Airport-Efficiency Measurement: Methodologies, Data Requirements, and Monte-Carlo Experiments

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1. Introduction

Unlike the standard economic theory assumes, the activity of airports yields various kinds of outputs, such as passenger loading/unloading, aircraft movements, and cargo handling. Meanwhile, these outputs share the same set of inputs: capital, labor, land, and other miscellaneous materials. Therefore, measuring the efficiency of the airport activities gives rise to several different methods that are suited to this joint-production characteristic. The paper briefly discusses the conventional methods such as partial-factor-productivity method, TFP method, and DEA-oriented methods, and points out respectively their virtue, as well as their built-in limitations in section 2.

As its main contribution, section 3 of this paper then develops the Endogenous-Weight TFP method, which is the method actually employed in the Airport-Benchmarking Project accomplished by the Air Transport Research Society [3]. This method will eliminate those limitations inherent to the conventional methods while retaining the virtue of them to the full extent. This new method is indeed general enough to be applied to any other joint-production industry other than airport operation. The paper also presents simulation results from Monte-Carlo experiments in an efficiency-ranking problem in the following section. Finally, section 5 concludes.

2. Conventional Methods

2.1. Total-Factor-Productivity (TFP) Method

One easy way of measuring the production efficiency would be the partial-factor productivity in which one each of input and output factors among multiple inputs

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and outputs are chosen to form a ratio. It is convenient and easy to compute, however, there are many values for one firm/period, such as fuel efficiency or labor efficiency, measured in terms of passenger or cargo volume, and thus it will never represent a firm's overall productivity/efficiency. This leads to the development of the methods that uniquely measure overall production efficiency.

Caves, Christensen, and Diewert [5] proposed multilateral-index method. Under the assumptions that the production structure is well described by the translog transformation function and that the technology is constant-return-to-scale, they have shown that the output index of one entity relative to a hypothetical representative entity can be expressed as a function of output levels and revenue shares, which they call the multilateral output index. Similar derivation shows that the input index can be expressed as a function of input levels of all entities, and their cost shares of each input.

Oum and Yu [9] utilizes this result in the following specification. Let k and j denote two different firms or time periods, then the gross TFP is computed as follows, as a comparison across firms or time periods:

$$\frac{TFP_k}{TFP_j} = \frac{Y_k/Y_j}{X_k/X_j} \tag{2.1}$$

where

$$Y_k = \prod_i \left(\frac{Y_{ik}}{\tilde{Y}_i}\right)^{\frac{R_{ik}+\bar{R}_i}{2}} \tag{2.2}$$

$$Y_j = \prod_i \left(\frac{Y_{ij}}{\tilde{Y}_i}\right)^{\frac{R_{ij}+\bar{R}_i}{2}} \tag{2.3}$$

$$X_k = \prod_i \left(\frac{X_{ik}}{\tilde{X}_i}\right)^{\frac{W_{lk}+\bar{W}_l}{2}} \tag{2.4}$$

$$X_j = \prod_l \left(\frac{X_{lj}}{\tilde{X}_l}\right)^{\frac{W_{lj}+\bar{W}_l}{2}} \tag{2.5}$$

and

Y_{ik} : output i for observation k
R_{ik} : revenue share of output i for observation k
$\bar{R}_i$: arithmatic mean of revenue shares of output i over all observations
$\tilde{Y}_i$: geometric mean of output i over all observation
X_{lk} : input l for observation k
W_{lk} : input-cost shares for input l for observation k
$\bar{W}_l$: arithmatic mean of input-cost shares of input l over all observations.

Required data are any two of

· revenue for each output for each firm/period
· price of each output for each firm/period
· quantity of each output for each firm/period

and any two of

· expenditure for each input for each firm/period
· price of each input for each firm/period
· quantity of each input for each firm/period.

Once the gross TFP index is computed as above, it is regressed against a set of explanatory variables to obtain the residual TFP. Two main objectives of this two-step method are; to exclude the effects of variations such as those that are beyond the managerial control; and to decompose the source of (in)efficiency of a particular firm. This procedure is often referred to as the two-step TFP method.

2.2. Data-Envelopment-Analysis (DEA) Method

The original idea of the DEA method comes from Farrell [8]. More recently, Pels, Nijkamp, and Rietveld [10] defines the production efficiency of a firm, whose input and output vectors are $(x_{1,0}, \cdots, x_{N,0})$ and $(y_{1,0}, \cdots, y_{M,0})$ respectively, as follows:

$$\min_{\lambda_1,\cdots,\lambda_L} \quad h_0 \tag{2.6}$$

$$s.t. \quad \sum_{l=1}^{L} \lambda_l y_{i,l} = y_{i,0}, \qquad \forall i = 1, \cdots, M \tag{2.7}$$

$$\sum_{l=1}^{L} \lambda_l x_{j,l} = h_0 x_{j,0}, \qquad \forall j = 1, \cdots, N \tag{2.8}$$

$$\lambda_1, \cdots, \lambda_L, h_0 \geq 0 \tag{2.9}$$

$$M + N \leq L + 1. \tag{2.10}$$

where L is the number of firms, N is the number of inputs, and M is the number of outputs. The idea is that λ's change to maximize the difference between the input vector of the firm and the linear combination of that of others, while holding the output vector identical. Therefore, the efficiency measured through this method is input-based. Also, since there is no restriction on the value of λ's, which is the weight for the linear combination, this method assumes constant return to scale. Figure 1 depicts the case of one input, one output, and two firms with subscripts 0 and l. In this situation, one may argue that local decreasing return to scale is prevailing, as the sum of the λ's, the weights in the linear combination, which is just one number λ, is greater than unity.

DEA method has an advantage that it does not require price (or, revenue/cost) data, however, it is sensitive to outliers and usually there are more than one most efficient observations. This multiplicity of efficient units becomes more serious when you impose variable or decreasing return to scale assumptions or when there are more inputs/outputs.[1]

[1] One way to alleviate, not to eliminate this problem is to use the principle-component method to reduce the number of variables.

Fare, Grosskopf, and Lovell [7] extends the DEA method introduced above, which is referred to as Malmquist Productivity Measurement. One difference of their method from the original DEA method is that it allows the decreasing return to scale. Another is that it decomposes measured inefficiency into several different reasons such as return to scale, allocation of inputs/outputs.

Let $\mathbf{u}^j$ and $\mathbf{x}^j$ be the output and input vectors of firm $j \in \{1, \ldots, J\}$ in R_+^M and R_+^N respectively. Also, let $\mathbf{M}$ and $\mathbf{N}$ be the output and input matrices of these J firms. Then under the constant-return assumption, the feasible set of input vectors given output, defined as $L(\mathbf{u}|\, C, S)$ is given as follows:

$$L(\mathbf{u}|\, C, S) = \left\{ \mathbf{x}|\, \mathbf{u} \leq \mathbf{zM}, \mathbf{x} \leq \mathbf{zN}, \mathbf{z} \in R_+^J \right\}, \quad \mathbf{u} \in R_+^M. \tag{2.11}$$

This situation is depicted in Figure 2 where there are only one each of input and output and two firms. In the case of non-increasing (decreasing) return to scale, (2.11) becomes as follows:

$$L(\mathbf{u}|\, N, S) = \left\{ \mathbf{x}|\, \mathbf{u} \leq \mathbf{zM}, \mathbf{x} \leq \mathbf{zN}, \mathbf{z} \in R_+^J, \sum_{j=1}^{J} z_j \leq 1 \right\}, \quad \mathbf{u} \in R_+^M. \tag{2.12}$$

Note the change in the notation: now the feasible set is denoted by $L(\mathbf{u}|\, N, S)$. Figure 3 depicts the situation of the same example as above. Finally, in the case of variable-return to scale, feasible set denoted by $L(\mathbf{u}|\, V, S)$ is given as

$$L(\mathbf{u}|\, V, S) = \left\{ \mathbf{x}|\, \mathbf{u} \leq \mathbf{zM}, \mathbf{x} \leq \mathbf{zN}, \mathbf{z} \in R_+^J, \sum_{j=1}^{J} z_j = 1 \right\}, \quad \mathbf{u} \in R_+^M. \tag{2.13}$$

Figure 4 depicts the situation where there are only one each of input and output and two firms. Note that the following is always true:

$$L(\mathbf{u}|\, V, S) \subset L(\mathbf{u}|\, N, S) \subset L(\mathbf{u}|\, C, S). \tag{2.14}$$

Define the efficiency measure of a firm whose input and output vectors are given as $\mathbf{x}$ and $\mathbf{u}$ respectively, denoted by F, as the following:

$$F = \arg\min_{\lambda} \lambda \mathbf{x} \tag{2.15}$$
$$s.t. \quad \lambda \mathbf{x} \in L(\mathbf{u}). \tag{2.16}$$

Note that F can take different values depending upon the assumption on the returns to scale. As shown in Figure 5, it is straightforward to see that

$$F|\, C, S \leq F|\, V, S. \tag{2.17}$$

Efficiency loss due to the return-to-scale assumption, denoted by S, is therefore always smaller than or equal to unity:

$$S \equiv \frac{F|\, C, S}{F|\, V, S} \leq 1. \tag{2.18}$$

Argument above is all referred to as the input-based productivity measure. Similar logic can be developed from the output point of view. That is to compare the level of output, given the input vector. This is referred to as the output-based productivity measure. Both of these input and output-based productivity measures are direct measurement. Alternative productivity measures are indirect ones, in the sense that they do not control output or input, but instead, controls total revenue or costs.

The first of the indirect measures uses the revenue, instead of the output vector, to define the feasible input set. This set is denoted by $IL(\mathbf{r}/R)$ where $\mathbf{r}$ is the output-price vector and R is the target revenue. There can be multiple output vectors that generate the same level of revenue, given the output-price vector. Therefore, IL always contains L as its subset:

$$L(\mathbf{u}) \subset IL(\mathbf{r}/R(\mathbf{u})). \tag{2.19}$$

This implies that the efficiency measured by using the $L(\mathbf{u})$ is higher than that from $IL(\mathbf{r}/R(\mathbf{u}))$: this difference captures the inefficiency in the choice of output combination.

Similar logic can be developed by controlling the cost, instead of the input vector. Doing this will capture the inefficiency arising from the allocation of inputs.

3. Endogenous-Weight TFP Method

3.1. Motivation

As we have seen above, TFP method uses revenue (or price) data to index the input and output index. DEA method does not have this high data requirement as it uses physical data only; however, it is extremely sensitive to outliers and typically, and it has multiple "best performers."

To surpass this dilemma, this section proposes another alternative method that does not require price or revenue data, and at the same time it does not have the sensitivity that DEA method has. The fundamental idea is to measure the production transformation function where there are multiple inputs and outputs. The specified model is very flexible in the sense that it endogenously determines the parameters such as returns to scale and elasticities of substitutions among inputs and outputs.

Since the method does not incorporate the price data, what it measures primary is the technical efficiency but not the price (or, allocation) efficiency. Once you obtain the price data, however, it is not a difficult task to measure the price efficiency using the measured production function.

3.2. Specification and Estimation

Suppose there are T observations. Let $X_{1,t}$ through $X_{m,t}$ and $Y_{1,t}$ through $Y_{n,t}$ be the tth observation on m inputs and n outputs respectively, both measured in physical units. Define $x_{i,t}$ and $y_{i,t}$ as standardized input and output for the tth observation as follows:[2]

$$x_{i,t} \equiv \exp\left(\frac{\ln X_{i,t} - \ln \tilde{X}_i}{s_{\ln X_i}}\right), \quad i = 1, \cdots, m; \quad t = 1, \cdots, T, \tag{3.1}$$

$$y_{i,t} \equiv \exp\left(\frac{\ln Y_{i,t} - \ln \tilde{Y}_i}{s_{\ln Y_i}}\right), \quad i = 1, \cdots, n; \quad t = 1, \cdots, T, \tag{3.2}$$

where $s_{\ln X_i}$ ($s_{\ln Y_i}$) is the sample standard deviation of $\ln X_i$ ($\ln Y_i$), and tilde ($\sim$) means the geometric mean, i.e.,

$$\ln \tilde{X}_i \equiv \frac{1}{T}\sum_{t=1}^{T} \ln X_{i,t}, \quad i = 1, \cdots, m, \tag{3.3}$$

$$\ln \tilde{Y}_i \equiv \frac{1}{T}\sum_{t=1}^{T} \ln Y_{i,t}, \quad i = 1, \cdots, n. \tag{3.4}$$

In what follows, subscript t is abreviated.

Multiple-input-multiple-output production function is defined as

$$f(x_1, \cdots, x_m, y_1, \cdots, y_n) = 0 \tag{3.5}$$

This equation defines $m+n-1$ dimensional (hyper-)surface in $m+n$ dimensional space. In other words, to each input vector x, a set of feasible production vectors (i.e., production-possibility frontier) corresponds. For simplicity, the production function is assumed to possess the separability:[3]

$$f(g(x_1, \cdots, x_m), y_1, \cdots, y_n) = 0. \tag{3.6}$$

This means that there exist isoquants, at any point of which the corresponding production-possibility frontier is identical.

[2]The followin alternative yields $\bar{x}_i$ and $\bar{y}_i$ being unity:

$$x_{i,t} \equiv \frac{\exp\left(\frac{\ln X_{i,t} - \ln \tilde{X}_i}{s_{\ln X_i}}\right)}{\frac{1}{T}\sum_{t=1}^{T}\exp\left(\frac{\ln X_{i,t} - \ln \tilde{X}_i}{s_{\ln X_i}}\right)}, \quad i = 1, \cdots, m$$

$$y_{i,t} \equiv \frac{\exp\left(\frac{\ln Y_{i,t} - \ln \tilde{Y}_i}{s_{\ln Y_i}}\right)}{\frac{1}{T}\sum_{t=1}^{T}\exp\left(\frac{\ln Y_{i,t} - \ln \tilde{Y}_i}{s_{\ln Y_i}}\right)}, \quad i = 1, \cdots, n.$$

[3]This separability assumption is solely for the simplicity. Without this assumption, (3.5) is estimated directly, after specifying some approprеate functional form.

Specify the production transformation function as a CES function as the following:

$$\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right)^{1/\gamma} = A\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)^{\delta/\rho}. \tag{3.7}$$

This CES specification is a variant of the standard specification, and is deliberately chosen to obtain a unique set of parameter estimates in the actual estimation.[4] Note that γ and ρ respectively represent the elasticity of substitution among outputs and inputs, and δ captures the return to scale. In order to explain the idiosyncracy of each entity, after taking the log of (3.7), a disturbance term u is added:

$$\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right) = \ln A + \frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right) + u \tag{3.8}$$

Assume that the disturbance term u has its expected value being zero.[5] Then the estimated parameters should minimize the sum of squared errors:

$$\min_{A,\gamma,\delta,\rho} F \equiv \sum_{t=1}^{T}\left[\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right) - \ln A - \frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)\right]^2. \tag{3.9}$$

Corresponding first-order conditions are as follows:

$$F_A = \frac{-2}{A}\sum_{t=1}^{T}\left[\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right) - \ln A - \frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)\right] \tag{3.10}$$

$$\begin{aligned} F_\gamma = {} & \frac{2}{\gamma}\sum_{t=1}^{T}\left\{\left[\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right) - \ln A - \frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)\right]\right. \\ & \left.\cdot\left[\left(\frac{\sum_{i=1}^{n} y_i^{\gamma}\ln y_i}{\sum_{i=1}^{n} y_i^{\gamma}}\right) - \frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right)\right]\right\} \end{aligned} \tag{3.11}$$

$$\begin{aligned} F_\delta = {} & \frac{-2}{\rho}\sum_{t=1}^{T}\left\{\left[\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right) - \ln A - \frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)\right]\right. \\ & \left.\cdot\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)\right\} \end{aligned} \tag{3.12}$$

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[4] Another possible specification would be the translog production function, however such production function is not valid for entire domain of inputs. See Bendt and Christensen [4] for details.

[5] Maximum-likelihood method requires a stronger assumption that the ratio of input and output indices follows the lognormal distribution (see, for example, Aitchison and Brown [2] for details):

$$\frac{\left(\frac{1}{n}\sum_{j=1}^{n} y_j^{\gamma}\right)^{1/\gamma}}{A\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)^{\delta/\rho}} \sim LN\left[0, \sigma^2\right].$$

Though this is a sufficient condition, it is not necessary for our method. Another alternative is to assume a combination of symmetric and truncated distributions. The asymmetry comes from the idea that the error distribution is a combination of white noise and firm-specific inefficiency, which can only take a negative value on output. See Aigner, Lovell, and Schmidt [1] for details.

$$
\begin{aligned}
F_\rho &= \frac{2\delta}{\rho}\sum_{t=1}^{T}\left\{\left[\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n}y_i^\gamma\right)-\ln A-\frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m}x_i^\rho\right)\right]\right.\\
&\qquad\left.\cdot\left[\frac{1}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m}x_i^\rho\right)-\left(\frac{\sum_{i=1}^{m}x_i^\rho\ln x_i}{\sum_{i=1}^{m}x_i^\rho}\right)\right]\right\}. \qquad (3.13)
\end{aligned}
$$

Solving the above first-order conditions can give the estimates, however, it is analytically difficult. The actual estimation therefore utilizes so-called "steepest-descent" method. After specifying the initial values of parameters to some appropriate level, search proceeds by using the above first-order conditions as a gradient of the objective function. Typically, second-order conditions (or, Hessian) are used to determine the step size: however, this may decrease the computation speed and thus greatly increase the execution time. To avoid this, it is better to set the step size manually, and variably.

3.3. Efficiency Measurement

3.3.1. Output-Oriented Efficiency Measure

Define the output-oriented production efficiency denoted by e_O as the following:

$$
e_O = \frac{\left(\frac{1}{n}\sum_{i=1}^{n}y_i^{\hat\gamma}\right)^{1/\hat\gamma}}{\hat A\left(\frac{1}{m}\sum_{i=1}^{m}x_i^{\hat\rho}\right)^{\hat\delta/\hat\rho}} \qquad (3.14)
$$

where hat (^) denotes the estimate of a parameter. The numerator of the right-hand side in (3.14) gives the actual output index; the denominator in turn gives the expected (or, "average") output index given the input vector. That is, e_O is the ratio of the distances from the origin to the PPF at the expected output index and the actual output vector. The ratio e_O being greater than unity implies efficiency while $e_O < 1$ implies inefficiency.

3.3.2. Input-Oriented Efficiency Measure

Define the input-oriented production efficiency denoted by e_I as the ratio of the distances from the origin to the isoquant at the expected input index and the actual input vector, where the expected input index is simply

$$
\left(\frac{1}{n}\sum_{i=1}^{n}y_i^{\hat\gamma}\right)^{1/\hat\gamma}. \qquad (3.15)
$$

This implies that e_I satisfies

$$
\left(\frac{1}{n}\sum_{i=1}^{n}y_i^{\hat\gamma}\right)^{1/\hat\gamma} = \hat A\left(\frac{1}{m}\sum_{i=1}^{m}(e_I x_i)^{\hat\rho}\right)^{\hat\delta/\hat\rho}, \qquad (3.16)
$$

which further implies that

$$e_I = \frac{\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\hat{\gamma}}\right)^{1/\hat{\delta}\hat{\gamma}}}{\hat{A}\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\hat{\rho}}\right)^{1/\hat{\rho}}}. \tag{3.17}$$

Again, e_I being greater than unity implies efficiency while $e_I < 1$ implies inefficiency.

3.4. Price/Allocation Efficiency Measurement

The previous subsections have shown that the endogenous-weight TFP method enables the measurement of production efficiency without referring to the price data. However, once the input-price data are obtained, it is relatively straightforward to find price efficiency using the estimated production function. Figure 6 illustrates the idea in two-input case. In Figure 6, a point A represents the input combination for one observation; the isoquant Y_a is the output level obtained by using the estimated parameters and output data for this observation in the left-hand side of (3.7); Y_1 represents the technically efficient (but not in price) output level expressed in terms of the input index obtained by using the estimated parameters and input data x_i's in the right-hand side of (3.7); and Y_0 represents the efficient (both in terms of technology and price) output level which is obtained as the solution of the following maximization problem:

$$\begin{aligned} &\max_{\mathbf{x}} \left[x_1^{\hat{\rho}} + \cdots + x_m^{\hat{\rho}}\right]^{\hat{\delta}} \\ &s.t. \qquad \mathbf{px} = \mathbf{px}_i \end{aligned} \tag{3.18}$$

where $\mathbf{x} = \{x_1, \cdots, x_m\}$, $\mathbf{p} = \{p_1, \cdots, p_2\}$, and $\mathbf{x}_i$ is the actual input vector of the ith observation. The difference (or ratio) between Y_a and Y_1 gives the technical (in)efficiency; the difference (or ratio) between Y_0 and Y_1 gives the price (in)efficiency; and the difference (or ratio) between Y_a and Y_0 gives the overall (in)efficiency. Similar logic can be applied to the outputs to obtain the allocation (in)efficiency.

4. Monte-Carlo Experiment

4.1. Experiment 1: the Two-Input-Two-Output Case

4.1.1. Data Generation

By using the equation (3.7), a sample of 100 observations on two inputs and two outputs is generated under a parameter setting of

$$A = \sqrt{2} \tag{4.1}$$

$$\delta = 1 \tag{4.2}$$

$$\gamma = 2 \tag{4.3}$$

$$\rho = 1, \tag{4.4}$$

while the probability distribution of the error is set as logit function:

$$\Pr\left(\left[\frac{1}{\gamma}\ln\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\gamma}\right) - \ln A - \frac{\delta}{\rho}\ln\left(\frac{1}{m}\sum_{i=1}^{m} x_i^{\rho}\right)\right] < Z\right) = \frac{e^{10(Z-1)}}{1+e^{10(Z-1)}} \tag{4.5}$$

where $n = m = 2$. Observations are sorted in terms of the error size which is interpreted as the efficiency, and are listed in Table 1. Note that the observation name reflects its "true" ranking.

4.1.2. Estimation Results

The estimation results of this Monte-Carlo experiment is as follows:

$$\hat{A} = 1.518 \quad (0.059) \tag{4.6}$$
$$\hat{\delta} = 1.075 \quad (0.064) \tag{4.7}$$
$$\hat{\gamma} = 2.143 \quad (0.325) \tag{4.8}$$
$$\hat{\rho} = 0.992 \quad (0.108)\,. \tag{4.9}$$

Bootstrapping yields the standard deviations that are given in brackets above.[6]

By using these parameter estimates, efficiency measures are calculated and presented in Table 2. Figure 7 shows the scatter diagram between the true and measured efficiency of these 100 observations. The correlation coefficient between the two is calculated as .977. By sorting observations according to this measured efficiency, the measured ranking is obtained and it is depicted against the true ranking in Figure 8. The rank correlation, which is the Spearman's order-correlation coefficient, is calculated to be .967.

4.2. Experiment 2: the Three-Input-Three-Output Case

4.2.1. Data Generation

The previous experiment has resulted in very high correlations both in efficiency itself and the ranking. One reason behind this result is that the data-generating mechanism is indeed the model specification that is used in estimation. Now, in order to verify the flexibility of this measurement methodology, an alternative specification, namely, non- CES-type functions is used in data generation. To have heterogeneity in the elasticity of substitution among inputs and outputs, more than two variables are necessary for each of input and output sides.

A sample of 50 observations on three inputs and three outputs is generated under a parameter setting of

$$\frac{1}{3}\left(y_1^2 + y_2^2 + y_3^1\right) = \frac{A}{3}\left(x_1^1 + x_2^{\frac{1}{2}} + x_3^{\frac{1}{2}}\right) \tag{4.10}$$

while the probability distribution of the error is again set as logit function. The generated data set is sorted in terms of efficiency, and listed in Table 3. Again, the observation name reflects its "true" ranking.

[6]See, for example, Efron [6] for the general explanation of the bootstrapping methods.

4.2.2. Estimation Results

There is not much meaning in listing the parameter estimates than using them in calculating the measured efficiency and ranking, as the estimated model (intentionally) mis-specifies the true data-generating mechanism. By using these parameter estimates, efficiency measures are calculated and presented in Table 4. Figure 9 shows the scatter diagram between the true and measured efficiency of these 50 observations, while Figure 10 is the same diagram after taking log of efficiency. The efficiency correlation coefficient is calculated as .939, and the Spearman's rank correlation coefficient is calculated to be .847. Figure 11 depicts the scatter diagram of true and measured rankings.

5. Conclusions

The endogenous-weight TFP method has the advantage of both index-TFP and DEA methods: it is not sensitive to outliers, and at the same time, data requirement is not strict, or in other words, there is no arbitrariness (or approximation) in the weight. Moreover, it gives a unique "best performer" unlike DEA-based methods.

Since all these methods define efficiency as the ratio between output and input indices, there may be a heteroschedasticity problem. That is, for smaller firms, efficiency measures may vary greatly, depending on the subtle difference in absolute level of output/input. This means, for smaller firms, the efficiency measures are more sensitive to the measurement error of the same magnitude than that for larger firms.

There are some factors that are beyond control of the airport authority. In order to better estimate the efficiency, after obtaining the gross TFP measurements, it should be regressed against a set of variables such as weather factor, proportion of government ownership, and so on, just as described in the two-step TFP method. Also, the airport performance is not only determined through the supply-side factors, but also demand factors as well. Incorporating such aspects into the model is desirable, however, at the same time this will face the data-availability problem.

As the results of Monte-Carlo experiments show, the rank correlation was lower in the second case where the true relationship is not CES. To better capture the true ranking, the model should be extended to accommodate non-CES type production functions. This is left for the future work.

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