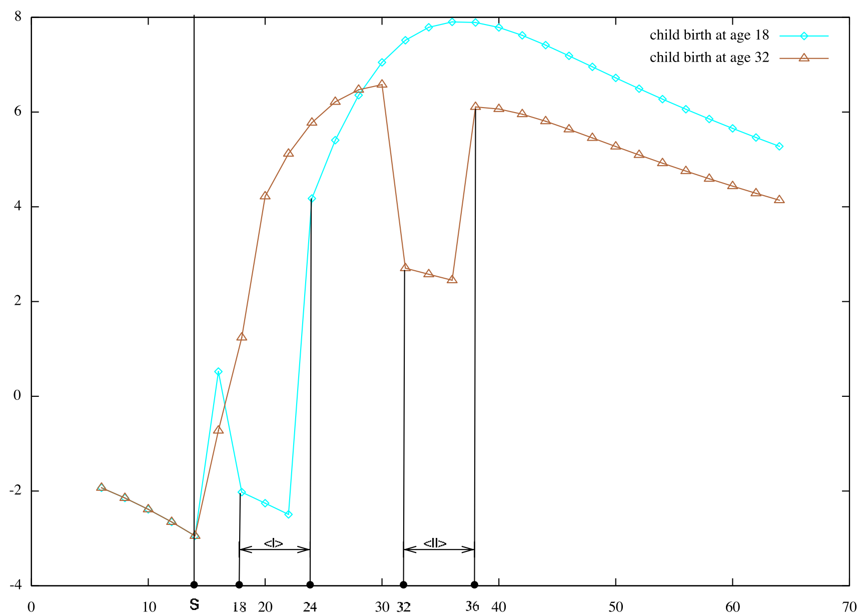

Figure 8: Earning profile over human capital over life cycle (TFP is in the middle range; year 1975)

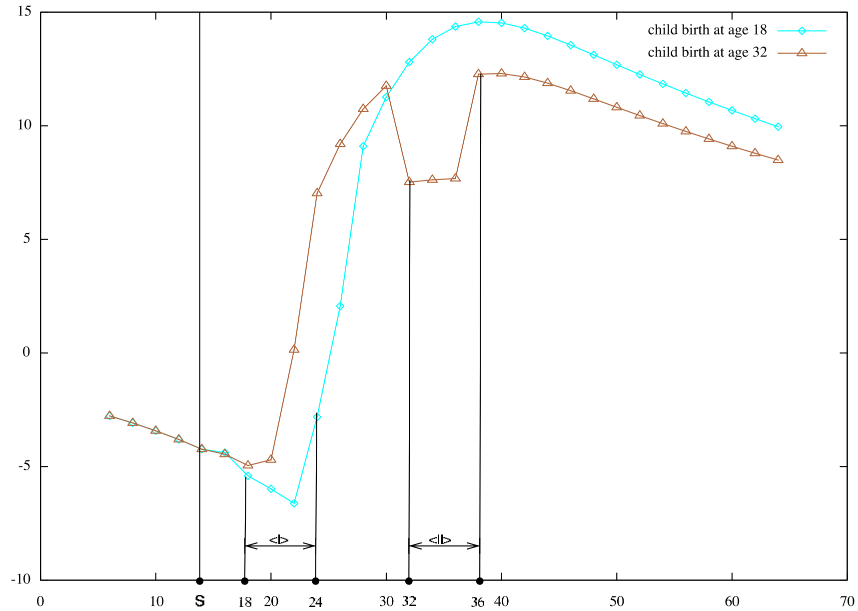

Figure 9: Earning profile over human capital over life cycle (TFP is high; year 2000)

On the other hand, there exists another force which delays childbirth timing. We first illustrate it by examining the Young and Old mother's human capital patterns (demonstrated in Figures 4 to 6) and the corresponding earning profiles (demonstrated in Figure 7 to 9). We find that although the Young mother's human capital is close or even higher than the Old mother's before age 30, the Young mother's per period earnings are significantly lower than the Old mother's. The variable causing the difference we would like to highlight here is the subsidy η_i. In our model only full time workers can obtain a subsidy for human capital investment. If women choose to have their children early, the time spent on child caring prevents her from being a full time worker ($i.e.$ $l_i + n_i > 0.5$). Moreover, this effect becomes stronger as TFP grows. When TFP is high, the required years of schooling ($n_i = 1$) and the periods of intensive on-the-job training (n_i close to 1) become longer (as shown in Figure 3). To have a child when young occupies time usage on n_i during these periods.[16] In this case, the Young mother delays the timing to becomes a full time worker, since she has to catch up the missing part of human capital accumulation during the motherhood. Given that, she has to pay the expenditure of human capital investments not only during motherhood, but also the periods after child rearing that she spends to catch up in human capital accumulation, at which $n_i > 0.5$.

[16] An extreme case is that the cohort 2000 choose have a child at age 18, while the supposed year of schooling should be until age 22.

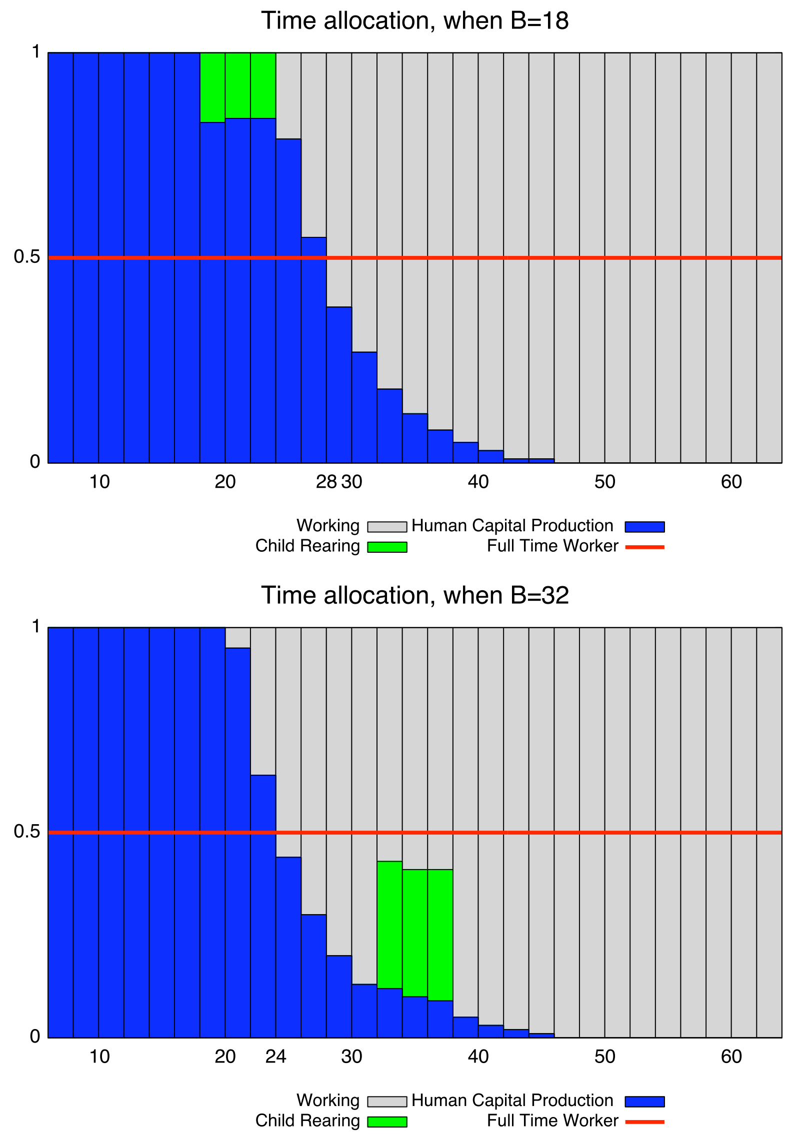

Figure 10: Time usage over life cycle (TFP is high; year 2000)

We use Figure 10 to demonstrate above statement. The Figure plots the time usage of two types of mother when TFP is high. If a Young mother chooses to have a child at age 18, her decisions l_i and n_i are squeezed by the time constraint (the inequality of (18) holds.). The loss of time during child rearing periods makes this mother spend a large proportion of time in human capital accumulation in the later periods, which leads her to become a full time worker later. For instance, in the case demonstrated in Figure 10, the Young mother becomes a full time worker four years later than the older mother.

The two driving forces discussed above have opposite effects. One force induces an agent to have a child early: after rearing her child a mother can concentrate on accumulating human capital and enjoying a higher future earning profile. On the other hand, having a child early makes the mother to become a full time worker later in the life and she will have to finance expenditure on human capital accumulation by herself for longer periods.

Another part of the shadow price of child rearing, which has been addressed much in the literature, is forgone earnings. Forgone earnings in our model is measured by the time spent on child times the wage profile $\{wh_i\}_{i=S}^{R}$. The human capital over the life cycle $\{h_i\}_{i=S}^{R}$ is endogenously determined by the choice of B. When mother chooses the optimal B, she does not only take into account forgone human capital, as discussed in above subsection, but also consider the forgone earnings. If the maternal periods locate in the low of $\{h_i\}_{i=S}^{R}$, a mother is willing to spend more time to rear a child with good quality due to a lower opportunity cost , and her consumption level also increases because of less forgone earnings. For instance, the Old Mother of 1950 cohort rear her child in periods with a relatively low wh_i among her life cycle. Comparing to the Young mother, her forgone earning is low. Given that the optimal decisions of B for the cohorts in 1950 to 1960 are at late fecund age, we find this forgone earning effect is important to determine the optimal childbirth timing.

4.2.3 The equilibrium result of optimal timing decision : an explanation of the U shape curve

The above discussion has demonstrated the implications and driving forces of the model. Based on these forces, we explain how the U shape pattern is formed in this subsection. We do so by examining the decisions making of three representative cohorts who have children at 1950, 1975, and 2000. To facilitate the explanation, we use Figures to examine the value of a mother's value function, $V(\cdot|B)$, the value obtained from the discounted consumption streams $U(\cdot|B) = \sum_{i=S}^{T} \beta^{i-S} u(c_i)$, and the discounted child value $\beta_c^{B+I-S} J(\cdot|B)$ of the representative agent over the entire fecund age. By exogenously fixing B, the Figures demonstrate how the timing decision changes the value of $U(\cdot|B)$, and $\beta_c^{B+I-S} J(\cdot|B)$, which contributes to the total value $V(\cdot|B)$. In the Figures, we present that differences of value functions for the agent whose age at child birth is B and those for agent with $B = 16$. (*i.e.* $V(\cdot|B) - V(\cdot|16)$, $U(\cdot|B) - U(\cdot|16)$, and $\beta_c^{B+I-S} J(\cdot|B) - \beta_c^{B+I-S} J(\cdot|16)$).

We start the explanation from the 1950 to 1960 cohorts. These cohorts' decisions are mainly determined by the shadow price of time spent on a child in terms of forgone earning. Since TFP is low, mothers' age profile of human capital is low. The optimal condition Eq.(18) implies that mothers would like to spend more time in investing her child. Therefore, the forgone earnings $\{wh_t l_t\}_{t=B}^{B+I}$ is high, comparing with the life time earnings $\{wh_t v_t\}_{t=I}^{R}$. This amout of "large" expenditure would be smaller if it is spent in the later life due to the income effect addressed earlier.

Moreover, due to the effects from forgone human capital accumulaiton, when a mother chooses a late age at childbirth, her human capital over the life cycle will be lower than otherwise and her earnings will be low late in her life (as shown in Figure 4 and 7). However, this decision makes her concentrate on working and obtain higher earnings early. Human capital accumulation can be completed to a certain level before having a child. During the child rearing periods, the human capital has begun decaying and is relatively low over the life cycle, as shown in Figure 4. The low human capital implies low forgone salary for each unit of time spent on child. On the one hand, she is willing to spend more time to rear a child to obtain a child with "high quality". On the other hand, her wealth is implicitly promoted by low forgone salary. She can use this wealth gain to increase her consumption level or to invest more in a child. This result can be observed in Figure 11. As B increases, the value of discounted child value $\beta_c^{B+I-S} J(\cdot|B)$ increases (the line with cross mark), but U decreases (the line with star mark). The change of J dominates that of U, *i.e.* $V(\cdot|B)$ is also increasing in B (the line with triangle mark). The higher increment of J comes from a lower shadow price of child rearing time in terms of forgone earnings. This result suggests that the loss from sacrificing mother's human capital accumulation (to have child late) is lower than the benefit from rearing a child with a lower opportunity cost.[17] The optimal decision is determined by the age at which the mother utilizes resources to obtain a child with "the best quality".

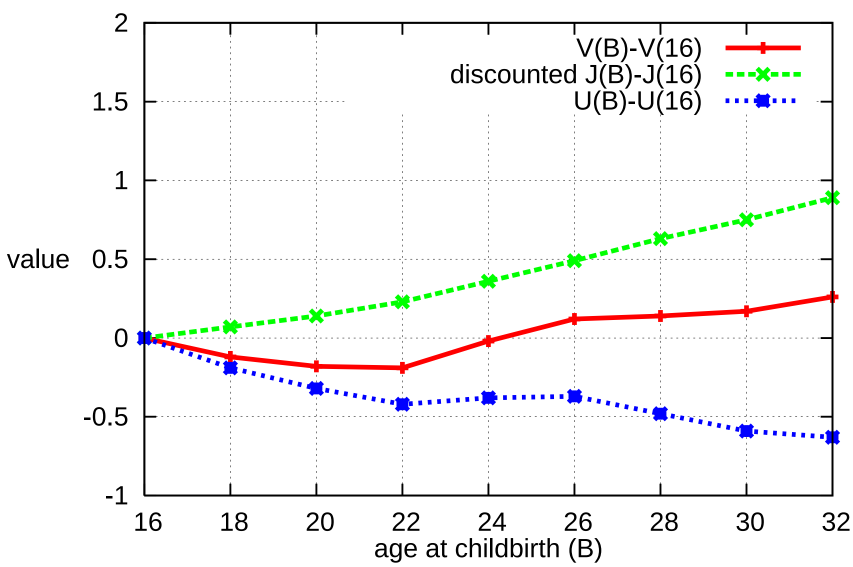

Figure 11: Values of a representative cohort by exogenously given birth timing B (TFP is low; year 1950)

[17]This conclusion still holds if we fixed mother's investment to be a constant $\overline{Q} = 7.0$. In this case $\beta_c^{B+I-S} J(\cdot|B)$ decreases and $V(\cdot|B)$ increases as B increases, and the increment of and $V(\cdot|B)$ dominates $\beta_c^{B+I-S} J(\cdot|B)$. This observation implies that to rear child in the late fecund age is lower, and the gain from the low cost is higher than the mother's loss of earning due to a low human capital in the late life cycle.

As TFP grows, during 1970-1980 the mother's optimal decision on childbirth timing becomes earlier. Comparing with earlier cohorts, mothers have higher age profile of human capital and spend less time in rearing child. *i.e.* A mother has higher earnings profile and endures less forgone earnings. This implies that child rearing expenditures is less relevant in mother's decision making. On the other hand, a mother cares more about life cycle earnings profile. In this stage, the effects from forgone human capital accumulation dominate a mother's decision. Specifically, a mother chooses to have childbirth earlier in order to obtain higher earnings profile. Figure 12 demonstrates such effect: a mother chooses to have child late would have a lower life time value V, which comes from a lower U. We can also find that this mother invest more on her child, comparing with the Young Mother ($\beta_c^{B+I-S}J(\cdot|B)$ is increasing in B). The reason is that late childbirth decision makes her human capital lower over the childbearing periods and she is willing to spend more time on her child. However, the gain from the child's value can not cover the loss of her value from consumption (U decreases more at a late age). On the other hand, to have a child at "too early" an age is not optimal because it squeezes the time to accumulate human capital into early working periods. These two forces determine the optimal age to be 26.

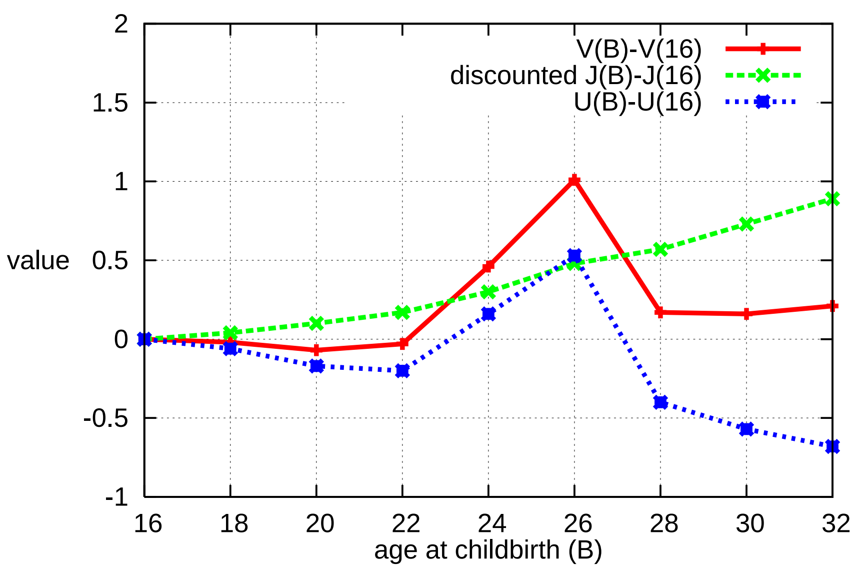

Figure 12: Values of a representative cohort by exogenously given birth timing B (TFP is in the middle range; year 1975)

As TFP continuously grows, the driving forces determining the middle cohorts decisions become less relevant, even though they still exist. A new consideration appears for the later cohorts when TFP is high enough. Early fecund periods (18-26) turn to be the most important periods of mother's human capital formation over the life cycle. The required periods for a mother to specialize ($n_i = 1$; *i.e.* years of schooling) and concentrate on human capital accumulation (n_i close to 1) become longer. Having a child early occupies time usage on human capital accumulation during these periods. Furthermore, this mother has to spend extra time (periods) on human capital accumulation to compensate the loss during maternal periods. As a result, a mother delays the timing to become a full time worker, she needs to pay for the expenditure of human capital investment for the prolonged "non-working" periods. On the other hand, since a mother's time usage is bounded by the time constraint, her time spent on child rearing is squeezed by the other usage, personal human capital investment. Thus, the child is poorly reared in these periods. Given that, as TFP grows, a mother's "golden time for her human capital formation" becomes longer, she postpones the optimal timing. We can verify the above statement by examining Figure 13. If a mother chooses to have a child at an age before 24, her utility derived from consumption and child value are similar among all the possible choices. However, if a mother chooses to have a child at later age, at which she has attained a certain level of human capital accumulation, she can increases both her value U(by becoming a full time worker early) and child value J (by spending more time on child rearing).

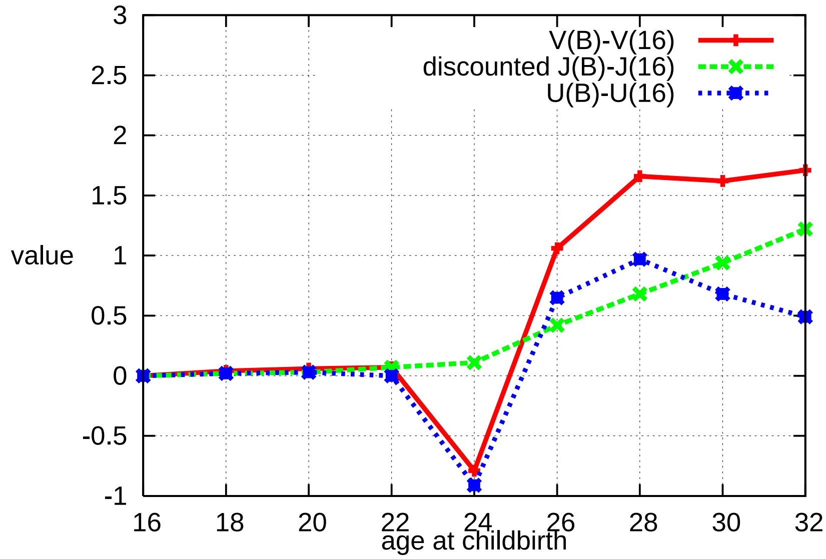

Figure 13: Values of a representative cohort by exogenously given birth timing B (TFP is high; year 2000)

On the other hand, the force from the forgone human capital is comparatively small, as we have concluded that the effects from forgone human capital decreases as TFP grows. However, they still affect a mother's decision, which can be verified by examining Figure 13. For the late childbirth decision (when B is between 28 to 32), as B increases, mothers' utility from consumption U decreases (due to the effects of forgone human capital), but J increases. The gain form child value cannot covers mothers' loss due to lower earnings profile.

To summarize, the U shape pattern is driven by the following process: When TFP is low (1950-1960), a late timing of childbirth is associated with a low shadow price of mother's time. A mother chooses the timing to rear a child with "the best quality". As TFP grows, the optimal timing of childbirth becomes earlier because of lower forgone human capital and higher earnings profile. As TFP continuously grows, optimal age of childbirth postpones because the required periods for a mother to concentrate on human capital accumulation becomes longer; the maternal periods should avoid occupying a mother's golden time for her human capital formation.

5 A Discussion: Other Effects not Implied in this Model

I have given an explanation for the U shape pattern of the timing of first childbirth after WW II. Different cohorts' optimal decisions are determined by the given TFP level and the implied driving forces from human capital acquisition and child rearing. One may be led to ask are there any other possible forces not addressed in this model but crucial to mothers' decision-making on childbirth timing? For instance, what effects did WW II have? And what about the number of children a mother plans to have? In this section I use U.S data to discuss the effects these factors on childbirth timing.

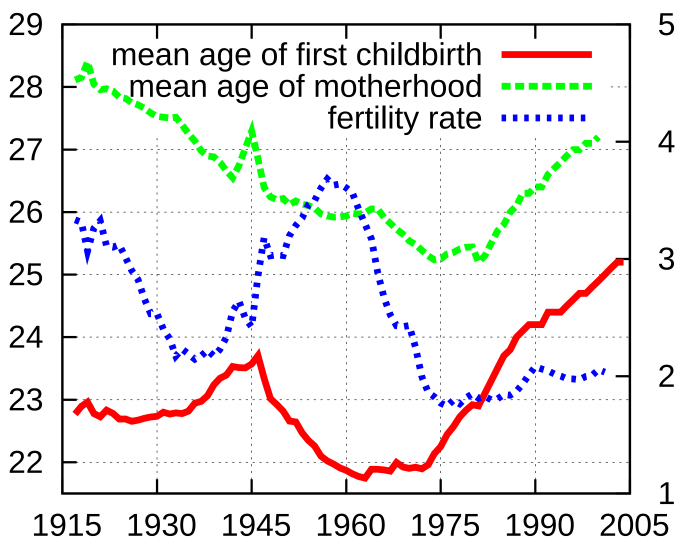

Figure 14: Summary of the fertility indicators in U.S. since 1917

Figure 14 plots the total period fertility rate, mean age of first childbirth, and mean age of motherhood (the average age of those mothers who have their childbirth in the given year) since 1917.[18] The right axis is the scale for the fertility rate and the left is the scale for mothers' age. It is clearly observed that the WW II seems to have had impacts on mother's age. Both mothers' age at first childbirth and age of motherhood rose during the WW II.

Why did this war cause an increase in the timing of childbearing? A straightforward conjecture is that the timing to have a child was postponed because most men were sent to the war. Or perhaps the uncertainty about the future caused potential mothers to decide to wait for a better circumstance. These underlying reasons are beyond the scope of this paper. The thing we are interested in is if the left-hand side of the U-shaped pattern is just a short-run phenomena driven by the war instead of the effects implied in this model. From both patterns regarding mothers' decision-making on timing shown in Figure 14, it is found that mothers still choose a relatively late timing to have childbirth before the war compared to the decisions around 1970. The choices made in 1917 are three years later than the choices made in 1975 especially for the mean age of motherhood. It is concluded that the decreasing trend still exists even without the effects from the war and this trend could still be explained by the forces implied in this model.

If the fertility rate is considered, the question one may ask is if the low of U-shaped pattern during 1970 is driven by the high fertility rate during the baby boom? The argument could be that given that mothers planned to have several children, so they decided to have a early childbirth in order to prepare for future children. In the U.S. data, it is observed that the deceasing trend starts from the the same period as the beginning of the baby-boom. Indeed, it is hard to reject this hypothesis. However, it can also be found that the early child birth decision proceeded for another 10 years during the period of baby bust (1960-1975). If the fertility rate is a crucial factor and negatively correlated to the timing decision as the argument stated earlier, it may be expected to see a reversed trend in the mother's timing decision during these periods, but it is not what we observed. On the other hand, consider the total fertility rate of European countries, as shown in Figure 15. During 1950-1960, total fertility rate slightly increased and then declined over time until 1980. In contrast to the case in the U.S., this decreasing trend coincides more with the trend of motherhood timing for the European countries.[19] Given this, it is even harder to tell the influences of fertility rate to motherhood timing.

[18] This data is produced by the National Institute of Child Health and Development. It is the earliest available dataset containing records for mother's age.

[19] In U.S. the low of the U-shaped pattern occurs around 1960

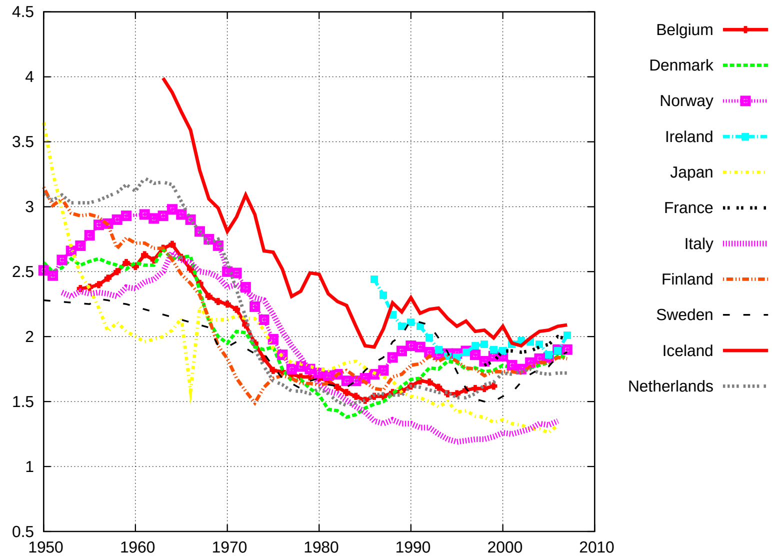

Figure 15: Total fertility rate

Indeed, there still exists many other possible factors that influence mothers' timing decisions but are not addressed in this study (*e.g.* birth control and household's joint decision). My focus here is to study this phenomenon of the history in a vacuum environment, by only considering the mothers optimal decision in response to TFP growth. This stylized fact is explained well in the context of this baseline model. On the other hand, since this model is not considered in a steady state, some implications may seem counterintuitive if the TFP grows continuously: mothers years of schooling, for instance, would continuously increase over time, as well the timing of motherhood. These issues, however, could still be solved by imposing other assumptions, *e.g.* allowing the productivity of human capital to grow as TFP grows. However, even though this model gives an explanation for this stylized fact, the predictive power of mothers' decision for many years later would be limited.

6 Conclusion

In this paper we examined the females' motherhood timing decisions over the time horizon in response to TFP growth, and gave an explanation of the stylized fact that the mean age of motherhood is a U shape curve over the past fifty years. We provided a new view considering women's joint decisions on human capital accumulation (years of schooling and on-the-job training), working, and child rearing over the life cycle. In contrast to the human capital story in the existing fertility timing/tempo literature that fertility decision results in experiences lost and thus decreases future earnings, we highlighted the decisions on human capital accumulation before motherhood and found it is crucial for the determination of the optimal timing of childbirth.

Some important features implied by our model are as follows: (1) Child rearing has significant effects on mother's human capital accumulation over the life cycle. When a mother thinks of having a child in the future, she would like to accumulate less human capital (*i.e.* receives less education or job training) until motherhood, since the time spent on child rearing decreases the return for her current investment. (2) As the birth timing is further away from a mother's age, the effect of child rearing time becomes weaker; moreover, the effect of child rearing on foregone human capital accumulation becomes weaker as TFP increases. (3) The forgone earnings of child rearing are associated with the human capital over the life cycle, while the optimal human capital investment over the life cycle is determined by the decision on childbirth timing. (4) If TFP is high enough, having a child at an earlier fecund age is not optimal decision. The reason is that to rear a child in early ages affects those periods in which woman originally would be in school or acquire intensive on-the-job training. This mother has to spend extra time (period) on human capital accumulation to compensate the loss during maternal periods; therefore, she goes to job market later and lost the subsidy of the job training paid by the firm.

These observations contribute to the U shape pattern of the childbirth timing over the last fifty years. When TFP is low (1950-1960), a mother chooses a late timing of childbirth. This decision makes her concentrate on working and obtain higher earnings early. Moreover, during the child rearing periods, the human capital has begun decaying and is relatively low over the life cycle. The low human capital implies low forgone salary for each unit of time spent on child. She can utilize the time to rear a child with the *best quality* without sacrificing her utility too much. As TFP grows, the optimal timing of childbirth becomes earlier. This decision makes a mother invest higher human capital over the life cycle, and obtain higher earnings profile, comparing to other childbirth timing. In addition, even though the implied forgone earnings during maternal periods are higher, but the gain from higher earnings over the life cycle is still higher than the loss. As TFP continuously grows, optimal timing of childbirth postpones. As the feature (4) stated in last paragraph, the required periods for a mother to concentrate on human capital accumulation becomes longer; the maternal periods should avoid occupying a mother's golden time for her human capital formation.

This framework sheds light on the relationship between economic growth and the tempo of birth. Empirical studies suggest that fertility rate is countercyclical in the U.S, which is driven by the birth timing decisions (Butz and Ward (1979)). This stylized fact regarding to the relationship between birth decisions and TFP fluctuation could be a future research direction. On the other hand, issues related to gender wage gap can also be studied within this framework. One main implication of this model is that the future fertility decision induces a mother to reduce human capital acquisition over the entire life cycle and as a result a female enjoys a lower earnings profile. A male do not face the same problem as long as he is not forced to leave labor market for fertility. An endogenously determined gender wage gap is implied. This mechanism differs from those in the existing literature.[20] Although some other factors (*e.g.* jointly household decision) need to be taken into account, this topic is worthy to be explored as another extension from this study.

Appendix

A. Summary of the Backward Solving Algorithm

The dynamic programming problem is solved by the followings steps, numerical methods are based on Judd (1998); Press (2007):

1. Step 1.Choose a B from mother's fecund periods$\left[\underline{U}, \overline{U}\right]$. For the given B, generate a upper bound and lower bound lifetime human capital sequences $\{h_i^M\}_{i=S}^R$, $\{h_i^m\}_{i=S}^R$, and determine the (possible) maximum year of schooling, S_p. By these information, we then build a two dimension grids $(L \times N)$. L is $\max(S_p, B+I)$, representing the length of time horizon. N is the number of points between the interval h_i^M and h_i^m, which are all the cases we plan to solve in each period. Each case is denoted as h_i^c, where $i \in [S, L]$ and $c \in \left[h_i^M, h_i^m\right]$

2. Step 2. Fix a value $\overline{Q}$ as the initial value (we will update this value later). Solve all the cases in the last period, $i = L$. Here, the "last period" is defined as that women finish rearing child and year of schooling, and will go to work after this period. That is,

$$n_j < 1, l_j = 0, \forall j \in [L+1, R]$$

   Last period problem is solved firstly, because once we know women will go to work at $L+1$, all the variables we concern h_{L+1}, n_L, l_L, x_L and $x_L^{'}$ can be solved by human capital production functions, (2), (3), (4) and equilibrium conditions (12), (13), (16), given $\overline{Q}$ and h_L^c.

3. Step 3. Back to solve all the cases, h_{L-1}^c, in period $L-1$. In this step, the difficulty is that when we solve the five endogenous variables listed above by using equilibrium

[20] Albanesi and Olivetti (2009) study this topic based on contract theory. Their mechanism is based on firms' imperfect information of home production, which leads to moral hazard and adverse selection problems in labor market.

condition (13) , we have to know the value of l_{i+1}, n_{i+1}, x_{i+1} and $F_{x_{i+1}}$. Given the solutions of all the cases in L we solved in step 2, we may interpolate these values. For example, for a given h^c_{L-1}, suppose women choose time n_i in accumulating her human capital, the the value of l_i, x_i, $x_i^{'}$ are determined by (12), (16), and (10). We can compute $\hat{h}_{i+1}$ by (2) and (3). By using "Spline interpolation" algorithm, we can get the value of $\hat{l}_{i+1}$, $\hat{n}_{i+1}$, $\hat{x}_{i+1}$ and $\hat{F}_{x_{i+1}}$.

4. Step 4. Apply step 3 to period $L-2, L-3, ..., S$ (period S only contains one case, h_S). Then for the given B and $\overline{Q}$ we can pin down the optimal solution: $\{h_{i+1}\}_{i=S}^{R}$, $\{n_i\}_{i=S}^{R}$, $\{x_i\}_{i=S}^{R}$, $\{l_i\}_{i=B}^{B+I-S}$, $\{x_i^{'}\}_{i=B}^{B+I-S}$. And c^* is determined by (12).

5. Step 5. Plug in the optimal solutions into (17) to check if the $\overline{Q}$ we pick in step 2 satisfy it or not. If not, pick another $\overline{Q}$, and repeat step 2 to step 5, until the LFS minus RHS of (17) is smaller than the tolerance level. Once optimal Q^* is obtained, mother's lifetime value $V(h_S, w)$ is determined by (6), given the birth happens at B chosen in step 1.

6. Step 6. Repeat step 1 to step 5 by choosing all the B in mother's fecund periods$\left[\underline{U}, \overline{U}\right]$. With the $V(h_S, w)$ on hand for each B, the optimal B^* is the B in $[S, \overline{U}]$ where $V(h_S, w)$ is maximized.

References

Albanesi, S., and C. Olivetti (2009): "Home production, market production and the gender wage gap: Incentives and expectations," *Review of Economic Dynamics*, 12(1), 80–107.

Ben-Porath, Y. (1967): "The production of human capital and the life cycle of earnings," *The Journal of Political Economy*, pp. 352–365.

Blackburn, M. K. L., D. E. Bloom, and D. Neumark (1993): "Fertility timing, wages, and human capital," *Journal of Population Economics*, 6(1), 1–30.

Bloom, D. E. (1982): "What's happening to the age at first birth in the United States? A study of recent cohorts," *Demography*, pp. 351–370.

——— (1984): "Delayed childbearing in the United States," *Population Research and Policy Review*, 3(2), 103–139.

BOSVELD, W. (1996): *The ageing of fertility in Europe: a comparative demographic-analytic study*. Amsterdam: Thesis Publishers.

BRYANT, W. K., AND C. D. ZICK (1996): "Are we investing less in the next generation? Historical trends in time spent caring for children," *Journal of Family and Economic Issues*, 17(3), 365–392.

BUTZ, W. P., AND M. P. WARD (1979): "The emergence of countercyclical US fertility," *American Economic Review*, 69(3), 318–328.

CHEN, K. Y., AND C. R. PLOTT (1998): "Nonlinear behavior in sealed bid first price auctions," *Games and Economic Behavior*, 25(1), 34–78.

CIGNO, A. (1991): *Economics of the Family*. Oxford University Press, USA.

CIGNO, A., AND J. ERMISCH (1989): "A microeconomic analysis of the timing of births," *European Economic Review*, 33(4), 737–760.

COX, J. C., AND R. L. OAXACA (1996): "Is bidding behavior consistent with bidding theory for private value auctions?," *Research in Experimental Economics*, 6, 131–148.

CUNHA, F., AND J. HECKMAN (2007): "The technology of skill formation," *American Economic Review*, 97(2), 31–47.

GOEREE, J. K., C. A. HOLT, AND T. R. PALFREY (2002): "Quantal response equilibrium and overbidding in private-value auctions," *Journal of Economic Theory*, 104(1), 247–272.

GUSTAFSSON, S. (2001): "Optimal age at motherhood. Theoretical and empirical considerations on postponement of maternity in Europe," *Journal of Population Economics*, 14(2), 225–247.

HAPPEL, S. K., J. K. HILL, AND S. A. LOW (1984): "An economic analysis of the timing of childbirth," *Population Studies: A Journal of Demography*, pp. 299–311.

HECKMAN, J. J., AND J. R. WALKER (1990): "The relationship between wages and income and the timing and spacing of births: Evidence from Swedish longitudinal data," *Econometrica: Journal of the Econometric Society*, pp. 1411–1441.

HOPFLINGER, F. (1984): "Cohort fertility in western europe: comparing fertility trends in recent birth cohorts'," *Genus*, 40(1-2), 19–46.

Hotz, V. J., J. A. Klerman, and R. J. Willis (1997): "The economics of fertility in developed countries," *Handbook of population and family economics*, 1, 275–347.

Judd, K. L. (1998): *Numerical Methods in Economics*. The MIT Press.

Manuelli, R., and A. Seshadri (2006): "Human capital and the wealth of nations," *manuscript, University of Wisconsin*.

——— (2009): "Explaining International Fertility Differences," *Quarterly Journal of Economics*, 124(2), 771–807.

Miller, A. (2005): "The effects of motherhood timing on career path," *manuscript, University of Virginia*.

Morgan, S. P. (1996): "Characteristic features of modern American fertility," *Population and Development Review*, 22, 19–63.

Press, W. H. (2007): *Numerical recipes: the art of scientific computing*. Cambridge University Press.

Rothe, I., J. Cassetty, and E. Boehnen (2001): "Estimates of Family Expenditures for Children: A Review of the Literature," *Report prepared for the Wisconsin Department of Workforce Development, Institute for Research on Poverty, University of Wisconsin–Madison*.

Sandberg, J. F., and S. L. Hofferth (2001): "Changes in children's time with parents: United States, 1981-1997," *Demography*, pp. 423–436.

Sayer, L. C., S. M. Bianchi, and J. P. Robinson (2004): "Are parents investing less in children? Trends in mothers' and fathers' time with children 1," *American Journal of Sociology*, 110(1), 1–43.

Walker, J. R. (1995): "The effect of public policies on recent Swedish fertility behavior," *Journal of Population Economics*, 8(3), 223–251.