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Trade and the Composition of Growth

Robert F. Kane
International University of Japan

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IUJ Research Institute
International University of Japan

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Abstract

This paper studies the relationship between innovation driven growth, distribution, and international trade. The model features two trade barriers: tariffs and distribution costs and three sources of growth: quality improvement, cost reduction, and product proliferation. This paper shows that distribution and manufacturing technologies have important interactions and are fundamentally linked. The distribution costs reduce the incentive to engage in cost reduction. Through this mechanism, trade has a compositional affect on economic growth. Tariffs affect both the extent of the market and the composition of the market. A reduction in tariffs increases market size and hence generates a temporary increase in quality growth and the entry rate. Because overseas sales are distribution intensive, the expansion of overseas sales drives a temporary reduction in manufacturing productivity growth. In contrast, if increased trade is driven by improvements to the distribution technology, both quality improvement and manufacturing productivity growth increase.

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1 Introduction

In this paper we study the relationship between growth, distribution, and international trade. The framework features an R&D driven growth model with costly distribution and three sources of innovation: quality improvement, cost reduction, and endogenous entry. We use the framework to address how trade and growth interact when distribution is modeled with a flexible empirically driven formulation.

In our environment, distribution is a distinct economic activity with its own technology. We model it as a downstream process. Subsequent to production, each good must be delivered to consumers. This is an important difference from the international economics literature. The literature typically adopts Samuelson's (Samuelson, 1954) well-known 'iceberg' specification: "To carry each good across the ocean you must pay some of the good itself." With this classic formulation, during the delivery process a fraction of each good shipped melts. This paper argues that modeling transportation flexibly—without the rigid structure of the iceberg specification—generates rich dynamic interactions between the transportation technology, trade, and growth.

To understand the mechanisms of this paper, it is important to understand the implications of the iceberg assumption. The iceberg specification implies that, up to a scaling constant, the transportation and manufacturing production functions are identical. The production functions for both activities not only feature identical factor intensities, returns to scale, etc,., but equally important, the distribution technology is linear in the manufacturing technology. This assumption implies that when Apple improves manufacturing methods, Apple's delivery capabilities increase by the same proportion. This is a strong and counterfactual implication. This property is particularly problematic for endogenous growth theory. The iceberg specification is also strongly rejected by the data. Hummels and Skiba (2004) and Irarrazabal, Moxnes, and Opromolla (2015) find strong evidence against the iceberg specification. Instead, they find strong evidence in favor of the per-unit (or specific) assumption which this paper adopts. The per-unit formulation allows for flexibility: the cost of delivery is no longer linear in manufacturing productivity. This allows our framework to shed light on the importance of distance.

Using meta-analysis Disdier and Head (2008) document the "Puzzling persistence of distance" on international trade. They show that the negative impact of distance rose and reached a peak around 1950 and remained high since then. This is surprising because the 1950's onward saw the containerization revolution. Directly to this point, Jacks and Pendakur (2010) find no evidence that the maritime transport revolution drove the late-nineteenth-century trade boom. In our environment, these facts are not puzzling. Unless the shipping technology improves, through better logistics, containerization, etc,. worldwide manufacturing productivity growth will *reduce* trade. Intuitively, if the world gets better at producing goods—without improving its capability of delivering them—the relative price of overseas goods increases. This is a dynamic analog to the Alchian-Allen theorem (Alchian and Allen, 1964) which we briefly review.

In its most basic form, the Alchian-Allen theorem establishes that per-unit frictions (tariffs and shipping costs) reduce the relative price of expensive goods. This mechanism explains why farmers "ship the good apples out" of local areas. The dynamic implications of this mechanism have largely been ignored. If farmers improve their manufacturing productivity and the cost of producing apples falls—unless the distribution technology improves at the same rate or faster—the relative burden of shipping costs increase. The increase in the relative burden of shipping costs increases the relative price of exported apples and hence reduces trade volume.

The preceding discussion focuses on the production and delivery of physical objects. The interaction between quality improvement and the distribution technology is different. Quality improvement allows consumers to get more enjoyment per physical object delivered. Consequently, quality improvement does not increase the relative burden of shipping costs. This difference between quality improvement and cost reduction drives some of the paper's results. Chiefly, tariffs have asymmetric affects on quality and productivity growth. A reduction in tariffs generates a temporary acceleration in quality growth. It also, however, generates a temporary reduction in manufacturing productivity growth. The mechanism is such that tariffs affect both aggregate market size and the composition of market size. A reduction in tariffs increases aggregate market size, but a larger proportion comes from the overseas market. Because overseas sales are distribution intensive, this reduces the incentive to engage in cost-reducing R&D. In the long-run, however, the steady state growth rate is invariant to tariffs because of endogenous entry.

The paper admits two classes of regimes. The first regime features an endogenous structure of costs. The steady state ratio of shipping to manufacturing costs is endogenously determined along with firm size. As the preceding paragraph suggests, the induced technical change provide an amplification mechanism to tariffs. A reduction in tariffs reduces manufacturing productivity growth and thus the relative shipping (distribution) cost declines which thus generates a further increase in trade. The second regime features the death of distance. Although the economy admits a steady state where the shipping and manufacturing technology grow at the same rate, it is also possible that the improvement in the distribution technology grows faster and the distribution costs asymptotically disappear. In this regime the model is capable of generating "takeoffs".

The paper is organized as follows. Section 2 presents the model. Section 3 solves the model. Section 4 presents the transitional dynamics and engages in comparative dynamics. Finally, section 5 concludes.

2 The model

We extend the framework of Peretto and Valente (2011) to include distribution and also introduce two sources of vertical innovation: quality improvement and cost reduction. There are two countries Home and Foreign which are denoted H and F respectively. There are a continuum of goods each of which are produced by a single firm and labor is the only physical resource. Firms engage in R&D to improve manufacturing productivity and product quality. Entrepreneurs create new goods which are produced by new firms. All variables are function of (continuous) time but we omit the time argument unless necessary to avoid confusion.

2.1 Households

Each country $J = H, F$ is populated by a representative household with lifetime utility

$$U^J = \int_0^\infty e^{-\rho t} L^J(t) \ln c^J(t)\, dt. \quad \rho > 0 \tag{1}$$

The household consists of L^J identical members and the time preference is ρ. The household's preferences are

$$c^J = \left(c_0^J\right)^{1-\alpha} \left(c_M^J\right)^{\alpha}, \tag{2}$$

where c_0^J denotes the consumption of nontradable goods and c_M^J is an index yielding utility from differentiated tradable goods. The sub-utility function of tradable goods is

$$c_M = \left[\int_0^{N^J} \left(\left(Q_i^J\right)^\theta \frac{X_i^{Jj}}{L^J}\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon\zeta}{\epsilon-1}} \left[\int_0^{N^k} \left(\left(Q_i^k\right)^\theta \frac{X_i^{kj}}{L^J}\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon(1-\zeta)}{\epsilon-1}}.$$

In this index, N^J is the mass of varieties produced in country J, and X_i^{Jj} and X_i^{kj} are the quantities of the i-th variety produced domestically and overseas respectively. Each variety has its own has an attached quality, Q_i^J. Lastly, $\zeta \in (0.5, 1)$ is a measure of domestic bias, $\epsilon > 1$ is the elasticity of substitution, and $\theta \geq 0$ governs quality's importance.

The representative household maximizes lifetime utility (1) subject to their flow budget constraint,

$$\dot{A}^J = w^J L^J + r^J A^J + T^J - E^J, \tag{3}$$

where A^J denotes aggregate asset holdings, w^J the wage, T^J is a lump sum rebate of tariffs revenue to consumers, and r^J is the interest rate in country J. The household's aggregate consumption expenditure is

$$E^J \equiv p_0^J C_0 + \int_0^{N^J} p_i^{Jj} X_i^{Jj} di + \int_0^{N^k} \left(1+\tau^J\right) p_i^{kj} X_i^{kj} di; \tag{4}$$

where p_i^{Jj} and p_i^{kj} are the prices of the i-th variety produced domestically and overseas respectively and τ^J is an ad-valorem tariff. Note that international and domestic prices differ because of per-unit (specific) distribution costs which we discuss momentarily. The households' maximization yields the savings plan

$$\frac{\dot{E}^J}{E^J} = r^J - \rho \tag{5}$$

and demands

$$p_0^J C_0^J = (1-\alpha) E^J; \tag{6}$$

$$X_i^{Jj} = \frac{\alpha\zeta E^J \left(Q_i^J\right)^{\theta(\epsilon-1)} \left(p_i^{Jj}\right)^{-\epsilon}}{\int_0^{N^J} \left(Q_i^J\right)^{\theta(\epsilon-1)} \left(p_i^{Jj}\right)^{1-\epsilon} di}, \quad X_i^{kj} = \frac{\alpha(1-\zeta) E^J \left(Q_i^k\right)^{\theta(\epsilon-1)} \left(p_i^{kj}\right)^{-\epsilon}}{\int_0^{N^k} \left(1+\tau^J\right) \left(Q_i^k\right)^{\theta(\epsilon-1)} \left(p_i^{kj}\right)^{1-\epsilon} di}. \tag{7}$$

2.2 Production and distribution technologies

The typical firm produces with technology

$$L_{Xi}^J = \left(Z_i^J\right)^{-\sigma} X_i^J + \phi \tag{8}$$

where L_{Xi}^J is the total amount of labor required to produce X_i^J units of good i in country J. The production costs consists of fixed component, ϕ and a variable component $\left(Z_i^J\right)^{-\sigma}$ where $\sigma > 0$ is the elasticity of manufacturing unit costs with respect to manufacturing knowledge Z_i^J.

In line with the production technology, we restrict attention to a distribution technology with labor as the sole input. For simplicity, we assume that distribution is undertaken in house.[1] Distribution is costly both domestically and overseas. To capture this fact we posit

[1] The iceberg formulation implicitly assumes in-house distribution.

the distribution technology

$$L_{Di}^{J} = sD_0 X_i^{Jj} + sD_1 X_i^{Jk}, \tag{9}$$

With this specification, every good produced requires labor to distribute them to consumers. Our aim of this specification is to capture a notion of distance. The parameter D_0 governs domestic delivery costs.[2] We assume overseas delivery is more costly and thus $D_1 > D_0$. Parameter s captures the state of the transportation technology which evolves according to

$$s_t = s_0 e^{-\varsigma t}, \tag{10}$$

and $\varsigma \geq 0$. The idea is that over time the economy becomes more efficient at delivering goods through improved logistics, etc,. The parameters D_0 and D_1 allow for non-neutral improvements to the shipping technology. For example, infrastructure may improve domestic distribution technology more than overseas shipments. In contrast, the containerization revolution may have improved the overseas shipping technology more than the domestic distribution.

2.3 Incumbent firms

Firms may engage in two types of innovation activities: quality improvement and manufacturing cost reduction. We posit the following technologies:

$$\dot{Q}_i^J = \beta_Q Q^J L_{Qi}^J,\ Q^J = \int_0^{N^J} Q_i^J di/N^J. \tag{11}$$

$$\dot{Z}_i^J = \beta_Z Z^J L_{Zi}^J,\ Z^J = \int_0^{N^J} Z_i^J di/N^J; \tag{12}$$

where L_{Qi}^J and L_{Zi}^J are the amount of labor employed in quality-improving and cost-reducing R&D by the i-th firm in country J and $\alpha_Q^J Q^J$ and $\alpha_Z^J Z^J$ are the efficiency of said labor. For simplicity, and to not confound our results, we abstract from cross-country spillovers.

Each firm faces an exogenous constant probability $\delta > 0$ of death. The typical firm maximizes the present discounted value of the net profit

$$V_i^J(0) = \int_0^{\infty} e^{-\int_0^t \left[r^J(s)+\delta\right]ds} \left[\Pi_i^J(t) - w^J(t)\left[\phi + L_{Zi}^J(t) + L_{Qi}^J(t)\right]\right] dt, \tag{13}$$

subject to the R&D technologies (11)-(12), and where the gross profit of a typical firm in

[2]The special case $B_0 = 0$, is of some interest because it imposes a common assumption in the literature in which only overseas shipments are costly.

country J is:

$$\Pi_i^J \equiv \underbrace{X_i^{Jj}\left\{p_i^{Jj} - w^J\left[\left(Z_i^J\right)^{-\sigma} + D_0 s\right]\right\}}_{\Pi_i^{Jj}} + \underbrace{X_i^{Jk}\left\{p_i^{Jk} - w^J\left[\left(Z_i^J\right)^{-\sigma} + D_1 s\right]\right\}}_{\Pi_i^{Jk}}. \tag{14}$$

The firm's gross profit can be decomposed into profits from domestic sales, Π_i^{Jj}, and profits from overseas sales, Π_i^{Jk}.

The firms optimization yields the prices

$$p_i^{Jj} = \frac{\epsilon w^J}{\epsilon - 1}\left[\left(Z_i^J\right)^{-\sigma} + D_0 s\right],\ p_i^{Jk} = \frac{\epsilon w^J}{\epsilon - 1}\left[\left(Z_i^J\right)^{-\sigma} + D_1 s\right]. \tag{15}$$

Because we have assumed distribution is under-taken in house, firms mark up their distribution cost. We can allow for different pricing strategies with no qualitative changes to the model's results. Before discussing the returns to innovation, it is useful to define the following elasticities:

$$\eta_Q \equiv \frac{\partial \ln \Pi_i^{Jj}}{\partial \ln Q_i} = \frac{\partial \ln \Pi_i^{Jk}}{\partial \ln Q_i} = \theta\left(\epsilon - 1\right); \tag{16}$$

$$\eta_{Zi}^{Jj} \equiv \frac{\partial \ln \Pi_i^{Jj}}{\partial \ln Z_i^J} = \sigma\left(\epsilon - 1\right)\left(\frac{\left(Z_i^J\right)^{-\sigma}}{\left(Z_i^J\right)^{-\sigma} + D_0 s}\right); \tag{17}$$

$$\eta_{Zi}^{Jk} \equiv \frac{\partial \ln \Pi_i^{Jk}}{\partial \ln Z_i} = \sigma\left(\epsilon - 1\right)\left(\frac{Z_i^{-\sigma}}{Z_i^{-\sigma} + D_1 s}\right). \tag{18}$$

Quality improvement is equally effective in raising domestic and overseas sales. In contrast, cost reduction raises is more effective in raising domestic sales: $\eta_{Zi}^{Jj} > \eta_{Zi}^{Jk}$. This plays an important role for the returns to innovation which we discuss below.

Lemma 1. *Let r_Q^J and r_Z^J denote the rate of return to cost reduction and quality improvement respectively in country $J = H, F$ and let $k \neq J$. In symmetric equilibrium,*

$$r^J = r_Q^J \equiv \frac{\beta_Q \eta_Q}{w^J}\Pi^J + \frac{\dot{w}^J}{w^J} - \delta - \frac{\dot{Q}^J}{Q^J}; \tag{19}$$

$$r^J = r_Z^J \equiv \frac{\beta_Z}{w^J}\left[\eta_Z^{Jj}\left(\chi\right)\frac{\Pi^{Jj}}{\Pi^J} + \eta_Z^{Jk}\left(\chi\right)\frac{\Pi^{Jk}}{\Pi^J}\right]\Pi^J - \delta + \frac{\dot{w}^J}{w^J} - \frac{\dot{Z}^J}{Z^J}; \tag{20}$$

where

$$\eta_Z^{Jj}\left(\chi\right) = \sigma\left(\epsilon - 1\right)\left(1 + D_0\chi^J\right)^{-1}; \tag{21}$$

$$\eta_Z^{Jk}\left(\chi\right) = \eta_{Zi}^E = \sigma\left(\epsilon - 1\right)\left(1 + D_1\chi^J\right)^{-1}; \tag{22}$$

$$\chi^J \equiv \frac{s}{(Z^J)^{-\sigma}}. \tag{23}$$

Proof: *See the appendix.*

The returns to innovation can be decomposed into the gains from the domestic and overseas market. As we have already discussed, the return to quality improvement is symmetric. It is also constant and invariant to quality, productivity, and the distribution technology. The return to quality improvement thus depends on the total profit—changes to its composition have no affect. The return to cost reduction is more complicated.

The return to cost reduction depends on profit shares: the share from domestic sales Π^{Jj}/Π and exports Π^{Jk}/Π. It also depends on the elasticities $\eta_Z^{Jj}(\chi)$ and $\eta_Z^{Jk}(\chi)$. To understand the mechanics, it is useful to first recall the Alchian-Allen theorem. It implies that per-unit frictions (tariffs and shipping costs) reduce the relative price of expensive goods. Consequently expensive items are traded more. Our model generates several dynamic predictions which build on Alchian and Allen's classic result.

An increase in manufacturing productivity reduces the share that manufacturing costs comprise of aggregate unit cost. Absent technical progress in distribution, eventually cost reduction cannot raise sales in either the domestic or overseas markets. Second, cost reduction is more effective in raising domestic than overseas sales: $\eta_Z^{Jj}(\chi) > \eta_Z^{Jk}(\chi)$. Because of this, the composition of profits matter. Holding all else constant, openness—which increases the share of profits from exports—reduces the return to cost reduction. To understand the mechanisms further, it is useful to rewrite the profits as

$$\Pi_i^{Jj} = \frac{\zeta \alpha E^J \left(\frac{\left(Z_i^J\right)^{-\sigma} + D_0 s}{\left(Q_i^J\right)^{\theta}} \right)^{1-\epsilon}}{\epsilon \int_0^{N^J} \left(\frac{\left(Z_i^J\right)^{-\sigma} + D_0 s}{\left(Q_i^J\right)^{\theta}} \right)^{1-\epsilon} di}; \tag{24}$$

$$\Pi_i^{Jk} = \frac{(1-\zeta)\, \alpha E^k \left(\frac{\left(Z_i^J\right)^{-\sigma} + D_1 s}{\left(Q_i^J\right)^{\theta}} \right)^{1-\epsilon}}{\epsilon \int_0^{N^k} \left(1+\tau^k\right) \left(\frac{\left(Z_i^J\right)^{-\sigma} + D_1 s}{\left(Q_i^J\right)^{\theta}} \right)^{1-\epsilon} di}. \tag{25}$$

The profits are isoelastic in quality. An improvement in quality not only reduces the quality adjusted cost of production, but also the quality adjusted cost of distributing the good to consumers. In contrast, manufacturing productivity growth fails to reduce the cost of delivery. Therefore, as manufacturing costs fall relative to the distribution cost, the distribution cost comprise a larger portion of prices. As manufacturing productivity improves, cost reduction becomes increasingly ineffective in raising sales. Because of this mechanism, and because overseas sales are distribution intensive, cost reduction is relatively less effective in improving overseas sales. Note that the preceding discussion crucially depends on our shipping cost specification. If $s = 0$, then productivity and quality are isomorphic.

2.4 Entry

Entrepreneurs hire labor to create new firms which serve the market. Following Peretto and Valente (2011) we assume that the entry costs are proportional to the value of production at the time of entry. Specifically, $V^J = \beta_N \boldsymbol{p_i^J} \cdot \boldsymbol{X_i^J}$ where $\boldsymbol{p_i^J}$ and $\boldsymbol{X_i^J}$ are price and quantity vectors consisting of domestic and overseas values. Let r_N^J denote the rate of return to entry in country J. Differentiating (13) with respect to time yields

$$r_N^J = \frac{\pi^J}{V^J} + \frac{\dot{V}^J}{V^J} - \delta. \tag{26}$$

3 General equilibrium

This section solves the model and presents the equilibrium dynamics. The equilibrium is symmetric within each country because all firms within the country make identical decisions and have the same productivity and quality. We thus drop the firm subscript.

3.1 Expenditure and interest rate

In this class of models, asset holdings consists of ownership shares of firms, consequently $A^J = N^J V^J$. This condition, combined with (3) and the trade balance condition yields equilibrium expenditure which which we characterize in the following lemma.

Lemma 2. *Consumption expenditure in Home and Foreign are, respectively,*

$$E^H = \frac{\left(1+\tau^H\right) L^H}{1-\rho\alpha\beta_N + \tau^H \left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}, \tag{27}$$

$$E^F = \frac{\left(1+\tau^F\right) w^F L^F}{1-\rho\alpha\beta_N + \tau^F \left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}, \tag{28}$$

where

$$w^F = \frac{L^H}{L^F}\left(\frac{1-\rho\alpha\beta_N + \tau^F \left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}{1-\rho\alpha\beta_N + \tau^H \left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}\right). \tag{29}$$

Proof: See the appendix.

Recall that Home's wage rate is the numeraire and hence $w^H = 1$. The constancy of nominal expenditure combined with the Euler equation (5) implies

$$r^J = \rho,\ J = H, F. \tag{30}$$

This property of the model generates substantial tractability and allows us to obtain sharp results for the underlying dynamic system.[3]

3.2 Quality and manufacturing productivity growth

Combining (19), (24), (25), (27), and (28), yields

$$\frac{\dot{Q}^J}{Q^J} = \begin{cases} \frac{\beta_Q \theta(\epsilon-1)\alpha L^J}{\epsilon N^J}\left(\frac{1+\zeta\tau^J}{1-\rho\beta\alpha+\tau^J[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N]}\right) - \rho - \delta & N < \bar{N}_Q^J \\ 0 & N > \bar{N}_Q^J \end{cases}. \tag{31}$$

Following similar steps, the growth rate of manufacturing productivity is

$$\frac{\dot{Z}^J}{Z^J} = \begin{cases} \frac{\beta_Z \sigma(\epsilon-1)\alpha L^J}{\epsilon N^J}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\alpha\beta_N+\tau^J[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N]}\right) - \rho - \delta & N < \bar{N}_Z^J \\ 0 & N > \bar{N}_Z^J \end{cases}. \tag{32}$$

As in standard in this class of models, the growth rates are decreasing in the mass of firms. Consequently there exists thresholds associated with zero innovation. The thresholds are, respectively,

$$\bar{N}_Q^J \equiv \frac{\beta_Q \theta\left(\epsilon-1\right)\alpha L^J}{\epsilon\left(\rho+\delta\right)}\left(\frac{1+\zeta\tau^J}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]}\right); \tag{33}$$

$$\bar{N}_Z^J\left(\chi^J\right) \equiv \frac{\beta_Q \sigma\left(\epsilon-1\right)\alpha L^J}{\epsilon\left(\rho+\delta\right)}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]}\right). \tag{34}$$

There are a few elements worthy of attention. First note that the relative distribution cost, $\chi = s/Z^{-\sigma}$ affects manufacturing productivity growth but not quality improvement. If manufacturing productivity rises faster than the economy's ability to transport goods, manufacturing costs become a smaller component of prices. Therefore productivity growth is decreasing in χ. We discuss tariffs in detail in section 4. It is noteworthy, however, that tariffs enter the growth rates differently. The reason is that the *composition* of sales matter

[3] Note that a restriction imposed later in the paper, equation (39), ensures that (27) and (28) are positive.

for cost reduction. Holding aggregate sales fixed, an increase in the proportion of overseas sales reduces the effectiveness of cost reduction.

3.3 Entry

In the appendix, we show that the growth of variety in country J is

$$g_N^J \equiv \frac{\dot{N}^J}{N^J} = \frac{\left(1+\zeta\tau\right)\left(1-\epsilon\beta_N\left(\rho+\delta\right)\right)-\epsilon N^J \frac{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}{\alpha L^J}\left[\phi+L_Z^J+L_Q^J\right]}{\epsilon\beta_N\left(1+\zeta\tau\right)}. \tag{35}$$

The economy can be in many different regimes with various non-negativity constraints on investment (33) and (33) binding. To prevent our analysis from becoming too taxonomic, we impose

$$\beta_Q\theta \geq \beta_Z\sigma. \tag{36}$$

Restriction (36) implies that whenever quality improvement is inactive, manufacturing cost-reduction is also inactive. This assumption has no baring on any results but reduces the amount of corner solutions required to present.

Substituting the R&D expenditures and taking the corner solutions into account yields

$$g_N^J = \begin{cases} \frac{\left(1+\zeta\tau^J\right)B_0-\sigma(\epsilon-1)\left(\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}\right)-\epsilon M\frac{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}{\alpha L^J}N^J}{\beta(1+\zeta\tau^J)} & N<\bar{N}_Q^J,\bar{N}_Z^J \\ \frac{\left(1+\zeta\tau^J\right)B_0-\epsilon\left[\phi-\left(\frac{\rho+\delta}{\beta_Q}\right)\right]\frac{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}{\alpha L^J}N^J}{\beta(1+\zeta\tau^J)} & \bar{N}_Z^J<N<\bar{N}_Q^J \\ -\delta & \tilde{N}<N \end{cases} \tag{37}$$

where $\tilde{N}$ is a threshold associated with zero gross entry and

$$M \equiv \frac{\phi\beta_Z\beta_Q-\left(\beta_Z+\beta_Q\right)\left(\rho+\delta\right)}{\beta_Z\beta_Q}>0; \tag{38}$$

$$B-\sigma\left(\epsilon-1\right)>0; \tag{39}$$

$$B \equiv 1-\beta_N\epsilon\left(\rho+\delta\right)-\theta\left(\epsilon-1\right). \tag{40}$$

Restriction (38) ensures that the net-entry rate is decreasing in N^J. Restriction (39) ensures that entry is profitable when the mass of firms is low enough in all possible regimes. Parameter restriction (39) ensures the steady state mass of firms is strictly larger than zero.

The threshold associated with zero gross entry ($\tilde{N}$) is cumbersome and thus relegated to the appendix. The more important threshold is the threshold associated with zero net entry. The $\dot{N}^J=0$ locus is

$$\bar{N}_N^J\left(\chi^J\right)=\begin{cases} L^J\alpha\left[\frac{\left(1+\zeta\tau^J\right)B-\sigma(\epsilon-1)\left(\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}\right)}{\epsilon M[1-\rho\alpha\beta_N+\tau^J[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N]]}\right] & N<\bar{N}_Z^J \\ L^J\alpha\left[\frac{\left(1+\zeta\tau^J\right)B}{\epsilon\left(\phi-\left(\frac{\rho+\delta}{\beta_Q}\right)\right)[1-\rho\alpha\beta_N+\tau^J[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N]]}\right] & \bar{N}_Z^J<N \end{cases}. \tag{41}$$

In this class of models, R&D expenditures are endogenous fixed sunk costs. As (35) demonstrates, when R&D expenditures are large, there is slow entry.[4] The rate of entry is decreasing in χ^J. When χ^J is large, manufacturing cost reduction is ineffective and firms engage in little of it. More interestingly, the tariffs affect the rate of entry. We will return to this point after solving for the steady state.

3.4 The dynamic system

Upon inspection, the dynamic structure depends on two important state variables: the mass of firms N^J and the ratio of the shipping and manufacturing technologies χ^J. The former evolves according to equation (37). The loci associated with zero net entry is (41). The latter state variable evolves according to

$$\frac{\dot{\chi^J}}{\chi^J}=\begin{cases}\sigma\left[\frac{\beta_Z\sigma(\epsilon-1)\alpha L^J}{\epsilon N^J}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\alpha\beta_N+\tau^J[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N]}\right)-\rho-\delta+\frac{1}{\sigma}\frac{\dot{s}}{s}\right] & N<\bar{N}_Z^J \\ \frac{\dot{s}}{s} & \bar{N}_Z^J<N\end{cases}. \tag{42}$$

The loci associated with a constant χ^J is

$$\dot{\chi^J}=0 \implies \bar{N}_\chi^J\left(\chi^J\right)=\frac{L^J\beta_Z\sigma\left(\epsilon-1\right)\alpha}{\epsilon\left(\rho+\delta+\frac{\varsigma}{\sigma}\right)}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]}\right).$$

In the steady state, the growth rate of both state variables (N^J and χ^J) must be constant which in turn requires both to converge constant, possibly zero, values.

4 The transition

In this section we show that the manufacturing and distribution technologies are intertwined. We show that this interaction yields new insight into the growth of trade. To help build in-

[4] See Sutton (2007) and Peretto (1996) for a thorough discussion.

sight, we first solve the model under the assumption $s_0 = 0$ which shuts down the distribution channel.

Lemma 3. *Suppose that $s_0 = 0$. The steady state is globally stable and features a constant mass of firms*

$$\left(N^J\right)^* = L^J \alpha \left(1 + \zeta \tau^J\right) \left[\frac{1 - \beta\left(\rho + \delta\right) - \theta\left(\epsilon - 1\right) - \sigma\left(\epsilon - 1\right)}{\epsilon M\left[1 - \rho\alpha\beta_N + \tau^J\left[1 - \alpha\left(1 - \zeta\right) - \rho\alpha\zeta\beta_N\right]\right]}\right],$$

and the growth rates of quality and manufacturing productivity are, respectively:

$$\left(g_Q^J\right)^* = \frac{\beta_Q \theta\left(\epsilon - 1\right) M}{1 - \left(\rho + \delta\right)\beta_N - \theta\left(\epsilon - 1\right) - \sigma\left(\epsilon - 1\right)} - \left(\rho + \delta\right);$$

$$\left(g_Z^J\right)^* = \frac{\beta_Z \sigma\left(\epsilon - 1\right) M}{1 - \left(\rho + \delta\right)\beta_N - \theta\left(\epsilon - 1\right) - \sigma\left(\epsilon - 1\right)} - \left(\rho + \delta\right).$$

Proof: See the appendix.

Lemma 3 establishes two results. First, the steady state growth rate is invariant to tariffs. Recall equations (49) and (50). Holding the mass of firms fixed, an increase in tariffs reduces spending on tradable goods and hence reduces sales. The reduction in sales not only reduces quality and manufacturing productivity growth, but also the incentive to create new goods. The net result is a temporary reduction to quality and productivity growth and a permanent reduction to the steady state mass of of firms. Second, the growth rates of quality and manufacturing productivity are qualitatively identical and only differ by technological parameters. In this section we show that, when $s_0 > 0$, these predictions are dramatically altered.

The model admits two regimes. The first regime features an endogenous structure of costs. Specifically the ratio of manufacturing to distribution costs is an endogenous variable which is jointly determined with the mass of firms. The second regime exhibits the "death of distance". In this regime, manufacturing productivity growth fails to keep pace with improvements in the distribution (shipping) technology. Asymptotically the economy behaves as if there is no distance between the two regions. In the long run, tariffs and home preferences are the only trade barriers.

4.1 The interior regime

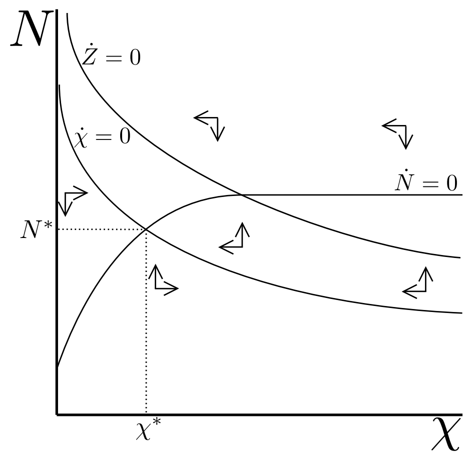


Figure 1: Interior steady state

Our next proposition aided by Figure 1 characterizes properties of the interior steady state.

Proposition 1. *Define*

$$K \equiv \frac{\beta_Z \sigma (\epsilon - 1) M}{B_0 - \sigma (\epsilon - 1)} - (\rho + \delta) > 0. \tag{43}$$

If

$$\sigma K > \varsigma > 0, \tag{44}$$

the interior steady state is globally stable. Given initial conditions $\left(\chi_0^J, N_0\right)$ *the economy converges to:*

$$\left(\chi^J\right)^* \equiv \underset{\chi^J}{argsolve} \left\{ \frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J} + \frac{(1-\zeta)}{1+D_1\chi^J} = \frac{\left(1+\zeta\tau^J\right)B}{\sigma(\epsilon-1)\left[\frac{M_0\beta_Z}{\rho+\delta+\frac{\varsigma}{\sigma}}+1\right]} \right\}; \tag{45}$$

$$\left(N^J\right)^* = \frac{L^J\alpha}{\epsilon M} \frac{\left(1+\zeta\tau^J\right)B - \sigma(\epsilon-1)\left(\frac{\left(1+\tau^J\right)\zeta}{1+D_0\left(\chi^J\right)^*} + \frac{(1-\zeta)}{1+D_1\left(\chi^J\right)^*}\right)}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}. \tag{46}$$

The steady-state dynamics of quality and productivity are

$$\left(Q^J\right)^* = Q_{SS}e^{g_Q^J t},\ g_Q^J \equiv \frac{\eta_Q^J\theta(\epsilon-1)}{\eta_Z^J}\left(\frac{M\eta_Z^J+\rho+\delta+\frac{\varsigma}{\sigma}}{B}\right) - \rho - \delta,\ Q_{SS}^J \equiv F\left(N_0^J, \chi_0^J, Q_0^J\right)$$

$$\left(Z^J\right)^* = Z_{SS}e^{g_Z^J t},\ g_Z^J \equiv \frac{\varsigma}{\sigma},\ Z_{SS}^J \equiv G\left(N_0^J, \chi_0^J, Z_0^J\right)$$

Proof: See the appendix.

Although there are two potential steady states, the parameter restrictions ensures the steady state featuring $\chi^J > 0$ is the unique global attractor. Note that the restriction $\varsigma > 0$ rules out hysteresis.[5] If $\varsigma = 0$, the properties of the dynamic system are slightly altered. The general insight, even in the special case in which $\varsigma = 0$, is that eventually manufacturing productivity must grow at the same rate as the distribution technology improves. This is a hard prediction of the model. The improvement of the distribution technology acts as a speed limit to manufacturing productivity. Quality improvement, on the other hand, may grow at a different rate.

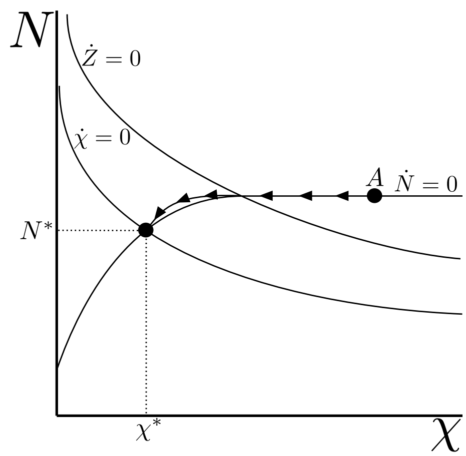
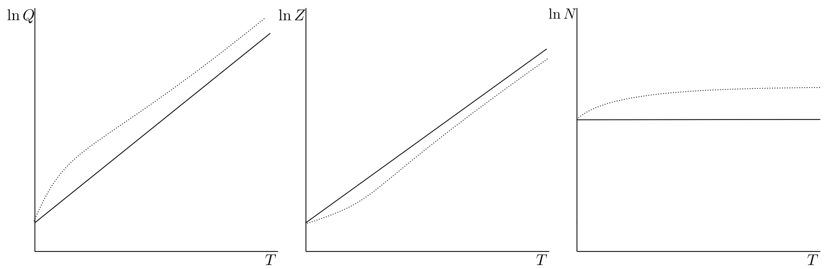


Figure 2: Transition path

An interesting property of the equilibrium is that the relative price of exports can either increase or decrease on the transition. Their evolution depends on initial conditions. In general, if χ is large, there is slow cost reduction and the burden of transportation costs decline over time. This decline in transportation costs can also generate 'takeoffs' which alter the dynamic system. Consider an economy with initial conditions, $A = (\chi_A, N_A)$. In this special case, the economy rides the $\dot{N} = 0$ locus until the transportation costs fall enough and χ crosses the $\dot{Z} = 0$ locus. Once this happens, firms begin cost reducing R&D. As χ continues to fall, firms engage in more cost reduction. Because it is a sunk cost, the increase in R&D expenditure reduces firm profitability and thus discourages entry. Hence after the $\dot{Z} = 0$ locus is crossed, firm size begins to rise. The increase in firm size also increases quality improvement. To summarize this sample transition path, the gradual reduction in transport costs increase innovation, both cost reduction and quality improvement, but there is a net decay in the mass of firms.

[5]Absent technical progress to distribution, the economy could enter a hysteresis region in which the non-negativity constraint on manufacturing cost reduction is binding.



(a) Quality (b) Productivity (c) Mass of firms

Figure 3: Reduction in tariffs

4.1.1 Comparative Dynamics

We now engage in comparative dynamics with respect to tariffs.

Corollary 1. *Assume the economy is initialized at the steady state, then:*

$$\frac{d\left(Q^J\right)^*}{d\tau^J} = \frac{dF\left(N_0^J, \chi_0^J, Q_0^J\right)}{d\tau^J} < 0; \tag{47}$$

$$\frac{d\left(Z^J\right)^*}{d\tau^J} = \frac{dG\left(N_0^J, \chi_0^J, Q_0^J\right)}{d\tau^J} > 0; \tag{48}$$

$$\frac{d\ln\left(N^J\right)^*}{d\tau^J} = -\frac{-\left(1-\alpha\right)\left(1-\zeta\right)}{\left(1+\zeta\tau^J\right)\left(1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]\right)} < 0.$$

Proof: See the following discussion.

The proof of equation (47) is straightforward. Differentiating equation (31) yields

$$\frac{d\left[\dot{Q}^J/Q^J\right]}{d\tau^J} = \frac{-\left(1-\zeta\right)\left(1-\alpha\right)\beta_Q\theta\left(\epsilon-1\right)\alpha L^J}{\left(1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]\right)^2\epsilon N^J} < 0. \tag{49}$$

Holding the mass of firms fixed, an increase in tariffs reduce trade and hence aggregate sales. This reduces the incentive to engage in quality improvement. The relationship between tariffs and manufacturing productivity growth is more complicated. Differentiating the growth rate of manufacturing productivity yields

$$\frac{d\left[\dot{Z}^J/Z^J\right]}{d\tau^J} = \frac{\left(1-\zeta\right)\left[\frac{\alpha\zeta\left(1-\rho\beta_N\right)}{1+D_0\chi^J} - \frac{1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N}{1+D_1\chi^J}\right]\beta_Z\sigma\left(\epsilon-1\right)L^J}{\left(1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]\right)^2\epsilon N^J}. \tag{50}$$

Similar to quality improvement, the increase in tariffs reduces overall sales and hence discourages cost reduction. It also, however, increases the relative importance of the domestic market. Because domestic sales are less distribution intensive, this increases the effectiveness of cost-reducing R&D—holding aggregate sales fixed. The mechanism is similar to the Alchian-Allen theorem: as the price of goods falls, the per-unit distribution cost becomes a larger component of price. Because the distribution cost comprises a larger proportion of the overseas price, cost-reduction is less effective at increasing profits from abroad. The sign of equation (50) depends on the term inside the brackets.

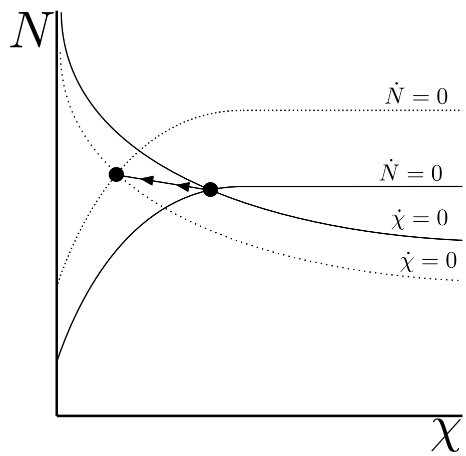


Figure 4: Reduction in tariffs

While tariffs may have an ambiguous affect on productivity growth for arbitrary initial conditions, they have an unambiguous impact when initialized at the steady state. This can be seen from the phase diagram. In terms of the phase diagram, the $\dot{N} = 0$ locus shifts up while the $\dot{\chi} = 0$ locus shifts down. The net result is a steady state featuring more firms and lower relative transportation cost. Applying the implicit function theorem to equation (45) yields

$$\frac{d\left(\chi^J\right)^*}{d\tau^J} = \frac{(D_1 - D_0)\frac{\zeta(1-\zeta)\left(\chi^J\right)^*}{1+\zeta\tau^J}}{\frac{\zeta(1+\tau^J)\left(1+D_1(\chi^J)^*\right)}{1+D_0\chi^J}D_0 + \frac{(1-\zeta)\left(1+D_0(\chi^J)^*\right)}{1+D_1\chi^J}D_1} > 0. \tag{51}$$

Equation (51) implies that an increase in tariffs increases the steady state relative distribution cost, χ. Recall that distribution costs decay exogenously at rate ς. This implies that the adjustment mechanism is productivity growth. On the transition path, an increase in tariffs always induces temporarily faster manufacturing productivity growth. Equation (51) also clearly demonstrates the driving force. The reason tariffs increase productivity growth is because overseas sales are distribution intensive, $D_1 > D_0$. If $D_1 = D_0$, tariffs would have no affect on steady state productivity growth.

This section explored the affects of tariffs. A reduction in tariffs—which generates an exogenous increase in openness—leads to a temporary increase in quality growth, a temporary decrease in quantity growth, and a permanently higher mass of firms. In the previous section we explored the transitional dynamics when shipping costs are relatively large. In that case, there is a gradual technologically driven increase in openness. In that case, there is a gradual rise in both quality and quantity growth growth, and a gradual reduction in the mass of firms. Although the comparison of these two results is not fully valid, the comparison shows the dynamic response to trade may depend on *why* trade rises.

4.2 The death of distance

This section explores the second regime, the death of distance. The next Proposition, aided by Figure 5, shows the condition in which the distance variables disappear from the steady state.

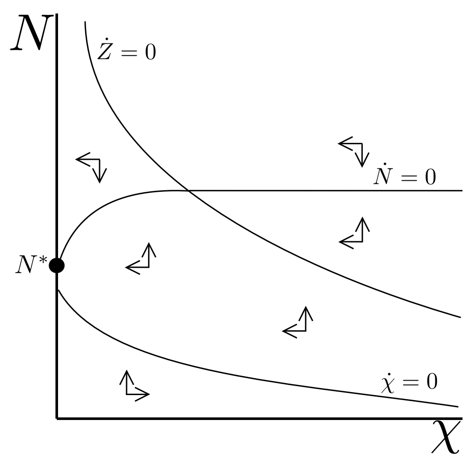


Figure 5: Death of distance

Proposition 2. *If $\varsigma > \sigma K$, the economy converges to $\chi^J = 0$. Asymptotically, the growth rates approach those presented in Lemma 3.*

Proof: See the appendix.

The rate of improvement of the distribution technology is a crucial parameter. In addition to affecting the interior steady state, if it is too large, it generates a bifurcation to the economy's dynamics. In Proposition 1, the condition $\sigma K > \varsigma$ ensures two things. First it ensures that the $\dot{\chi} = 0$ and $\dot{N} = 0$ intersect. It also ensures that the trivial steady state with $\chi = 0$ is unstable. If $\varsigma > \sigma K$, the loci no longer intersect and the trivial steady state becomes stable. Along the transition path, χ eventually must eventually decline and asymptotically approaches zero.

In this regime, tariffs are uniformly bad for growth. The positive relationship between tariffs and productivity growth only holds for the interior steady state. If χ becomes arbitrarily small, as it must under the conditions of Proposition 2, tariffs are unambiguously bad for productivity growth.

5 Conclusion

This paper presents a parsimonious model of trade and growth which features endogenous entry, quality improvement, and cost reduction. They key insight is that innovation decisions are not only affected by the extent of the market, but also its composition. Tariffs affect market size but also the composition by affecting the relative price of domestic goods. This paper predicts that a reduction in tariffs temporarily increases quality growth and permanently expands the mass of firms. The reduction in tariffs also, however, temporarily reduces manufacturing productivity growth.

Trade thus affects the composition of growth. The gains from trade, more variety and better goods real but difficult to measure. The temporary reduction in productivity growth is much easier to observe. Of course the model abstracts from cross country spillovers and other mechanisms that can increase productivity growth. Nonetheless, the forces presented in this paper suggest that the dynamic gains are biased towards the intangible. This builds on a point suggested by Romer (1994)—see also Feenstra (1992)—but is driven by a different mechanism. While the gains from trade are difficult to measure, they are real and likely substantial. The paper also shows that, if trade is technologically driven by a gradual reduction in transportation cost, trade and growth both increase. Intuitively, the reduction in trade costs increase the efficiency of cost reduction and reduces the price of exports.

References

Alchian, A. A., and W. R. Allen (1964): *University Economics*. Wadsworth Publishing Company.

Disdier, A.-C., and K. Head (2008): "The Puzzling Persistence of the Distance Effect on Bilateral Trade," *The Review of Economics and Statistics*, 90(1), 37–48.

Feenstra, R. C. (1992): "How Costly Is Protectionism?," *Journal of Economic Perspectives*, 6(3), 159–178.

Hummels, D., and A. Skiba (2004): "Shipping the Good Apples Out? An Empirical Confirmation of the Alchian-Allen Conjecture," *Journal of Political Economy*, 112(6), 1384–1402.

Irarrazabal, A., A. Moxnes, and L. D. Opromolla (2015): "The Tip of the Iceberg: A Quantitative Framework for Estimating Trade Costs," *The Review of Economics and Statistics*, 97(4), 777–792.

Jacks, D. S., and K. Pendakur (2010): "Global Trade and the Maritime Transport Revolution," *The Review of Economics and Statistics*, 92(4), 745–755.

Peretto, P., and S. Valente (2011): "Resources, innovation and growth in the global economy," *Journal of Monetary Economics*, 58, 387–399.

Peretto, P. F. (1996): "Sunk Costs, Market Structure, and Growth," *International Economic Review*, 37(4), 895–923.

Romer, P. (1994): "New goods, old theory, and the welfare costs of trade restrictions," *Journal of Development Economics*, 43(1), 5–38.

Samuelson, P. A. (1954): "The Transfer Problem and Transport Costs, II: Analysis of Effects of Trade Impediments," *The Economic Journal*, 64, 264–289.

Sutton, J. (2007): *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration*, vol. 1. The MIT Press, 1 edn.

Appendix

For convenience the appendix is self contained.

Lemma 1

The firms Hamiltonian is

$$H = \Pi_i^J - w^J(t)\left[\phi + L_{Zi}^J(t) + L_{Qi}^J(t)\right] + \lambda_{Qi}\beta_Q Q^J L_{Qi}^J + \lambda_{Zi}\beta_Z Z^J L_{Zi}^J$$

The first order conditions are

$$\frac{w^J(t)}{\beta_Q Q^J} = \lambda_{Qi}; \tag{52}$$

$$\frac{w^J(t)}{\beta_Z Z^J} = \lambda_{Zi}; \tag{53}$$

$$r = \frac{H_{Qi}}{\lambda_{Qi}} + \frac{\dot{\lambda}_{Qi}}{\lambda_{Qi}}; \tag{54}$$

$$r = \frac{H_{Zi}}{\lambda_{Zi}} + \frac{\dot{\lambda}_{Zi}}{\lambda_{Zi}}; \tag{55}$$

where

$$H_{Qi} = \frac{\partial \Pi_i^J}{\partial Q_i} = \frac{\partial \Pi_i^{Jj}}{\partial Q_i} + \frac{\partial \Pi_i^{Jk}}{\partial Q_i}; \tag{56}$$

$$H_{Zi} = \frac{\partial \Pi_i^J}{\partial Z_i} = \frac{\partial \Pi_i^{Jj}}{\partial Z_i} + \frac{\partial \Pi_i^{Jk}}{\partial Z_i}. \tag{57}$$

Combining the first order conditions yields

$$r_Q^J = \frac{\beta_Q}{w^J(t)}\left[\frac{\partial \ln \Pi_i^{Jj}}{\partial \ln Q_i^J}\Pi_i^{Jj} + \frac{\partial \ln \Pi_i^{Jk}}{\partial \ln Q_i^J}\Pi_i^{Jk}\right] + \frac{\dot{w}}{w} - \frac{\dot{Q}^J}{Q^J} = \frac{\beta_Q}{w^J(t)}\eta_Q \Pi_i^J + \frac{\dot{w}}{w} - \frac{\dot{Q}^J}{Q^J}; \tag{58}$$

$$r_Z^J = \frac{\beta_Z}{w^J(t)}\left[\frac{\partial \ln \Pi_i^{Jj}}{\partial \ln Z_i^J}\Pi_i^{Jj} + \frac{\partial \ln \Pi_i^{Jk}}{\partial \ln Z_i^J}\Pi_i^{Jk}\right] + \frac{\dot{w}}{w} - \frac{\dot{Z}^J}{Z^J}. \tag{59}$$

Lemma 2

Note that trade balance requires

$$\int_0^{N^k} \frac{\alpha(1-\zeta)E^J\left(Q_i^k\right)^{\theta(\epsilon-1)}\left(p_i^{kj}\right)^{1-\epsilon} di}{\int_0^{N^k}\left(1+\tau^J\right)\left(Q_i^k\right)^{\theta(\epsilon-1)}\left(p_i^{kj}\right)^{1-\epsilon} di} = \int_0^{N^J} \frac{\alpha(1-\zeta)E^k\left(Q_i^J\right)^{\theta(\epsilon-1)}\left(p_i^{kj}\right)^{1-\epsilon}}{\int_0^{N^J}\left(1+\tau^k\right)\left(Q_i^J\right)^{\theta(\epsilon-1)}\left(p_i^{kj}\right)^{1-\epsilon} di} di,$$

and hence

$$\frac{E^J}{E^k} = \frac{1+\tau^J}{1+\tau^k}. \tag{60}$$

The household's budget constraint, repeated for convenience, is

$$\dot{A}^J = w^J L^J + r^J A^J + T^J - E^J.$$

The tariffs rebates are

$$T^J = E^J \frac{\tau^J \alpha (1-\zeta)}{1+\tau^J}. \tag{61}$$

Note that asset holdings consists of the ownership of firms and hence $A^J = N^J V^J$ where $V^J = \beta_N \boldsymbol{p_i^J} \cdot \boldsymbol{X_i^J}$. The equilibrium value of the firm is

$$V^J = \frac{\alpha\beta_N}{N^J}\left[\frac{\zeta\left(1+\tau^k\right)E^J + (1-\zeta)E^k}{1+\tau^k}\right].$$

Using the trade balance condition and the value of the firms yields:

$$A^J = \alpha\beta_N\left[\frac{\zeta\left(1+\tau^k\right)E^J + (1-\zeta)E^k}{1+\tau^k}\right] = \alpha\beta_N\left[\frac{\zeta\left(1+\tau^J\right)+(1-\zeta)}{1+\tau^J}\right]E^J. \tag{62}$$

The household's budget constraint can thus be rewritten as

$$\frac{\dot{E}^J}{E^J} = \frac{w^J L^J}{\alpha\beta_N\left[\frac{\zeta(1+\tau^J)+(1-\zeta)}{1+\tau^J}\right]E^J} + r^J + \frac{\tau^J\alpha(1-\zeta)}{\alpha\beta_N\left[\zeta\left(1+\tau^J\right)+(1-\zeta)\right]} - \frac{\left(1+\tau^J\right)}{\alpha\beta_N\left[\zeta\left(1+\tau^J\right)+(1-\zeta)\right]}$$

Hence

$$0 = \frac{w^J L^J\left(1+\tau^J\right)}{\alpha\beta_N\left[\zeta\left(1+\tau^J\right)+(1-\zeta)\right]E^J} + \rho + \frac{\tau^J\alpha(1-\zeta)}{\alpha\beta_N\left[\zeta\left(1+\tau^J\right)+(1-\zeta)\right]} - \frac{\left(1+\tau^J\right)}{\alpha\beta_N\left[\zeta\left(1+\tau^J\right)+(1-\zeta)\right]}$$

Therefore

$$E^J = \frac{w^J L^J\left(1+\tau^J\right)}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}. \tag{63}$$

Setting Home's wage as the numeraire, $w^H = 1$, yields

$$E^H = \frac{L^H\left(1+\tau^H\right)}{1-\rho\alpha\beta_N+\tau^H\left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}.$$

Using the trade balance condition (60) and (63) yields

$$\frac{1+\tau^H}{1+\tau^F} = \frac{L^H\left(1+\tau^H\right)}{w^F L^F\left(1+\tau^F\right)} \frac{1-\rho\alpha\beta_N+\tau^F\left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}{1-\rho\alpha\beta_N+\tau^H\left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}$$

and hence

$$w^F = \frac{L^H}{L^F}\left(\frac{1-\rho\alpha\beta_N+\tau^F\left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}{1-\rho\alpha\beta_N+\tau^H\left[1-\alpha\left(1-\zeta\left(1-\rho\beta_N\right)\right)\right]}\right).$$

Proposition 1 and 2

The Jacobian is

$$J = \begin{pmatrix} F_\chi & F_N \\ G_\chi & G_N \end{pmatrix}$$

where

$$F\left(\chi,N\right) = \chi\sigma\left[\frac{\beta_Z\left(\epsilon-1\right)\alpha L^J}{\epsilon N^J}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]}\right)-\rho-\delta-\frac{\varsigma}{\sigma}\right];$$

$$G\left(\chi,N\right) = N\frac{\left(1+\zeta\tau^J\right)B-\sigma\left(\epsilon-1\right)\left(\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}\right)-M\epsilon N^J\frac{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}{L^J\alpha}}{\beta_N\left(1+\zeta\tau^J\right)}.$$

The Jacobian's elements are

$$\begin{aligned} F_\chi = {} & \sigma\left[\frac{\beta_Z\left(\epsilon-1\right)\alpha L^J}{\epsilon N^J}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\beta\alpha+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\beta\alpha\zeta\right]}\right)-\rho-\delta-\frac{\varsigma}{\sigma}\right] \\ & -\chi\sigma\frac{\beta_Z\left(\epsilon-1\right)\alpha L^J\left(\frac{\left(1+\tau^J\right)\zeta D_0}{\left(1+D_0\chi^J\right)^2}+\frac{(1-\zeta)D_1}{\left(1+D_1\chi^J\right)^2}\right)}{\epsilon N^J\left[1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]\right]} \end{aligned}$$

$$F_N = -\chi\sigma\frac{\beta_Z\left(\epsilon-1\right)\alpha L^J}{\epsilon\left(N^J\right)^2}\left(\frac{\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha\left(1-\zeta\right)-\rho\alpha\zeta\beta_N\right]}\right);$$

$$G_\chi = N\frac{\sigma\left(\epsilon-1\right)\left(\frac{\left(1+\tau^J\right)\zeta D_0}{\left(1+D_0\chi^J\right)^2}+\frac{(1-\zeta)D_1}{\left(1+D_1\chi^J\right)^2}\right)}{\beta\left(1+\zeta\tau^J\right)};$$

$$G_N = \frac{\left(1+\zeta\tau^J\right)B-\sigma\left(\epsilon-1\right)\left(\frac{\left(1+\tau^J\right)\zeta}{1+D_0\chi^J}+\frac{(1-\zeta)}{1+D_1\chi^J}\right)-2M\epsilon N^J\frac{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}{L^J\alpha}}{\beta\left(1+\zeta\tau^J\right)}$$

The first $\chi = 0$ and $N > 0$ implies

$$\frac{\alpha L^J \left(1+\zeta\tau^J\right)}{\epsilon M}\left(\frac{B-\sigma(\epsilon-1)}{1-\rho\alpha\beta_N+\tau^J\left[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N\right]}\right) = N^*$$

In this case

$$F_\chi = \sigma\left[\frac{\beta_Z M(\epsilon-1)}{B-\sigma(\epsilon-1)} - \rho - \delta - \frac{\varsigma}{\sigma}\right]$$

$$F_N = 0$$

$$G_\chi = N\frac{\sigma(\epsilon-1)\left(\left(1+\tau^J\right)\zeta D_0 + (1-\zeta)D_1\right)}{\beta_N\left(1+\zeta\tau^J\right)}$$

$$G_N = -N\frac{M\epsilon\frac{1-\rho\alpha\beta_N+\tau^J[1-\alpha(1-\zeta)-\rho\alpha\zeta\beta_N]}{L^J\alpha}}{\beta_N\left(1+\zeta\tau^J\right)}$$

The trace, $F_\chi + G_N$, is ambiguous. The determinant is

$$F_X G_N - G_\chi F_N = F_X G_N$$

If F_χ is positive, determinant is negative and hence unstable. If F_χ is negative, determinant is positive. The trace is negative and hence it is stable. Therefore, if

$$\sigma K > \varsigma > 0, \tag{64}$$

where $K \equiv \frac{\beta_Z\sigma(\epsilon-1)M}{B_0-\sigma(\epsilon-1)} - (\rho+\delta)$, the interior steady state is the only stable steady state.