

# Multidimensional Networks: Creating a New Framework of Organization Structure for Complex Strategies

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## Abstract

Firms today typically face and must coordinate multiple dimensions of competition. This paper extends the conventional notion of organization structure and recaptures it as an aggregation of differentiated networks specialized for multiple dimensions of strategic attention. Graph-theoretic languages commonly used for network representations are not sufficient to capture such aggregations. In this study, we develop an extension of directed graphs called *modal graphs* to represent and analyze the multidimensional complexity of organization structure. In modal graphs, elementary building blocks are *coordination clauses*, each of which is qualified by a strategic focus called a *modality*, a concept adapted from sentential modalities in modal logic. Individual networks of the firm's structure are then given by compositions of related coordination clauses. While modal graphs enable massively networked structures, it is still desirable to designate a dominant dimension as the conventional reporting hierarchy. We identify a class of modal graphs and develop soundness and completeness results for desirable reporting hierarchies with respect to this class. Namely, we construct a procedure and show that the procedure identifies *only* desirable reporting hierarchies (soundness) and *all* the desirable reporting hierarchies (completeness).

## 1. Introduction

In the decades of inquiry into the workings of organizational effectiveness, there has been a vast accumulation of theoretical and empirical research results on possible factors of organizational effectiveness and their interplay. Invariably throughout this inquiry, organization structure has been a primary factor of effectiveness, and its "fit" with other factors has been vigorously investigated through various frameworks of fit contingency and configuration (e.g., Chandler 1962, Galbraith 1973, Miller 1986, Nadler and Tushman 1997, Donaldson 2001, Burton et al. 2002, Burton and Obel 2005). This paper aims to contribute to this long line of research on organizational effectiveness through an enhanced notion of organization structure both in terms of its conceptual scope and its representational rigor.

Initially, the conceptual scope of organization structure was largely confined to hierarchies of vertical reporting relationships. While hierarchical grouping generally facilitates intra-unit communication and interaction, it also creates barriers for inter-unit coordination. Firms have hence devised various means of formal horizontal linkages across vertical boundaries (Galbraith 1973, Davis and Lawrence 1977, Nadler and Tushman 1997). More recently, extensive research attention has been given to boundary spanning relationships of a more spontaneous, emergent nature (Tushman and Katz 1980, Perrone et al. 2003, Carlile 2004, Kellogg et al. 2006). An interesting line of research here is the interplay between formal structures and informal, spontaneous networks of relationships. Fenton and Pettigrew (2000), for instance, reports a case in which informal, spontaneous networks developed into formally institutionalized structures. Inspired by Giddens' theory of structuration (Giddens 1984), there has been an active line of research on the dynamics of formal-informal interplay (e.g., Desanctis and Poole 1994, Orlikowski 2000, 2002). Unfortunately, however, the conventional notion of organization structure sits at a distance too far from the spontaneous-emergent camp of organization studies to benefit from insights on the recursive dynamics of individual actors and institutionalized structures. What is desirable here, then, is an extended notion of organization structure that captures networks of relationships of distinct flavors: vertical, horizontal, or emergent.

In his agenda "from command-and-control to coordinate-and-cultivate", Malone (2004, Part III) proposes "coordinate-and-cultivate" as a general framework of management in which generally formal "command-and-control" is merely a special case. Thus, the coordinate-and-cultivate framework represents "the whole range of possibilities of management, from the completely centralized to the completely decentralized" (Malone 2004, p.12). Along a similar line, we view the structure of an organization as the total aggregation of all relationships arising in its intended or emergent coordination activities.

This integrative view of organization structure is, however, not entirely new. "Differentiated networks" (Ghoshal and Bartlett 1990, Nohria and Ghoshal 1997) and "structured networks" (Goold and Campbell 2002), for instance, share similar views of organization structure. A point to note here is that the motivation behind their extended views of organization structure is to properly capture the increasing complexity of organization structure that firms struggle to implement in response to accelerating competition and resulting complexity of strategies. More specifically, firms, in many situations, are forced to place simultaneous emphasis on multiple, often conflicting strategic priorities: e.g., global efficiency, local responsiveness and world-wide learning (Bartlett et al. 2003), exploitation and exploration (March 1991), search and stability (Rivkin and Siggelkow 2003). We refer to such strategic priorities as *strategic dimensions* in

order to acknowledge and emphasize the view that they are *orthogonal* to each other in the sense that any one of them cannot be replaced by a combination of others, yet they are not necessarily mutually exclusive and hence can be pursued and implemented simultaneously (O'Reilly and Tushman 2004, Gupta et al. 2006). The notion of “requisite complexity” (Nohria and Ghoshal 1997, p.173) then suggests that multidimensional complexity of the firm’s strategy unavoidably results in similarly multidimensional complexity of the firm’s organization structure.

A dimension of organization structure here manifests in a network of coordination relationships specialized for the corresponding strategic dimension. For instance, the firm’s geographic dimension is intended for its local responsiveness while its product dimension aims to implement its global efficiency. Thus, our view of organization structure, which we refer to as *multidimensional organization structure* in this paper, is an aggregation of multiple networks each of which is specialized for a specific strategic focus. This notion of organization structure also reflects the fact that firms have been exploring various organizational forms of multidimensional nature such as matrix structure (Davis and Lawrence 1977), overlay units (Goold and Campbell 2002), front-back structures (Galbraith 2005) and nested networks (Fenton and Pettigrew 2000, p.93).

Now the question is how multidimensional organization structure can be properly represented and analyzed. Its complexity far exceeds what conventional means of representation and analysis such as organizational charts can offer. There have been some attempts to devise a “language” to better cope with the complexity beyond typical organizational charts (Goold and Campbell 2002, Mintzberg and Van de Heyden 1999). But they too lack rigor and expressiveness. In the style of social network analysis, the differentiated network approach uses a conventional graph-theoretic notation to represent network structures of organizations (Ghoshal and Bartlett 1990). Unfortunately, however, the monolithic nature of the notation fails to differentiate the heterogeneity of multiple subnetworks within a single “differentiated” network (Ghoshal and Bartlett 1990, Figure 1).

In social network analysis, directed graphs are the most commonly used notation for network representation. Again, their flat, undifferentiated structure is not capable of capturing the subnet heterogeneity of multidimensional organization structure. However, the multiplexity of interpersonal relationships has been long acknowledged in the social network community (Kilduff and Tsai 2003, p.33; Borgatti and Foster 2003). For instance, two people might be related as neighbors in the family life setting, colleagues in the workplace setting, and club

members in the setting of some professional society. An extension of directed graphs called multivariate directed graphs (Wasserman and Faust 1994, p.75), more generally known as multigraphs (Bang-Jensen and Gutin, p.18), can be used to capture a collection of different types of networks within a single representation.

In this paper, we develop another extension of directed graphs called *modal graphs*. Modal graphs are closely related to multigraphs, but they use formal constructs called *modalities* to explicitly represent multiple dimensions of organization structure and strategy. As will be discussed in the next section, the notion of modality is adapted from sentential modalities in modal logic (Blackburn et al. 2001). Extended through modality constructs, modal graphs can represent and analyze a multi-layered collection of networks, each specialized for a distinct strategic focus.

Thus, summarizing our approach, the extended conceptual scope of organization structure adopted in this paper is what we call multidimensional organization structure, and the formalism of modal graphs implements its representational and analytical rigor. In the next section, we formally define modal graphs, and give some examples to illustrate their formal constructs and how they can be used to represent multilayer networks. We then develop some mathematical properties of modal graphs and present duality results, which are useful for analysis of multidimensional organization structure. Although modal graphs are very expressive in capturing network complexity and flexibility, one still would like to ensure when and how conventional reporting hierarchies, given modal graphs, can be identified and imposed. We thus identify a class of modal graphs, and obtain soundness and completeness results for this class. Namely, we develop a procedure and show that the procedure constructs *only* desirable reporting hierarchies (soundness) and *all* the desirable reporting hierarchies (completeness). Finally we conclude with a discussion on applications of modal graphs.

## 2. Modal Graphs

In this section, we introduce modal graphs, a language for representation and analysis of multidimensional networks of formal and informal workplace relationships. At the workplace, people coordinate their activities to accomplish a larger task. Typically, the person in charge of a larger task coordinates activities of those in charge of its smaller subtasks. Thus, we consider a collection of positions and coordination relationships among them induced by task-subtask relationships. More specifically, given positions  $p$  and  $q$  in charge of a task and its subtask, respectively, we say  $q$  *answers to*  $p$ . Such “answer to” relationships can be formal and highly

centralized as in reporting relationships, or informal and greatly decentralized. At the extreme end of decentralization, a self-coordinating team may accomplish a shared task through informal communication among its members, without answering to anyone. To capture this type of situations within a uniform formal notation, we consider *empty positions*, and view members of a self-coordinating team as answering to a shared empty position. In addition, we consider such coordination relationships, induced by task-subtask relationships, occur in specific strategic contexts, which we refer to as *modalities*. More formally, we have the following definition.

**Definition 1.** A modal graph is a tuple  $(P, M, R, Z)$  where

- $P$  is a set of *positions*;
- $M$  is a set of *modalities*;
- $R$  is a set of ordered pairs of the form  $(p, a) \rightarrow (q, a)$  where  $p, q \in P$ ,  $p \neq q$  and  $a \in M$ . When  $R$  has a pair  $(p, a) \rightarrow (q, a)$ , we say that  $p$  answers to  $q$  through  $a$ . We call pairs in  $R$  *answer relationships*. When  $p$  answers to  $q$  through  $a$ ,  $a$  is a *north modality* of  $p$  and it is a *south modality* of  $q$ . We assume that for each  $p \in P$ , the set of north modalities of  $p$  is disjoint from the set of south modalities of  $p$ ; and
- $Z$  is a subset of  $P$ , consisting of designated positions called *empty positions*.

Figures 1, 2 and 3 show some examples of modal graphs. As in these examples, we follow some notational conventions. Throughout this paper, letters  $p, q$ , and  $r$ , with or without subscripts, denote positions, while  $a, b, c$ , and  $d$ , with or without subscripts, represent modalities. For formal simplicity, we assume that an empty position has exactly one modality, and we use  $\varphi$ , subscripted with its modality, to denote an empty position. Lines in the modal graphs in the Figures represent answer relationships. Although lines do not have arrowheads to indicate asymmetrical answer relationships, the convention adopted is that when there is a line from a position to another position upward, the first position answers to the second position. Thus, for instance, in Figure 1,  $p_3$  answers to  $p_1$ , that in turn answers to  $p_0$ .

Figure 1 is a schematic representation of a case in which the secondary dimension, indicated by modalities  $b_i$ , is a partial mirror image of the primary dimension, indicated by modalities  $a_i$ . Often, the primary dimension is business units, and the secondary dimension is geography. At DuPont (Galbraith 2000, p.105-106), for instance, two of the business unit executives located in Europe and Asia are expected to develop regional ties with business communities, governments and politicians, while at a lower level, site managers in particular locations (e.g., a manufacturing site in Luxembourg) serves as country managers (e.g., the Luxembourg country manager).

Figure 1 A Modal Graph for Partial-Mirror Coordination

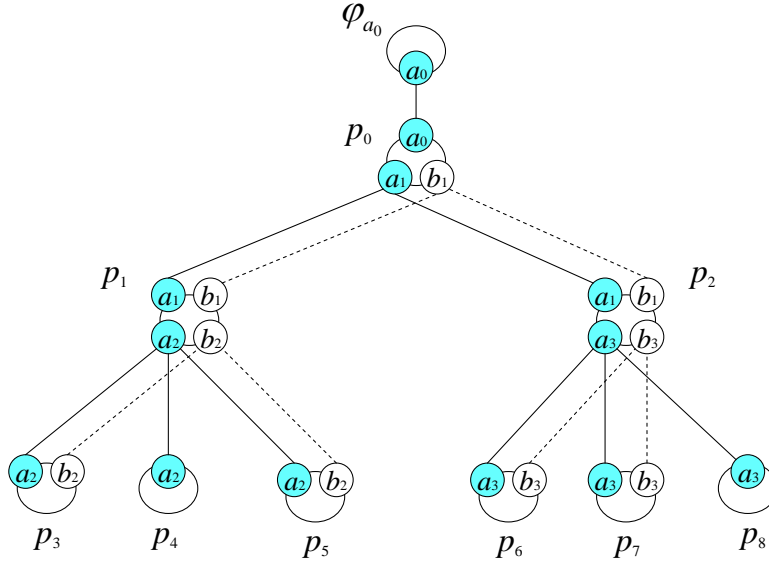
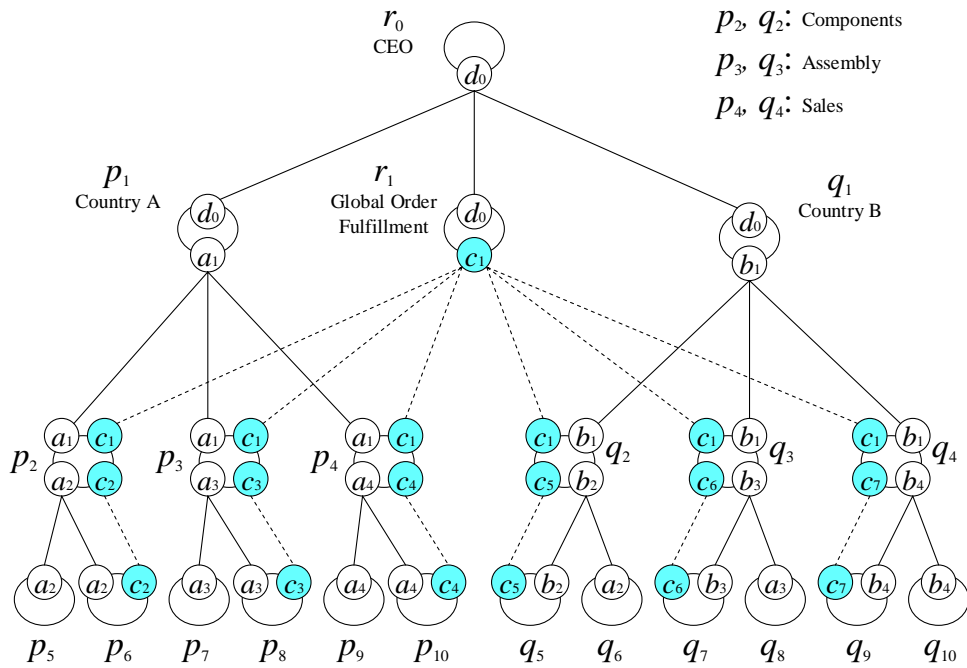


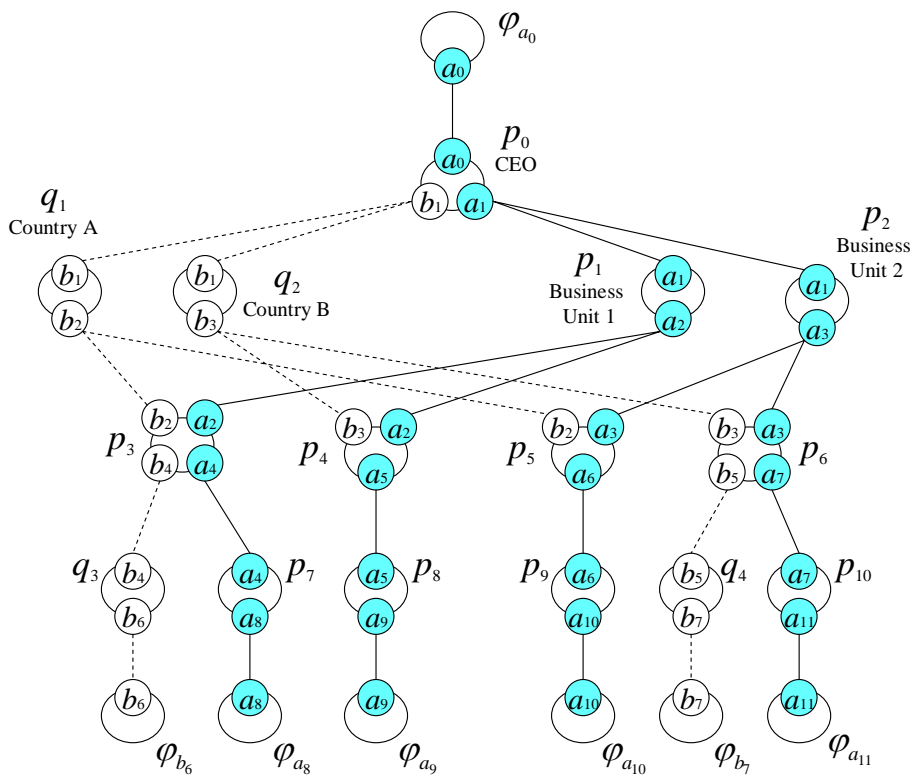
Figure 2 A Modal Graph for Indirect Homogeneous Coordination



Regarding north and south modalities in Definition 1, the position  $p_0$  in Figure 1, for instance, has one north modality  $a_0$  and two south modalities  $a_1$  and  $b_1$ . A north modality of a position is intended to serve as a larger, integrative strategic context in which south modalities of the position can be interpreted. Thus,  $a_0$  at  $p_0$  is intended to give a unifying context for a business unit modality  $a_1$  and a geographic modality  $b_1$ .

Figure 2 schematically illustrates a coordination structure at a U.S. medical equipment company (Galbraith 2000, p.167-174). In this case, structurally homogeneous units located in different countries are coordinated for global efficiency through an extensive, cross-functional, cross-regional team (the unit indicated by modalities  $c_i$ ). These country units are specialized for specific product lines, but these product lines are for global distribution. Hence, these units must be coordinated for order aggregation, production planning, scale-sensitive shared components, etc. The coordinating team, thus, has a large diversity, consisting of members from different regions (such as countries A and B), different functions (such as components, assembly and sales), and the headquarters. Some coordination relationships are maintained virtually through computer-mediated communications.

Figure 3 A Modal Graph for Direct Heterogeneous Coordination



As in positions  $p_2$  and  $q_2$  in Figure 2, when a position has two or more north modalities, it is called a *multimodal position*. Positions such as  $p_1$  and  $p_2$  in Figure 1 are also multimodal positions. While  $p_2$  and  $q_2$  in Figure 2 are the case of so-called two-boss positions,  $p_1$  and  $p_2$  in Figure 1 are not two-boss positions due to the mirror-image nature of the two dimensions.

Figure 3 schematically shows a case in which units with different dimensions (the dimension of business units indicated by modalities  $a_i$  and the geography dimension indicated by modalities  $b_i$ ) are directly interlinked without a separate coordinating unit. This case is, however, different from a balanced matrix, and the dimension of business units represents the dominant dimension. ABB, among many other firms, has shifted from a balanced matrix to this type of coordination structure, which Ruigrok et al. (2000) calls the network multidivisional organization.

Figure 3 also illustrates a few other constructs. Given a modal graph  $\delta$  and a position  $p$  in  $\delta$ , we say that  $p$  is a *boundary position* in  $\delta$ , if  $p$  answers to no positions, or no position answers to  $p$  in  $\delta$ .  $\delta$  is *closed* if every boundary position in  $\delta$  is an empty position. Note that the modal graph in Figure 3 is closed whereas the modal graphs in Figures 1 and 2 are not closed. Additionally, we call an empty position with a south modality a *cap position*, and an empty position with a north modality a *base position*. Similarly, the modality of a cap position is a cap modality, and the modality of a base position is a base modality. The position  $\varphi_{a_0}$  in Figure 3 is a cap position, and positions such as  $\varphi_{b_6}$  and  $\varphi_{a_{10}}$  in Figure 3 are base positions. Also in Figure 3,  $a_0$  is a cap modality while  $b_6$  and  $a_{10}$  are base modalities.

Now we introduce elementary building blocks for composing modal graphs.

**Definition 2.** Given a modal graph  $\delta$ , the set of all answer relationships in  $\delta$  having the same modality is called a *coordination clause* of  $\delta$ . Given a coordination clause, a position having a south modality is called a *head* of the clause, and a position having a north modality an *anchor* of the clause. A coordination clause is *regular* if it has a single head.  $\delta$  is *regular* when every coordination clause in it is regular.

Figure 4 shows coordination clauses of the modal graph in Figure 1. All of them are regular clauses, and hence the modal graph in Figure 1 is regular. Similarly, it is easy to verify that the modal graphs in Figures 2 and 3 are also regular. In this study, we only discuss regular



coordination clauses and regular modal graphs.

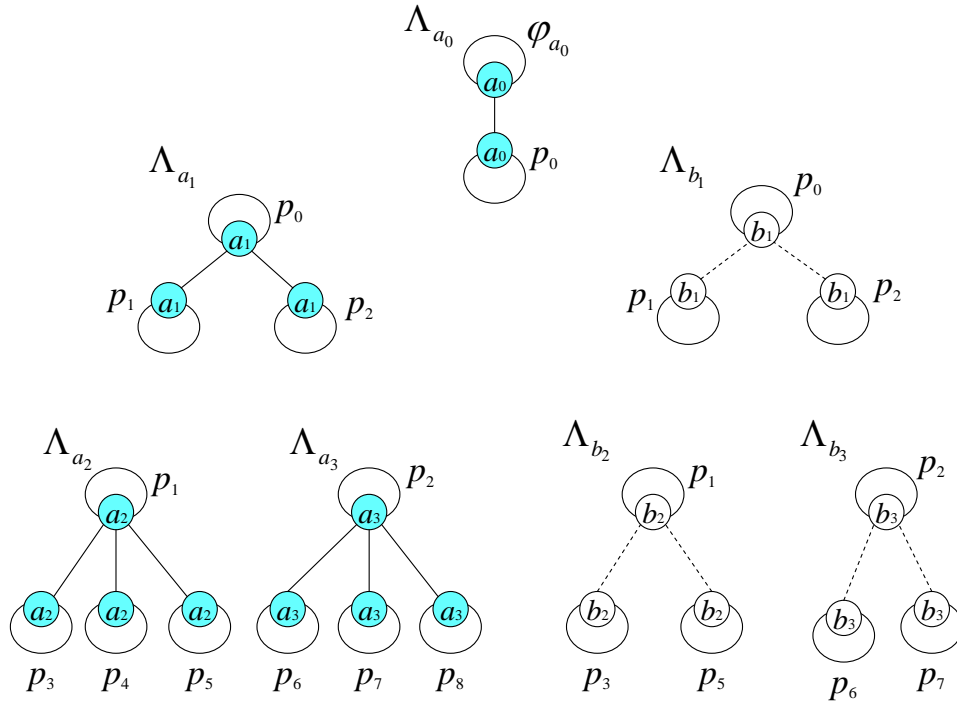
Given a clause of the form  $\{(p_1, a) \rightarrow (p_0, a), (p_2, a) \rightarrow (p_0, a), \dots, (p_n, a) \rightarrow (p_0, a)\}$ , it is sometimes convenient to write it as  $p_0 \xleftarrow{a} p_1, p_2, \dots, p_n$ .

There are essentially three types of clauses:

- $p_0 \xleftarrow{a} p_1, p_2, \dots, p_n$  with  $n \geq 1$  where  $p_0$  and some  $p_i$  are non-empty. These clauses are called *full clauses*.
- $\varphi_a \xleftarrow{a} p_1, p_2, \dots, p_n$  with  $n \geq 1$  where some  $p_i$  are non-empty. These clauses are called *cap clauses*.
- $p \xleftarrow{a} \varphi_a$  where  $p$  is non-empty. These clauses are called *base clauses*.

A clause with  $a$  as its modality is called  $a$ -clause. We write  $\Lambda_a$  to denote  $a$ -clause.

Figure 4 Coordination Clauses of the Modal Graph in Figure 1



As stated earlier, the notion of modality in this paper is loosely related to sentential modalities in modal logic (Blackburn et al. 2001). In modal logic, a modality is a concept that qualifies logical sentences: *e.g.*, “it is possible that ...”, “it is necessary that ...”, “it is obligatory that ...” and “it is permitted that ...”. Similarly, a modality in this paper refers to a strategic focus, or orientation that qualifies coordination clauses, which are elementary “sentences” for building modal graphs: *e.g.*, efficiency focus in production, customer responsiveness in sales, a sense of direction in technology development, and commitment to large-scale effectiveness in supply chain management.

In the next section, we discuss how coordination clauses can be composed. We will then be able to more explicitly specify what we mean by networks with different strategic dimensions within a single organization structure.

### 3. Composition of Coordination Clauses

In this and following sections, many of the constructs we develop for analysis of modal graphs are directed graphs. We summarize here some preliminary concepts of directed graphs.

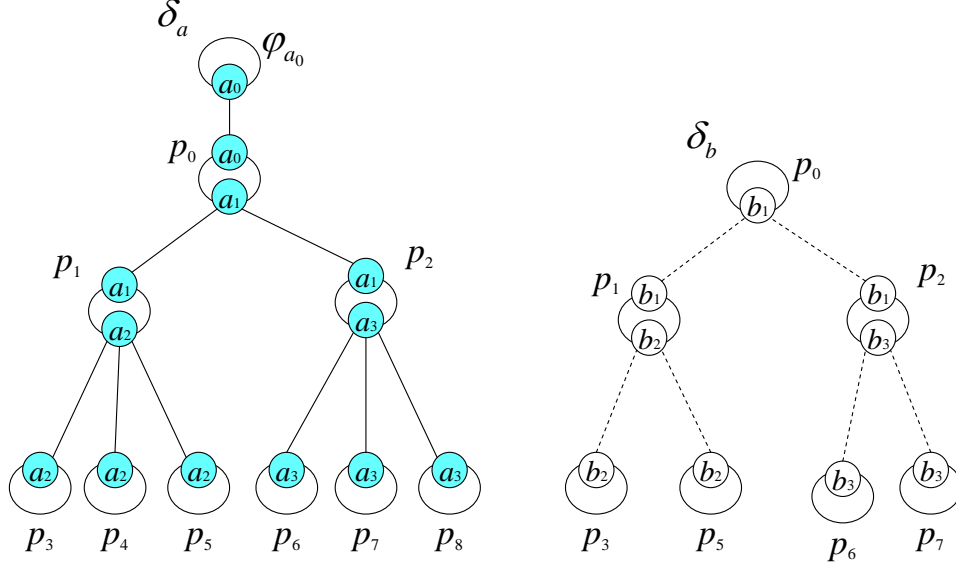
A directed graph is a pair  $(V, E)$  where  $V$  is a set of *vertices* and  $E$  is a set of ordered pairs of distinct vertices called *directed edges*. A directed edge  $(x_1, x_2)$  is written  $x_1 \rightarrow x_2$ .  $x_1$  is called an *in-neighbor* or *child* of  $x_2$ , and  $x_2$  an *out-neighbor* or a *parent* of  $x_1$ . Given a directed graph  $G$ , we write  $V(G)$  to denote the set of all vertices of  $G$ , and  $E(G)$  to denote the set of all directed edges of  $G$ . When a directed graph  $G$  has a chain of directed edges of the form

$$(x_1, x_2), (x_2, x_3), \dots, (x_{n-1}, x_n)$$

we write  $x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow \dots \rightarrow x_{n-1} \rightarrow x_n$  for the chain. When  $G$  has such a chain from  $x_1$  to  $x_n$ , we say that  $x_n$  is *reachable* from  $x_1$  in  $G$ , and write  $x_1 \leq_G x_n$ , or simply  $x_1 \leq x_n$  when  $G$  is clear from the context. For the reason of formal convenience, we assume  $x$  is reachable from  $x$  (and hence  $x \leq x$ ). Given distinct  $x_i$  and  $x_j$ , when they are

mutually reachable from each other, a chain from  $x_i$  to itself through  $x_j$  is a *cyclic chain*.  $G$  is *cyclic* if  $G$  has one or more cyclic chains. Otherwise,  $G$  is *acyclic*. We say  $G$  is *rooted* if there exists  $x \in V(G)$  such that for every  $y \in V(G)$ ,  $y \leq_G x$ .  $x$  is called a *root* of  $G$ . Note that a rooted directed graph has a unique root when it is acyclic, and that the unique root has no parents. A rooted, acyclic directed graph is a *tree* if every vertex other than its root has exactly a single parent.

Figure 5 Modal Graphs Whose Union is the Modal Graph in Figure 1



Now, given a modal graph  $\delta$ , we consider a tree-structured collection of clauses in  $\delta$ , which we call a *clausal composition* in  $\delta$ . Namely, a clausal composition in  $\delta$  is a tree  $\pi$  where  $V(\pi)$  is a set of clauses in  $\delta$ , and  $E(\pi)$  a set of relationships between clauses that capture some proper sense of composing clauses, as we will see. We first introduce *graph union*, which is the main mechanism of composing clauses.

**Definition 3.** Let  $\delta_i = (P_i, M_i, R_i, Z_i)$  be modal graphs for  $i = 1, \dots, n$ , with  $M_i \cap M_j = \emptyset$  for  $i \neq j$ . The *union* of modal graphs  $\delta_i$ , written  $\delta_1 \cup \delta_2 \cup \dots \cup \delta_n$ , is a tuple  $(P, M, R, Z)$  given by:

- $P = \bigcup_{i=1, \dots, n} P_i$
- $M = \bigcup_{i=1, \dots, n} M_i$
- $R = \bigcup_{i=1, \dots, n} R_i$

$$\bullet \quad Z = \bigcup_{i=1, \dots, n} Z_i$$

We say that a union of modal graphs whose modality sets are mutually disjoint, as in the above definition, is a *modally disjoint union* of modal graphs. It is straightforward to verify the following:

**Proposition 1 (Graph Union).** A modally disjoint union of modal graphs is a modal graph.

Proof (see the appendix).

For an example, see the two modal graphs,  $\delta_a$  and  $\delta_b$ , in Figure 5. They are modally disjoint, and their union is the modal graph in Figure 1.

**Definition 4.** Let  $\delta$  be a modal graph. A non-empty position in  $\delta$  is said to be *free* in  $\delta$  if it has no north modalities. Let  $\Lambda$  be a coordination clause. We say that  $\Lambda$  *accepts*  $\delta$  if the modality of  $\Lambda$  is new to  $\delta$  and if every free position of  $\delta$  is an anchor of  $\Lambda$ .  $\Lambda$  is an *interface* for a collection of modal graphs if it accepts every modal graph in the collection.

For an example, see Figure 4. The clause  $\Lambda_{a_1}$  is an interface for modal graphs  $\Lambda_{a_2}$  and  $\Lambda_{a_3}$  (which happen to be in clausal form).

Now we are ready to give a formal definition of clausal composition. The definition is recursive.

**Definition 5.** Let  $\delta$  be a modal graph. If  $\Lambda$  is a coordination clause in  $\delta$ ,  $\Lambda$  is a (trivial) *clausal composition in  $\delta$* , which *denotes* the modal graph  $\Lambda$ . Let  $\pi_1, \pi_2, \dots, \pi_n$  be clausal compositions in  $\delta$  such that modal graphs  $\bar{\pi}_1, \bar{\pi}_2, \dots, \bar{\pi}_n$  denoted by  $\pi_1, \pi_2, \dots, \pi_n$ , respectively, have pair-wise disjoint sets of modalities. Let  $\Lambda$  be a coordination clause in  $\delta$  which is an interface for  $\{\bar{\pi}_1, \bar{\pi}_2, \dots, \bar{\pi}_n\}$ . Then the expression  $\Lambda(\pi_1, \pi_2, \dots, \pi_n)$  is a *clausal composition in  $\delta$* , which *denotes* the modal graph  $\Lambda \cup \bar{\pi}_1 \cup \bar{\pi}_2 \cup \dots \cup \bar{\pi}_n$ . A clausal composition of a modal graph  $\delta$  is a clausal composition in  $\delta$  that denotes  $\delta$ .

Note that given a clausal composition in  $\delta$ , every clause in the composition appears exactly once in the composition. The recursive nature of the definition captures tree structure of clausal compositions. For a simple example, consider the following composition, referring to Figure 4.

$$\pi = \Lambda_{a_0}(\Lambda_{a_1}(\Lambda_{a_2}, \Lambda_{a_3}), \Lambda_{b_1}(\Lambda_{b_2}, \Lambda_{b_3})).$$

$\pi$  denotes the modal graph in Figure 1.  $\pi$  can be seen as a tree graph given by:

$$V(\pi) = \{\Lambda_{a_0}, \Lambda_{a_1}, \Lambda_{a_2}, \Lambda_{a_3}, \Lambda_{b_1}, \Lambda_{b_2}, \Lambda_{b_3}\}$$

$$E(\pi) = \{\Lambda_{a_1} \rightarrow \Lambda_{a_0}, \Lambda_{b_1} \rightarrow \Lambda_{a_0}, \Lambda_{a_2} \rightarrow \Lambda_{a_1}, \Lambda_{a_3} \rightarrow \Lambda_{a_1}, \Lambda_{b_2} \rightarrow \Lambda_{b_1}, \Lambda_{b_3} \rightarrow \Lambda_{b_1}\}$$

Note that compositions  $\Lambda_{a_0}(\Lambda_{a_1}(\Lambda_{a_2}, \Lambda_{a_3}))$  and  $\Lambda_{b_1}(\Lambda_{b_2}, \Lambda_{b_3})$  denote, respectively, the modal graphs  $\delta_a$  and  $\delta_b$  in Figure 5. Thus, given a modal graph, its component networks can be captured as clausal compositions in the modal graph.

#### 4. Positional and Modal Views and their Duality

When a modal graph  $(P, M, R, Z)$  has answer relationships  $(p, a) \rightarrow (q, a)$ , pairs such as  $(p, a)$  and  $(q, a)$  are called *fronts* of the modal graph. As  $a$  is a north modality of  $p$ ,  $(p, a)$  is called a *north front*, and sometimes written  $(p, a)^N$ . Similarly,  $(q, a)$  is called a *south front*, and sometimes written  $(q, a)^S$ . Every front in a modal graph is either a north front or a south front, but not both at the same time, due to the modal separation condition given in Definition 1.

Given a modal graph  $\delta$ , let  $X_\delta$  be the set of all fronts in  $\delta$ . We define an equivalence relation,  $\approx_p$ , on  $X_\delta$ . Namely,  $x \approx_p y$  if  $x$  and  $y$  has the same position. We write  $[q]$  to denote the equivalence class of a position  $q$  under the relation  $\approx_p$ , i.e., the set of all fronts having  $q$  as the position. The set of all such equivalence classes of  $X_\delta$  is called the *quotient set* of  $X_\delta$  under the relation  $\approx_p$ , and written  $X_\delta / \approx_p$ . Similarly, we define another equivalence relation,  $\approx_m$ , on  $X_\delta$ . Namely,  $x \approx_m y$  if  $x$  and  $y$  has the same modality. We write  $[a]$  to denote the equivalence class of a modality  $a$  under the relation  $\approx_m$ , i.e., the set of all fronts having  $a$  as the modality. We write  $X_\delta / \approx_m$  to denote the quotient set of  $X_\delta$  under  $\approx_m$ , i.e., the set of all equivalence classes of  $X_\delta$  under  $\approx_m$ .

**Definition 6.** Let  $\delta$  be a modal graph, and  $X_\delta$  be the set of all fronts of  $\delta$ . The *position quotient* of  $\delta$ , written  $\mathbf{pQ}(\delta)$ , is a directed graph given by:

$$V(\mathbf{pQ}(\delta)) = X_\delta / \approx_p$$

$$E(\mathbf{pQ}(\delta)) = \{[q] \xrightarrow{a} [p] \mid (q, a)^N \in [q], (p, a)^S \in [p]\}$$

Similarly, the *modal quotient* of  $\delta$ , written  $\mathbf{mQ}(\delta)$ , is a directed graph given by:

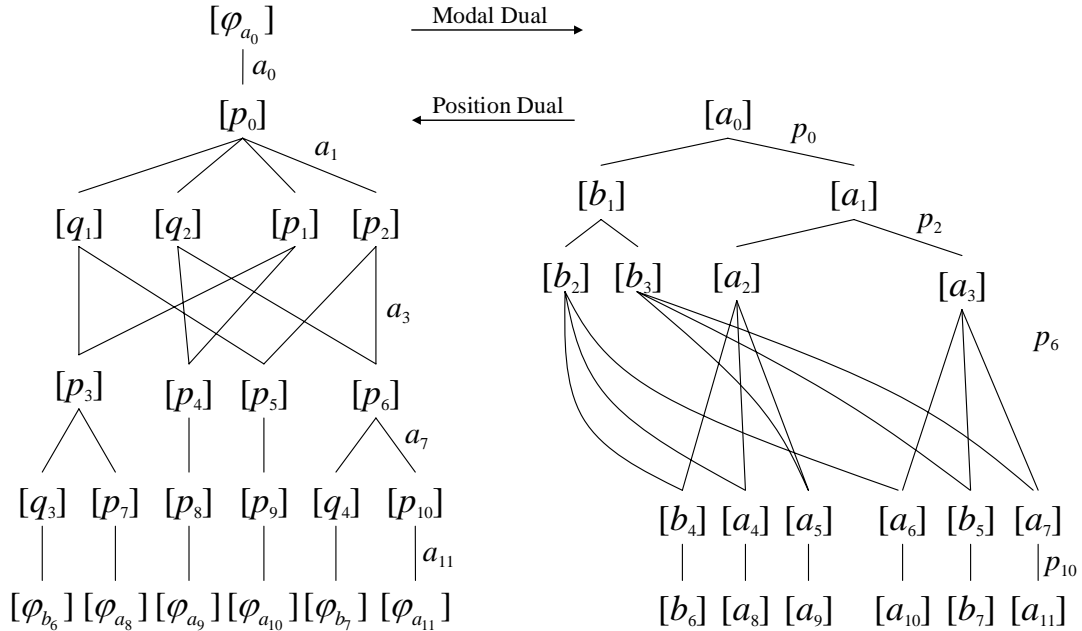
$$V(\mathbf{mQ}(\delta)) = X_\delta / \approx_m$$

$$E(\mathbf{mQ}(\delta)) = \{[b] \xrightarrow{p} [a] \mid (p, b)^S \in [b], (p, a)^N \in [a]\}$$

For examples, see Figure 6. The left graph and the right graph in Figure 6 are, respectively, the position quotient and the modal quotient of the modal graph in Figure 3. Note that if  $[b] \xrightarrow{p} [a_1]$  and  $[b] \xrightarrow{q} [a_2]$  in  $\mathbf{mQ}(\delta)$  for regular  $\delta$ , we have  $p = q$ . Also note that the definition of modal graphs allows a pair of distinct vertices in a position quotient to have two or more edges. Such graphs are known as *multigraphs* (Bang-Jensen and Gutin 2001).

Given a cap position  $\varphi_a$  and a base position  $\varphi_b$ , we also call  $[\varphi_a]$  and  $[\varphi_b]$  a cap position and a base position, respectively. Similarly, given a cap modality  $a$  and a base modality  $b$ , we also call  $[a]$  and  $[b]$  a cap modality and a base modality, respectively.

Figure 6 Position and Modal Quotients of the Modal Graph in Figure 3



**Definition 7.** Let  $\delta$  be a modal graph. The *modal dual* of  $\mathbf{pQ}(\delta)$ , written  $\Delta^m(\mathbf{pQ}(\delta))$ , is a directed graph given by:

$$V(\Delta^m(\mathbf{pQ}(\delta))) = \{[a] \mid [q] \xrightarrow{a} [p] \in E(\mathbf{pQ}(\delta))\}$$

$$E(\Delta^m(\mathbf{pQ}(\delta))) = \{[a_1] \xrightarrow{p} [a_2] \mid [q_1] \xrightarrow{a_1} [p] \xrightarrow{a_2} [q_2] \text{ in } \mathbf{pQ}(\delta)\}$$

Similarly, the *position dual* of  $\mathbf{mQ}(\delta)$ , written  $\Delta^p(\mathbf{mQ}(\delta))$ , is a directed graph given by:

$$V(\Delta^p(\mathbf{mQ}(\delta))) =$$

$$\{[p] \mid [b] \xrightarrow{p} [a] \in E(\mathbf{mQ}(\delta))\} \cup \{[\varphi_a] \mid [a] \text{ is a cap or a base modality in } \mathbf{mQ}(\delta)\}$$

$$E(\Delta^p(\mathbf{mQ}(\delta))) = \{[p_1] \xrightarrow{a} [p_2] \mid [b_1] \xrightarrow{p_1} [a] \xrightarrow{p_2} [b_2] \text{ in } \mathbf{mQ}(\delta)\} \cup$$

$$\{[p] \xrightarrow{a} [\varphi_a] \mid [b] \xrightarrow{p} [a] \in E(\mathbf{mQ}(\delta)) \text{ and } [a] \text{ is a cap modality in } \mathbf{mQ}(\delta)\} \cup$$

$$\{[\varphi_b] \xrightarrow{b} [p] \mid [b] \xrightarrow{p} [a] \in E(\mathbf{mQ}(\delta)) \text{ and } [b] \text{ is a base modality in } \mathbf{mQ}(\delta)\}$$

For examples, see Figure 6. The left graph is the position dual of the right graph, and the right graph is the modal dual of the left graph.

**Proposition 2 (Quotient Duality).** Let  $\delta$  be a closed modal graph.

$$(1) \quad \Delta^m(\mathbf{pQ}(\delta)) = \mathbf{mQ}(\delta)$$

$$(2) \quad \Delta^p(\mathbf{mQ}(\delta)) = \mathbf{pQ}(\delta)$$

Proof (see the appendix).

## 5. Reporting Compositions

With the massively connected, multidimensional network structure, it seems to be still essential for the firm to retain one dimension for hierarchical reporting control. This reporting dimension should be complete in the sense that every position in the structure appears in the network under this dimension. The reporting dimension should also respect the unity of command, i.e., every position has exactly a single position to report to, except the top position. We first characterize a general class of modal graphs for which one can always find such a reporting dimension.

**Definition 8.** Let  $\delta$  be a modal graph.  $\mathbf{pQ}(\delta)$  is *properly rooted* if it has a cap position  $[\varphi_a]$  which is reachable from every position other than cap positions.  $[\varphi_a]$  is called a *proper root* of  $\mathbf{pQ}(\delta)$ . Similarly,  $\mathbf{mQ}(\delta)$  is *properly rooted* if it has a cap modality  $[a]$  which is reachable from every modality other than cap modalities.  $[a]$  is called a *proper root* of  $\mathbf{mQ}(\delta)$ .

**Proposition 3 (Dual-Preservation Properties).** Let  $\delta$  be a modal graph.

- (1)  $\mathbf{pQ}(\delta)$  is acyclic if and only if  $\mathbf{mQ}(\delta)$  is acyclic.
- (2) Assume  $\delta$  is closed. Then  $\mathbf{pQ}(\delta)$  is properly rooted if and only if  $\mathbf{mQ}(\delta)$  is properly rooted.

Proof (see the appendix).

In light of Proposition 3, we say a modal graph is *acyclic* if its position quotient is acyclic. Similarly, we say that a modal graph is *properly rooted* if its position quotient is properly rooted. We do not necessarily claim that answer relationships be acyclic. But, as cyclic modal graphs require additional formal constructs to study their properties, in this paper, we focus on the class of acyclic modal graphs and properties specific to this class.

**Definition 9.** Let  $\delta$  be a properly rooted modal graph, and  $[\varphi_a]$  a proper root of  $\mathbf{pQ}(\delta)$ . A *position hierarchy* of  $\mathbf{pQ}(\delta)$  rooted at  $[\varphi_a]$  is a position quotient obtained from  $\mathbf{pQ}(\delta)$  by the following procedure:

Step 1: Remove every cap position other than  $[\varphi_a]$  and its incoming edges: *i.e.*, for every cap clause  $\varphi_b \xleftarrow{b} q_1, q_2, \dots, q_n$ , remove  $(\varphi_b, b)^S$  and  $(q_i, b)^N$  for each  $1 \leq i \leq n$  from  $[\varphi_b]$  and  $[q_i]$  for each  $1 \leq i \leq n$ , respectively.

Step 2: For every non-empty position  $[q]$  having two or more outgoing edges after Step 1, remove all but one of those edges and add new positions of the form  $[\varphi_{a_i}]$  where  $a_i$  is the label of a removed edge: *i.e.*, for each collection of edges  $\{[q] \xrightarrow{a_i} [p_i]\}_{1 \leq i \leq n}$ , with  $n \geq 2$ , select any integer  $k$  between 1 and  $n$ , and remove  $(q, a_i)^N$  from  $[q]$  for all  $i \neq k$ , and add  $[\varphi_{a_i}] = \{(\varphi_{a_i}, a_i)^N\}$  for all  $i \neq k$  as new vertices.

For an example, see Figure 7. The left graph is a position hierarchy of the position quotient in Figure 6. Note that the graph is a tree, and that every position in the original modal graph in Figure 3, which derives the position quotient in Figure 6, appears exactly once in the position hierarchy. Hence, the hierarchy is complete and respects the unity of command.

**Definition 10.** Let  $\delta$  be a properly rooted modal graph, and  $[a]$  be a proper root of  $\mathbf{mQ}(\delta)$ . A



modal hierarchy of  $m\mathcal{Q}(\delta)$  rooted at  $[a]$  is a modal quotient obtained from  $m\mathcal{Q}(\delta)$  by the following procedure:

Step 1: Remove every cap modal class other than  $[a]$  and its incoming edges: *i.e.*, if  $[b]$  is a cap modal class other than  $[a]$ , remove the vertex  $[b]$  from the set of vertices  $X_\delta / \approx_m$  (and consequently all incoming edges to  $[b]$ ).

Step 2: For every vertex  $[c]$  having two or more outgoing edges after Step 1, remove all but one of those edges, and make each parent of  $[c]$  with a removed edge an base modality: *i.e.*,

for each collection of edges  $\{[x] \xrightarrow{p} [a_i]\}_{1 \leq i \leq n}$  with  $n \geq 2$  and  $[x]$  possibly varying

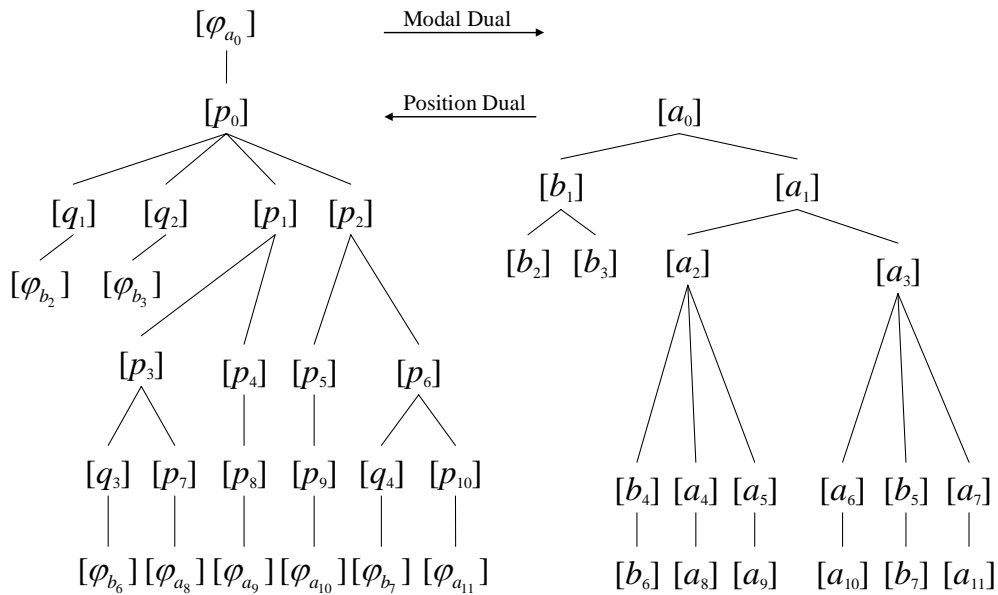
over two or more modalities, select any integer  $k$  between 1 and  $n$ , and remove  $(p, a_i)^N$

from  $[a_i]$  for each  $i \neq k$ , and make  $[a_i]$  a base modality by adding a base front  $(\varphi_{a_i}, a_i)^N$  to it

for each  $i \neq k$ .

For an example, see Figure 7. The right graph is a modal hierarch of the modal quotient in Figure 6.

Figure 7 A Position Hierarchy (left) and a Modal Hierarchy (right) of the Position Quotient and the Modal Quotient in Figure 6, Respectively



**Proposition 4.** Let  $\delta$  an acyclic, closed and properly rooted modal graph. Let  $G$  be a position hierarchy of  $\mathbf{pQ}(\delta)$ , and  $H$  a modal hierarchy of  $\mathbf{mQ}(\delta)$ .

- (1)  $G$  is a tree, and for every non-empty position  $[p]$  in  $\mathbf{pQ}(\delta)$ ,  $[p] \in V(G)$ .
- (2)  $H$  is a tree, and for every modality  $[b]$  in  $\mathbf{mQ}(\delta)$  other than cap modalities,  $[b] \in V(H)$ .

Proof (see the appendix).

**Proposition 5 (Hierarchy Duality).** Let  $\delta$  an acyclic, closed and properly rooted modal graph.

- (1) The modal dual of a position hierarchy of  $\mathbf{pQ}(\delta)$  is a modal hierarchy of  $\mathbf{mQ}(\delta)$ .
- (2) The position dual of a modal hierarchy of  $\mathbf{mQ}(\delta)$  is a position hierarchy of  $\mathbf{pQ}(\delta)$ .

Proof (see the appendix).

Note that in Figure 7, one graph is the dual of the other.

**Corollary.** Let  $\delta$  an acyclic, closed and properly rooted modal graph. There is a bijection between the collection of position hierarchies of  $\mathbf{pQ}(\delta)$  and the collection of modal hierarchies of  $\mathbf{mQ}(\delta)$ .

**Definition 11.** Let  $\delta$  be a closed, acyclic and properly rooted modal graph, and  $\pi$  a clausal composition of  $\delta$ .  $\pi$  is a *reporting composition* of  $\delta$  if there exists a position hierarchy  $G$  of  $\mathbf{pQ}(\delta)$  such that for every pair of a position  $p$  and a non-empty position  $q$  in  $\delta$ ,  $[r] \xrightarrow{b} [q] \xrightarrow{a} [p]$  in  $G$  if and only if  $\Lambda_b \rightarrow \Lambda_a \in E(\pi)$  where  $p$  and  $q$  are the heads of  $\Lambda_a$  and  $\Lambda_b$ , respectively.

Informally, thus, a reporting composition is a clausal composition where the heads of its clauses respect reporting relationships.

We say that a reporting composition  $\pi$  of  $\delta$  is *standardized* if every cap clause in  $\delta$  other than the root clause of  $\pi$  appears as a child clause of the root clause in  $\pi$ .

**Definition 12.** Let  $\delta$  be a properly rooted modal graph,  $[a]$  a proper root of  $\mathbf{mQ}(\delta)$ , and  $H$  a modal hierarchy of  $\mathbf{mQ}(\delta)$  rooted at  $[a]$ . A *clausal image* of  $H$ , written  $\lambda(H)$ , is a directed graph given by:

$V(\lambda(H))$  is the set of all clauses in  $\delta$ ,

$$E(\lambda(H)) = \{\Lambda_x \rightarrow \Lambda_y \mid [x] \rightarrow [y] \in E(H)\} \cup \\ \{\Lambda_x \rightarrow \Lambda_a \mid \Lambda_x \text{ is a cap clause in } \delta \text{ other than } \Lambda_a\}$$

**Lemma.** Let  $\delta$  be a properly rooted modal graph. The clausal image of a modal hierarchy of  $m\mathcal{Q}(\delta)$  is a clausal composition of  $\delta$ .

Proof (see the appendix).

As an example, note that the clausal image of the modal hierarchy in Figure 7 is a clausal composition of the modal graph in Figure 3.

**Proposition 6 (Soundness and Completeness).** Let  $\delta$  be a closed, acyclic and properly rooted modal graph.

- (1) For every modal hierarchy of  $m\mathcal{Q}(\delta)$ , its clausal image is a reporting composition of  $\delta$ .
- (2) Every reporting composition of  $\delta$  in the standardized form is a clausal image of some modal hierarchy of  $m\mathcal{Q}(\delta)$ .

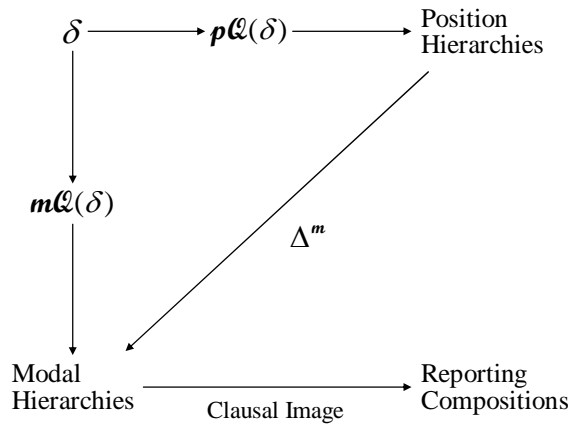
Proof (see the appendix).

Figure 8 summarizes the construction of reporting compositions. The point to note is that given a modal graph, its reporting compositions can be derived either through the position view via the position quotient or through the modal view via the modal quotient.

## 6. Conclusions

In this paper, the firm is seen as an aggregation of differentiated networks, each with a specific strategic dimension. These differentiated networks within the firm's structure are explicitly captured as clausal compositions. Once a dominant dimension is identified as the firm's reporting structure, the firm can modify the remaining networks, or add new ones while retaining its reporting structure intact. With rapidly increasing advances in computer-mediated communication, the firm could massively network itself for enhanced coordination and abilities to internalize emergent strategic dimensions. Given this possibility, an area of contribution towards improved methodologies for organizational effectiveness is the typology of coordination structures. The examples in Figures 1, 2 and 3 briefly illustrate partial-mirror coordination, indirect homogeneous coordination, and direct heterogeneous coordination. This collection is only an initial, illustrative sampling. A more comprehensive classification is possible and beneficial.

Figure 8 Two Paths for Deriving Reporting Compositions of a Modal Graph



Related to the typology of coordination structures is an active line of research on boundary spanning coordination. As Carlile (2004, p.566) states, if “[...] instead of seeing the firm as a bundle of resources (Barney 1991), it can be more completely described as a bundle of different types of boundaries [...]”, it is important to develop systematic knowledge of “different types of boundaries” and how they are structured to form a “bundle”. Coordination structures that capture both formal and informal relationships, as in modal graphs, offer a means of study to gain further insights on the recursive dynamics of individual actors and institutionalized structures.

## Appendix: Proofs of Lemmas and Propositions

**Proposition 1 (Graph Union).** A modally disjoint union of modal graphs is a modal graph.

**Proof.** We first introduce a notation. Given a modal graph with  $R$  as its set of answer relationships, define the following sets:

- $R^N = \{x \mid x \rightarrow y \in R\}$
- $R^S = \{y \mid x \rightarrow y \in R\}$

Note that  $R^N \cap R^S = \emptyset$  if and only if for each position of the modal graph the set of north modalities of the position is disjoint from the set of south modalities of the position. Let

$\delta_i = (P_i, M_i, R_i, Z_i)$  be modal graphs for  $i = 1, \dots, n$ , with  $M_i \cap M_j = \emptyset$  for  $i \neq j$ , and

$\delta = (P, M, R, Z)$  their union. It suffices to show  $R^N \cap R^S = \emptyset$ . Suppose  $R^N \cap R^S \neq \emptyset$

and let  $(p, a) \in R^N \cap R^S$ . Then  $a \in M_k$  and  $a \notin \bigcup_{i \neq k} M_i$ . Note that  $R^N = \bigcup_{1 \leq i \leq n} R_i^N$ . Thus,

$(p, a) \notin \bigcup_{i \neq k} R_i^N$  and  $(p, a) \in R_k^N$ . Similarly,  $R^S = \bigcup_{1 \leq i \leq n} R_i^S$ , and hence  $(p, a) \notin \bigcup_{i \neq k} R_i^S$  and

$(p, a) \in R_k^S$ . We obtain a contradiction  $(p, a) \in R_k^N \cap R_k^S$ .

□

**Proposition 2 (Quotient Duality).** Let  $\delta$  be a closed modal graph.

- (1)  $\Delta^m(\mathbf{p}\mathcal{Q}(\delta)) = \mathbf{m}\mathcal{Q}(\delta)$
- (2)  $\Delta^p(\mathbf{m}\mathcal{Q}(\delta)) = \mathbf{p}\mathcal{Q}(\delta)$

**Proof for (1).** Let  $\delta = (P, M, R, Z)$ .

$[a] \in V(\mathbf{m}\mathcal{Q}(\delta))$  if and only if

$[a] \in X_\delta / \approx_m$  if and only if

For some  $q, p \in P$ ,  $(q, a) \rightarrow (p, a) \in R$  if and only if

$[q] \xrightarrow{a} [p] \in E(\mathbf{p}\mathcal{Q}(\delta))$  if and only if

$[a] \in V(\Delta^m(\mathbf{p}\mathcal{Q}(\delta)))$

$[a_1] \xrightarrow{p} [a_2] \in E(\mathbf{m}\mathcal{Q}(\delta))$  if and only if

$(p, a_1)^S \in [a_1]$  and  $(p, a_2)^N \in [a_2]$  if and only if

For some  $q_1, q_2 \in P$ ,  $(q_1, a_1) \rightarrow (p, a_1), (p, a_2) \rightarrow (q_2, a_2) \in R$  as  $\delta$  is closed if and only if

$[q_1] \xrightarrow{a_1} [p] \xrightarrow{a_2} [q_2]$  in  $\mathbf{p}\mathcal{Q}(\delta)$  if and only if

$$[a_1] \xrightarrow{p} [a_2] \in E(\Delta^m(\mathbf{pQ}(\delta))).$$

□

**Proof for (2).** Let  $\delta = (P, M, R, Z)$ . We first show  $V(\Delta^p(\mathbf{mQ}(\delta))) = V(\mathbf{pQ}(\delta))$ .

For an empty position  $\varphi_a$ ,

$$[\varphi_a] \in V(\mathbf{pQ}(\delta)) \text{ if and only if}$$

$$[\varphi_a] \text{ contains a front } (\varphi_a, a) \text{ if and only if}$$

$$[a] \text{ contains a front } (\varphi_a, a) \text{ if and only if}$$

$$[a] \text{ is a cap or base in } \mathbf{mQ}(\delta) \text{ if and only if}$$

$$[\varphi_a] \in V(\Delta^p(\mathbf{mQ}(\delta))).$$

For a non-empty position  $p$ ,

$$[p] \in V(\mathbf{pQ}(\delta)) \text{ if and only if}$$

$p$  has some north modality  $a$  and some south modality  $b$  as  $\delta$  is closed if and only if

$$[b] \xrightarrow{p} [a] \in E(\mathbf{mQ}(\delta)) \text{ if and only if}$$

$$[p] \in V(\Delta^p(\mathbf{mQ}(\delta))).$$

Now we show  $E(\Delta^p(\mathbf{mQ}(\delta))) = E(\mathbf{pQ}(\delta))$ .

For  $[q] \xrightarrow{a} [\varphi_a]$  where  $q$  is a non-empty position,

$$[q] \xrightarrow{a} [\varphi_a] \in E(\mathbf{pQ}(\delta)) \text{ if and only if}$$

$$[b] \xrightarrow{q} [a] \in E(\mathbf{mQ}(\delta)) \text{ for some } [b] \text{ where } [a] \text{ is a cap in } \mathbf{mQ}(\delta) \text{ if and only if}$$

$$[q] \xrightarrow{a} [\varphi_a] \in E(\Delta^p(\mathbf{mQ}(\delta))).$$

For  $[\varphi_b] \xrightarrow{b} [p]$  where  $p$  is a non-empty position,

$$[\varphi_b] \xrightarrow{b} [p] \in E(\mathbf{pQ}(\delta)) \text{ if and only if}$$

$$[b] \xrightarrow{p} [a] \in E(\mathbf{mQ}(\delta)) \text{ for some } [a] \text{ where } [b] \text{ is a base in } \mathbf{mQ}(\delta) \text{ if and only if}$$

$$[\varphi_b] \xrightarrow{b} [p] \in E(\Delta^p(\mathbf{mQ}(\delta))).$$

For  $[q] \xrightarrow{b} [p]$  where  $p$  and  $q$  are both non-empty positions,

$$[q] \xrightarrow{b} [p] \in E(\mathbf{pQ}(\delta)) \text{ if and only if}$$

$$[c] \xrightarrow{q} [b] \xrightarrow{p} [a] \text{ in } \mathbf{mQ}(\delta) \text{ for some } [a] \text{ and } [c] \text{ as } \delta \text{ is closed if and only if}$$

$$[q] \xrightarrow{b} [p] \in E(\Delta^p(\mathbf{mQ}(\delta))).$$

□

**Proposition 3 (Dual-Preservation Properties).** Let  $\delta$  be a modal graph.

- (1)  $\mathbf{pQ}(\delta)$  is acyclic if and only if  $\mathbf{mQ}(\delta)$  is acyclic.
- (2) Assume  $\delta$  is closed. Then  $\mathbf{pQ}(\delta)$  is properly rooted if and only if  $\mathbf{mQ}(\delta)$  is properly rooted.

Proof for (1). We first introduce a definition as an aid for proof. We say that a modal graph is *cyclic* if it has a chain of answer relationships of the following form:

$$(p_0, a_0) \rightarrow (p_1, a_0), (p_1, a_1) \rightarrow (p_2, a_1), \dots, (p_{n-1}, a_{n-1}) \rightarrow (p_n, a_{n-1}), (p_n, a_n) \rightarrow (p_0, a_n)$$

Note that  $\mathbf{pQ}(\delta)$  is cyclic if and only if  $\delta$  is cyclic. Note also that  $\mathbf{mQ}(\delta)$  is cyclic if and only if  $\delta$  is cyclic.

□

Proof for (2). Similar to the proof above, we first introduce a definition. We say that a modal graph is *properly rooted* if it has an empty position  $\varphi_a$  such that for every non-empty position  $q_0$ , it has a chain of answer relationships of the following form:

$$(q_0, b_1) \rightarrow (q_1, b_1), (q_1, b_2) \rightarrow (q_2, b_2), \dots, (q_{n-1}, b_n) \rightarrow (q_n, b_n), (q_n, a) \rightarrow (\varphi_a, a)$$

Note that  $\mathbf{pQ}(\delta)$  is properly rooted if and only if  $\delta$  is properly rooted. It remains to show that  $\delta$  is properly rooted if and only if  $\mathbf{mQ}(\delta)$  is properly rooted. Suppose that  $\delta$  is properly rooted at  $\varphi_a$ . We show that  $\mathbf{mQ}(\delta)$  is properly rooted at  $[a]$ . Let  $[b_0]$  be a non-cap in  $\mathbf{mQ}(\delta)$ . Then  $b_0$  is a south modality of some non-empty position. Call it  $q_0$ . As  $q_0$  is non-empty, by the supposition,  $\delta$  has a chain of answer relationships of the form specified above. Thus  $[a]$  is reachable from  $[b_1]$ . But as  $b_0$  is a south modality of  $q_0$ ,  $[b_0] \xrightarrow{q_0} [b_1]$  in  $\mathbf{mQ}(\delta)$ . Thus,  $[a]$  is reachable from  $[b_0]$ . Now suppose  $\mathbf{mQ}(\delta)$  is properly rooted, and let  $[a]$  be its proper root. Since  $[a]$  is a cap in  $\mathbf{mQ}(\delta)$ , it contains a cap front  $(\varphi_a, a)^S$ . We show that  $\delta$  is properly rooted at  $\varphi_a$ . Let  $q_0$  be a non-empty position in  $\delta$ . Since  $\delta$  is closed,  $q_0$  has a south modality. Call it  $b_0$ . Since  $[b_0]$  is not a cap in  $\mathbf{mQ}(\delta)$ ,  $[a]$  is reachable from  $[b_0]$ , and  $\mathbf{mQ}(\delta)$  has a chain of the following form:

$$[b_0] \xrightarrow{q_0} [b_1] \xrightarrow{q_1} \dots \xrightarrow{q_{n-1}} [b_n] \xrightarrow{q_n} [a]$$

Since  $[a]$  contains  $(\varphi_a, a)^S$ , we obtain a chain of answer relationships of the form specified above. Thus,  $\delta$  is properly rooted. We have shown that  $\mathbf{mQ}(\delta)$  is properly rooted if and only

if  $\delta$  is properly rooted. □

**Proposition 4 (Quotient Trees).** Let  $\delta$  an acyclic, closed and properly rooted modal graph. Let  $G$  be a position hierarchy of  $\mathbf{pQ}(\delta)$  rooted at  $[\varphi_a]$ , and  $H$  a modal hierarchy of  $\mathbf{mQ}(\delta)$  rooted at  $[a]$ .

- (1)  $G$  is a tree, and for every non-empty position  $[p]$  in  $\mathbf{pQ}(\delta)$ ,  $[p] \in V(G)$ .
- (2)  $H$  is a tree, and for every modality  $[b]$  in  $\mathbf{mQ}(\delta)$  other than cap modalities,  $[b] \in V(H)$ .

Proof for (1). We first show  $G$  is a tree. Since  $\mathbf{pQ}(\delta)$  is acyclic, by the construction of  $G$ ,  $G$  is acyclic. Note that  $[\varphi_a] \in V(G)$ . As  $[\varphi_a]$  is a cap, it has no parents. It suffices to show that every vertex of  $G$  other than  $[\varphi_a]$  has exactly a single parent: as  $G$  is acyclic,  $[\varphi_a]$  would then be reachable from every vertex of  $G$ . Let  $[q]$  be a vertex of  $G$  other than  $[\varphi_a]$ . Since  $[q]$  is not a cap,  $[\varphi_a]$  is reachable from it in  $\mathbf{pQ}(\delta)$ . By the construction of  $G$ ,  $[q]$  has exactly a single parent in  $G$ . Thus, we conclude  $G$  is a tree. Let  $[q]$  be a non-empty position in  $\mathbf{pQ}(\delta)$ . Whenever north fronts are removed from  $[q]$  in the construction of  $G$ ,  $[q]$  retains one north front. So,  $[q]$  remains to be a vertex in  $G$ . □

Proof for (2). We first show  $H$  is a tree. Since  $\mathbf{mQ}(\delta)$  is acyclic, by the construction of  $H$ ,  $H$  is acyclic. Note that  $[a] \in V(H)$ . As  $[a]$  is a cap, it has no parents. It suffices to show that every vertex of  $H$  other than  $[a]$  has exactly a single parent: as  $H$  is acyclic,  $[a]$  would then be reachable from every vertex of  $H$ . Let  $[b]$  be a vertex of  $H$  other than  $[a]$ . Since  $[b]$  is not a cap,  $[a]$  is reachable from it in  $\mathbf{mQ}(\delta)$ . By the construction of  $H$ ,  $[b]$  has exactly a single parent in  $H$ . Thus, we conclude  $H$  is a tree. Let  $[b]$  be a non-cap modality in  $\mathbf{mQ}(\delta)$ . Whenever a north front is removed from  $[b]$  in the construction of  $H$ , a base front is added to it. So,  $[b]$  remains to be a vertex in  $H$ . □

**Proposition 5 (Hierarchy Duality).** Let  $\delta$  an acyclic, closed and properly rooted modal graph.

- (1) The modal dual of a position hierarchy of  $\mathbf{pQ}(\delta)$  is a modal hierarchy of  $\mathbf{mQ}(\delta)$ .
- (2) The position dual of a modal hierarchy of  $\mathbf{mQ}(\delta)$  is a position hierarchy of  $\mathbf{pQ}(\delta)$ .

Proof for (1) and (2). Let  $G$  be a position hierarchy of  $\mathbf{pQ}(\delta)$  rooted at  $[\varphi_a]$ , and  $H$  a modal hierarchy of  $\mathbf{mQ}(\delta)$  rooted at  $[a]$ . Let  $\delta'$  be a modal graph obtained from  $\delta$  by removing all cap



clauses other than  $a$ -clause. Note that  $\delta'$  is closed as  $\delta$  is closed and properly rooted. Note also that the construction of  $\delta'$  is equivalent to the Step 1 in Definitions 10 and 11. Let  $\sigma$  be a function that selects, given a multimodal position of  $\delta'$ , all but one of its north modalities.

Let  $\delta^a$  be a modal graph given by:

$$X_{\delta^a} = X_{\delta'} - \bigcup_{q \in P'} \{(q, a_i)^N\}_{a_i \in \sigma(q)} \cup \bigcup_{q \in P'} \{(\varphi_{a_i}, a_i)^N\}_{a_i \in \sigma(q)}$$

where  $P'$  is the set of multimodal positions of  $\delta'$ . Note that the construction of  $\delta^a$  from  $\delta'$  corresponds to the Step 2 in Definitions 10 and 11. Thus, we have  $G = \mathbf{pQ}(\delta^a)$  and  $H = \mathbf{mQ}(\delta^a)$ . Note also that  $\delta^a$  is closed. By Proposition 2 (Quotient Duality), we obtain

- $\Delta^m(G) = \Delta^m(\mathbf{pQ}(\delta^a)) = \mathbf{mQ}(\delta^a) = H$
- $\Delta^p(H) = \Delta^p(\mathbf{mQ}(\delta^a)) = \mathbf{pQ}(\delta^a) = G$

□

**Lemma 1.** Let  $\delta$  be a properly rooted modal graph. The clausal image of a modal hierarchy of  $\mathbf{mQ}(\delta)$  is a clausal composition of  $\delta$ .

*Proof.* Let  $H$  be a modal hierarchy of  $\mathbf{mQ}(\delta)$  rooted at  $[a]$ , and  $\lambda(H)$  the clausal image of  $H$ . First note that  $\lambda(H)$  is a tree. We must show that every clause in  $\lambda(H)$  is an interface for its subtrees. Given a clause in  $\lambda(H)$ , let  $T$  be the subtree rooted at the clause. We show that the root clause of  $T$  is an interface for its subtrees by induction on the height of  $T$ . Suppose the height of  $T$  is 0. Then  $T = \Lambda_a$ . Trivially,  $\Lambda_a$  is an interface for its subtrees as it has no subtrees. Now suppose that the height of  $T$  is  $n$ , and that the claim holds for every subtree with height less than  $n$ . Let  $T = \Lambda_a(\pi_1, \pi_2, \dots, \pi_n)$  where each  $\pi_i$  is, by the induction hypothesis, a subtree in which its root clause is an interface of its subtrees. Note that  $\pi_i$  has at most one free position, and that if it has a free position, it is the head of its root clause. If  $\pi_i$  has no free position,  $\Lambda_a$  trivially accepts it. Let  $\Lambda_{b_i}$  be the root clause of  $\pi_i$ , and suppose that the head  $p$  of the clause is free. But  $[b_i] \xrightarrow{p} [a]$  in  $H$ , and hence  $\Lambda_a$  accepts  $\pi_i$ . We have shown that  $\Lambda_a$  is an interface of its subtrees. We conclude  $\lambda(H)$  is a clausal composition.

□

**Proposition 6 (Soundness and Completeness).** Let  $\delta$  be a closed, acyclic and properly rooted modal graph.

- (1) For every modal hierarchy of  $\mathbf{mQ}(\delta)$ , its clausal image is a reporting composition of  $\delta$ .
- (2) Every reporting composition of  $\delta$  in the standardized form is a clausal image of some modal

hierarchy of  $\mathbf{mQ}(\delta)$ .

Proof for (1). Let  $H$  be a modal hierarchy of  $\mathbf{mQ}(\delta)$ , and  $\lambda(H)$  a clausal image. By Lemma 1,  $\lambda(H)$  is a clausal composition of  $\delta$ . We show that  $\lambda(H)$  is a reporting composition associated with the position hierarchy  $\Delta^p(H)$ . Let  $p$  be a position, and  $q$  a non-empty position in  $\delta$ . Then we have,

$[r] \xrightarrow{b} [q] \xrightarrow{a} [p]$  in  $\Delta^p(H)$  if and only if  
 $H$  has  $[b] \xrightarrow{q} [a]$  or  $[b] \xrightarrow{q} [a] \xrightarrow{p} [c]$  (depending on whether  $p$  is an empty position or not) if and only if  
 $\Lambda_b \rightarrow \Lambda_a \in E(\lambda(H))$  where  $\Lambda_a$  and  $\Lambda_b$  have heads  $p$  and  $q$ , respectively.

□

Proof for (2). Let  $\pi$  be a reporting composition of  $\delta$  in the standardized form. Let  $G$  be a position hierarchy of  $\mathbf{pQ}(\delta)$  such that for every pair of a position  $p$  and a non-empty position  $q$  in  $\delta$ ,  $[r] \xrightarrow{b} [q] \xrightarrow{a} [p]$  in  $G$  if and only if  $\Lambda_b \rightarrow \Lambda_a \in E(\pi)$  where  $p$  and  $q$  are the heads of  $\Lambda_a$  and  $\Lambda_b$ , respectively. We show that  $\pi$  is a clausal image of  $\Delta^m(G)$ . We have,

$[b] \xrightarrow{q} [a] \in E(\Delta^m(G))$  if and only if  
 $[r] \xrightarrow{b} [q] \xrightarrow{a} [p]$  in  $G$  if and only if  
 $\Lambda_b \rightarrow \Lambda_a \in E(\pi)$  where a position  $p$  and a non-empty position  $q$  are the heads of  $\Lambda_a$  and  $\Lambda_b$ , respectively.

As  $\pi$  is in the standardized form, every cap clause other than the root clause is a child of the root clause. Thus,  $\pi$  is a clausal image of  $\Delta^m(G)$ .

□

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