

Promotion of Eco-Products and Environmental Regulation with Learning-by-Doing

Makoto Kakinaka
International University of Japan

and

Koji Kotani
Yokohama National University

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Graduate School of International Relations
International University of Japan

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Makoto Kakinaka

Graduate School of International Relations, International University of Japan

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Koji Kotani

Faculty of Environment and Information Science, Yokohama National University

Abstract

This paper studies a dynamic model of eco-product planning, where an eco-product supplied by a single producer is differentiated from a conventional product generating negative externalities, and the production technology of the eco-product is characterized by learning-by-doing. The result states that the learning effect causes the eco-product to be more promoted and brings about more favorable outcomes on social welfare. This study also examines how the environmental regulation on the conventional product, associated with a price distortion, affects the promotion of the eco-product, consumer surplus, the single producer's profit, and negative externalities. It is shown that the impact of the environmental regulation is similar to that of a rise in the learning effect. Furthermore, whether or not the environmental regulation should be adopted is highly dependent on the degree of the learning effect. In the presence of a large learning potential, the environmental regulation may not only promote the eco-product effectively but also improve social welfare through intensifying the learning effect.

Keyword: eco-product, environmental regulation, product differentiation, learning-by-doing, optimal control

JFL classification: Q55, Q58

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1 Introduction

“Eco-products,” “green markets,” or “environmental-friendly commodities,” etc. . . Many terms have been coined and commonly used in a society to represent an impure public good that jointly plays the roles of both private and public goods.¹ This is due to the fact that a variety of eco-products have been introduced into markets as environmental awareness increases, and the importance of the promotion of eco-products is widely recognized to improve the environment (Conrad (2005)). One critical feature of eco-products is that they do not generate negative externalities, such as air pollution, from production and consumption, while a conventional product casts some degree of negative externalities.²

At the same time, various environmental regulations on a conventional product generating negative externalities have been implemented by government authorities to mitigate environmental problems. Such regulations usually affects the resulting output of not only a conventional product but also an eco-product through changing the activities of private agents, though this issue has rarely been studied in the literature. Thus, for policy makers who intend to promote an eco-product, it should be important to understand the relation between the promotion of an eco-product and environmental regulations. This paper studies such related issues, where an eco-product is supplied by a single producer, and the production of a conventional product is influenced by the governmental regulation.

There is a series of economic literature which examines the implications of products generating negative externalities and various environmental policy measures in the presence of environmentally aware consumers (see, e.g., Arora and Gangopadhyay (1995), Cremer and Thisse (1999), Moraga-Gonzalez and Padron-Fumero (2002), Bansal and Gangopadhyay (2003), Eriksson (2004), and Conrad (2005)). Employing a model with product differentiation that originates from consumers’ environmental awareness, these researchers discuss firms’ incentives in choosing the quality or technology that sets the environmental-friendliness of polluting products.

This paper also considers consumers’ environmental awareness as a source of product differentiation, but from a different angle which, to our best knowledge, has never been taken in any pervious work. We develop a model where an eco-product supplied by a single producer is differentiated from a conventional product supplied in a perfectly competitive market. Instead of a situation in which producers set the

¹This paper uses these terminologies interchangeably. However, we mainly employ an “eco-product.”

²In reality, however, eco-products also generate a certain level of negative externalities, but in this paper we assume that the adverse effects on the environment or humans are negligible for simplicity.

environmental technology or the degree of the environmental-friendliness of their products, this study explicitly considers an eco-product that never generates negative externalities and a conventional product that generates negative externalities, irrespective of the production technology. This specification may be consistent with a situation where a single producer first invents and introduces an eco-product into a market and then competes with a pre-existing conventional product due to their substitutability. Our objective in this research is to derive some important implications related to the promotion of the eco-product and the environmental regulation by modelling this type of situation.

For our purpose, we introduce some additional distinctions between this and previous studies. The first distinction is to introduce learning-by-doing in the production of an eco-product as an additional building block. Several authors acknowledge that current production enhances future productivity through learning, and this dynamic learning effect is an essential engine of technological progress in individual firms as well as at national levels.³ In this paper, the learning effect is considered important in the production process of eco-products and is internalized in the technology of the single producer without any spillover. To capture this, it is assumed that the technology of the eco-product is characterized with instantaneous constant returns to scale but its marginal cost declines over time due to dynamic economies of scale associated with learning-by-doing.

It is also widely admitted that the learning effect is not a unique source for process innovation in individual firms as other authors claim (see, e.g., Bellas (1998) for an example of environmental-related technology). Research and development for a new production technology is also an important source of process innovation. However, there is sufficient evidence that the learning effect plays a crucial role in cost reduction (see, e.g., Bellas (1998) and Argote and Epple (1990)). Since the nature of learning is dynamic, analyzing the production of eco-products in a framework of a dynamic model is worthwhile to examine how the learning effect affects the resulting outcome and to find some important implications.

The second distinction is that our analysis considers standards or targets for environmental quality, like emission standards, as environmental regulations rather than incentive schemes, like tax and subsidy. Cropper and Oates (1999) state that since in reality there are some problems of measurement or other informational obstacles to implement first-best incentive policies, the determination of actual environmental policies consists of two steps: the first step is that standards or targets for environmental quality are

³For early work on learning-by-doing, see, e.g., Arrow (1962), Spence (1981), Fudenberg and Tirole (1983), Romer (1986), and Stokey (1988).

determined; and the second is that a regulatory system with economic incentive schemes is arranged so that these standards are efficiently satisfied. From a practical point of view, this paper pays attention to the first stage of setting the environmental targets or standards as a government policy tool, while the exploration of an equivalent basis in the second step is left for a future research.⁴

This paper first develops a static pricing model of a single firm producing an eco-product as a base case, where its production technology is not associated with the learning effect. With this approach, we discuss a wide set of implications in the eco-product market and suggest the possibility that the environmental regulation on the conventional product induces social welfare improvement through promoting the substitutable eco-product under certain conditions. More specifically, it is shown that the government's optimal policy is to adopt the environmental regulation when the marginal cost for an eco-product is high enough.

Building upon the static model, this study next develops a dynamic pricing model for an eco-product under the learning effect.⁵ We keep the basic structure, but the main difference is that the marginal cost for an eco-product changes over time due to the learning effect. This dynamic model again shows that the environmental regulation promotes the market share of an eco-product. Moreover, when there is a relatively large learning effect, the environmental regulation is more likely to improve social welfare in the long run. The larger the learning potential is, the more significant the government's role is. This is because the environmental regulation on the conventional product promotes the current production of an eco-product, which in turn intensifies the learning effect, reduces the marginal cost of the eco-product, and induces a relatively large welfare gain in the future.

These results in this paper have some connection with a contentious debate of whether tightening environmental standards is good or not for a society. The claim made by Porter (1991) and Porter and van der Linde (1995) is that tightening environmental standards may trigger technological innovation within firms so that it may be better for everyone in the long run. On the other hand, Palmer, Oates, and Portney (1995) question this viewpoint and conclusion. Though this paper focuses on technological progress in the production of an eco-product through learning-by-doing and deals somewhat with a specific situation compared to the general case they describe, our result suggests that the environmental regulation

⁴There are many studies on the role of environmental standards, like emission standards, in the discussions of various contexts, such as abatement technology and compliance. See, e.g., Arora and Gangopadhyay (1995) and Stranlund (1997).

⁵Miravete (2003) develops a similar model to ours in the discussion of time-consistent protection for a single producer with learning-by-doing in the context of international trade.

promotes technological innovation by intensifying the dynamic learning effect. As a result, it may improve social welfare in the dynamic sense. We believe that this study provides one of the exemplary dynamic models that support the conjecture that tightening environmental standards makes a society better off, and this point could be counted as one of the main contributions.

In the next section, we elaborate on the basic elements of the model which are common both in the static and dynamic model. The section is followed by a presentation of the static pricing model and the main results. In the third section, the dynamic pricing model with the learning effect is developed. We compare the results with those derived in the static model. In the next section, the optimal decision made by the government is examined, and some numerical examples are shown to illustrate the implications of the model. In the final section, we offer some conclusions. Note that all proofs are in the Appendix.

2 The Model

We consider an economy with a numeraire sector and an industry consisting of two sectors, an eco-product and a conventional product. The model is written in continuous time. The conventional product is produced in a perfectly competitive market with a constant marginal cost, which is fixed over time. In contrast, the eco-product is produced by a single producer without new entry. These products are substitutes. The technology of the eco-product is characterized by instantaneous constant returns to scale, but its marginal cost declines with output due to dynamic economies of scale through learning-by-doing, as explained later. In this study, the only state variable is the level of marginal cost of an eco-product.

The difference between the conventional product and the eco-product is that the former entails negative externalities from consumption or production, while the latter does not. To control pollution, the government may impose an environmental regulation on the production of the conventional product. For simplicity, it is assumed that the government simply decides whether or not to adopt the regulation. The regulation precludes negative externalities but causes the producers to incur an additional marginal cost. Let $\phi \in \{0, 1\}$ denote the binary choice such that $\phi = 1$ if the government adopts the regulation, and $\phi = 0$ otherwise. Specifically, we assume that the constant marginal cost for the conventional product is given by $\tau(\phi)$ such that $\tau(0) = \underline{\tau}$ and $\tau(1) = \underline{\tau} + \xi$, and the pollution level per unit of the output of the conventional product is given by $\epsilon(\phi)$ such that $\epsilon(0) = \varepsilon$ and $\epsilon(1) = 0$, where $0 < \underline{\tau} < \underline{\tau} + \xi < 1$ and $\varepsilon > 0$. Notice that if the environmental regulation is adopted, there is no pollution. The environmental regulation in this

study may be interpreted as the adoption of environmental standards on the conventional product which reduce pollution emission to such a low level that negative externalities become negligible. This is because new standards for environmental quality are usually determined by the criterion of “permissible concentration” or “acceptable” (see, e.g., Kolstad (2003) and Baumol and Oates (1988)). As ξ becomes larger, the environmental regulation causes a larger additional increase in the marginal cost for the eco-product. Moreover, as ε becomes larger, the production of the conventional product induces a larger amount of pollution without the environmental regulation. Thus, this specification captures the trade-off relationship between an increase in the marginal cost for the conventional product with the environmental regulation and an increase in negative externalities through pollution without the environmental regulation.

Let x and y denote the consumption of an eco-product sold by a single producer and the consumption of a conventional product in a perfectly competitive market. Following Singh and Vives (1984), we assume that there is a representative consumer whose preference is described by a quasi-linear utility with respect to a numeraire, $q + u(x, y)$, where q is the consumption of the numeraire and $u(x, y)$ is a quadratic sub-utility with symmetric cross-effects with $u(x, y) = (x + y) - (x^2 + y^2 + 2\gamma xy)/2$. The parameter $\gamma \in [0, 1)$ measures the degree of product differentiation between the eco-product and the conventional product. The sufficient condition $\Delta = 1 - \gamma^2 > 0$ ensures that the sub-utility function $u(x, y)$ is strictly concave. If $\gamma = 0$ the conventional product and the eco-product are viewed as independent, but as $\gamma \rightarrow 1$ these products become closer to perfect substitutes.

At each instantaneous time, the representative consumer maximizes the utility subject to his budget constraint $I = q + px + \tau(\phi)y$, where p is the price of the eco-product decided by the single producer. Notice that the price of the conventional product is equal to its marginal cost $\tau(\phi)$ due to perfect competition. The first-order conditions yield the following demand functions:

$$x^*(p, \phi) = \frac{1 - \gamma(1 - \tau(\phi))}{1 - \gamma^2} - \frac{1}{1 - \gamma^2}p; \quad y^*(p, \phi) = \frac{(1 - \tau(\phi)) - \gamma}{1 - \gamma^2} + \frac{\gamma}{1 - \gamma^2}p. \quad (1)$$

The demand for the eco-product is decreasing in p while the demand for the conventional product is increasing in p , i.e., $x_p^* < 0$ and $y_p^* \geq 0$ with equality if $\gamma = 0$. Since $\tau(0) = \underline{\tau} < \underline{\tau} + \xi = \tau(1)$, the environmental regulation increases the demand for the eco-product but decreases the demand for the conventional product, i.e., $x^*(p, 0) < x^*(p, 1)$ and $y^*(p, 0) > y^*(p, 1)$. Using demands (1), we obtain instantaneous consumer surplus, $S(p, \phi) \equiv u(x^*(p, \phi), y^*(p, \phi)) - px^*(p, \phi) - \tau(\phi)y^*(p, \phi)$, and instantaneous

profit for a single producer, $\pi(p, \phi, c) \equiv (p - c)x^*(p, \phi)$, where c is the marginal cost for the eco-product.⁶ To understand the relationship between the entire industry and the eco-product consumption, we respectively define the market size of the entire industry and the market share of the eco-product by $K(p, \phi) \equiv x^*(p, \phi) + y^*(p, \phi)$ and $X(p, \phi) \equiv \frac{x^*(p, \phi)}{K(p, \phi)} \in [0, 1]$. The market size is simply represented by the sum of the eco-product and the conventional product consumption, and the market share is represented by the eco-product consumption divided by the market size.

For simplicity, we assume that the instantaneous negative externalities from the conventional product are given by $E(p, \phi) = \Gamma(\epsilon(\phi)y^*(p, \phi))$, where Γ is strictly increasing and strictly convex with $\Gamma(0) = 0$ and $\Gamma(\epsilon) = \infty$.⁷ It depends only on the total pollution, which is represented by the pollution level per unit of the output of the conventional product times the conventional product consumption, $\epsilon(\phi)y^*(p, \phi)$. Then, instantaneous social welfare is defined as $T(p, \phi, c) \equiv \pi(p, \phi, c) + S(p, \phi) - E(p, \phi)$.

The central assumption is that the technology of an eco-product exhibits instantaneous constant returns to scale, but its marginal cost, c , is reduced over time as the single producer accumulates output, i.e., the level of marginal cost is reduced due to the accumulation of experience. We also consider the depreciation of experience, or in an alternative interpretation, the existence of potential adjustment costs in the accumulation of such experience.⁸ The reduction in the marginal cost is described as the following state equation:

$$\dot{c} = -\lambda[x^*(p, \phi) - \sigma(c)], \quad (2)$$

where $\lambda \geq 0$ represents the marginal cost reduction effect per unit of output, and $\sigma(c)$ captures a situation where the value of experience depreciates over time so that it is decreasing in c . The parameter λ can be interpreted as the dynamic learning effect. For simplicity, it is assumed that $\sigma(c) = \eta - \delta c$ with $\eta > 0$ and $\delta > 0$ and that the marginal cost of the eco-product in the initial period is not large enough so that $c_0 < \eta/\delta$. Notice that when there is no dynamic learning effect, i.e., $\lambda = 0$, our problem is reduced to a static single producer's problem with two differentiated products, in which one is produced by a single

⁶Consumer surplus in our definition does not include the negative externalities that come from pollution associated with the consumption and production of the conventional product. Such negative externalities are captured separately in this paper.

⁷The assumption $\Gamma(\epsilon) = \infty$ captures a situation in which when large consumption of the conventional product enough, the negative externalities from the conventional product are extremely high so that the society never accepts such a high level of negative externalities.

⁸Several authors claim that depreciation of knowledge is also important in reality and show some empirical evidence. See, e.g., Argote, Beckman, and Epple (1990) and Benkard (2000).

producer and the other is produced in a perfectly competitive market.

To analyze the pricing of the eco-product and the environmental regulation, we consider an economy which extends over the following two steps. In step 1, the government decides whether or not to adopt the environmental regulation on the production of the conventional product. We assume that the government cannot change its policy once it makes its decision. The decision affects the constant marginal cost for the conventional product, $\tau(\phi)$, and the pollution level per unit of the output of the conventional product, $\epsilon(\phi)$. In step 2, taking the government's decision, ϕ , as given, the single producer producing the eco-product decides the pricing schedule over time so that the present value of profits is maximized. Then, consumption and production of the eco-product and the conventional product take place in each time.

In this paper, we only consider the case that the government commits itself to the new environmental standard as the environmental regulation for a sufficiently long time, once it is fixed. Certainly, some authors suggest that there may be strategic interactions between government and industry, and such interactions may induce a frequent change of environmental standards over time.⁹ Although we admit such a problem, it seems worthwhile to examine the case of the government's commitment since it seems historically common that once targets or standards are fixed, they are sustained for a sufficiently long time such as 5 years, 10 years or much longer.¹⁰

3 Static Pricing Model

This section examines a static case where the single producer chooses the optimal price of the eco-product taking the environmental regulation ϕ and the marginal cost c as given. A static case is equivalent to one in which there is no dynamic learning effect, i.e., $\lambda = 0$. In this case, since the marginal cost for the eco-product is constant over time, the single producer simply maximizes its instantaneous profit $\pi(p, \phi, c)$ with respect to p , taking ϕ and c as given. The first-order condition yields the optimal price of the eco-product:

$$p_M(\phi, c) = \frac{c + \tau(\phi)\gamma + 1 - \gamma}{2}. \quad (3)$$

⁹Kolstad (2003) explains such possibility in chapter 11, and Yao (1988) develops a model of dynamic interactions between the government and the automobile industry in the discussion of emission controls.

¹⁰For example, the Safe Drinking Water Act (SDWA), the Clean Air Act set by the US EPA or most environmental standards implemented by the Japanese Ministry of the Environment were last amended 10 years ago. For reference, see the web-sites of these public organizations.

Using (3), we obtain the consumption of the eco-product and the conventional product, consumer surplus, and the single producer's profit: $x_M(\phi, c) \equiv x^*(p_M(\phi, c), \phi)$, $y_M(\phi, c) \equiv y^*(p_M(\phi, c), \phi)$, $S_M(\phi, c) \equiv S(p_M(\phi, c), \phi)$ and $\pi_M(\phi, c) \equiv \pi(p_M(\phi, c), \phi, c)$, respectively. Moreover, we obtain the market size of the entire industry, the market share of the eco-product, and negative externalities: $K_M(\phi, c) \equiv K(p_M(\phi, c), \phi)$, $X_M(\phi, c) \equiv X(p_M(\phi, c), \phi)$ and $E_M(\phi, c) \equiv E(p_M(\phi, c), \phi)$. In order to make our analysis simple, in the rest of this paper, we assume that the solution is interior in the sense that the consumption levels of the eco-product and the conventional product are positive. Based on the above equations, we first discuss the impacts of technological progress and the environmental regulation and then examine the optimal policy of whether or not the regulation should be adopted.

3.1 Technological Improvement

This subsection examines technological improvement of the production of the eco-product, which exhibits a decline in its marginal cost, when there is no dynamic learning effect, i.e., $\lambda = 0$. Then, we deduce the following results.

Proposition 1 (Technological Improvement) *Suppose that $\lambda = 0$. For any $\phi \in \{0, 1\}$, technological progress in the production of the eco-product (1) decreases the optimal eco-product price; (2) increases the output of the eco-product; (3) decreases the output of the conventional product and the negative externalities; (4) increases consumer surplus as well as the profit for the single producer; and (5) increases the market size of the industry as well as the market share of the eco-product.*

The results of Proposition 1 are straightforward. Technological progress in the eco-product not only substitutes the conventional product into the eco-product in the economy through a decline in the price of the eco-product but also improves social welfare through an increase in both consumer surplus and through the profit for the single producer and a decrease in the negative externalities.

3.2 Environmental Regulation

In this subsection, we consider the impact of the environmental regulation on the economy without a dynamic learning effect. The environmental regulation affects the optimal price of the eco-product by raising the market price of the conventional product. Then, we deduce the following results.

Proposition 2 (Environmental Regulation) *Suppose that $\lambda = 0$. For any c , the environmental regulation (1) increases the optimal price of the eco-product; (2) increases the output of the eco-product; (3) decreases the output of the conventional product and the negative externalities; (4) decreases consumer surplus but increases the profit for the single producer; and (5) decreases the market size of the industry but increases the market share of the eco-product.*

The environmental regulation allows the single producer of the eco-product to take advantage through increasing the production cost of the conventional product. Similar to the case of technological improvement in Proposition 1, the environmental regulation promotes the eco-product and decreases the conventional product and the negative externalities. However, the effects on the price of the eco-product, consumer surplus, and the market size of the industry take the opposite direction of technological improvement. In this case, promoting the eco-product can be achieved at the expense of consumer surplus. In contrast to Proposition 1 where technological progress always improves social welfare, whether the regulation improves social welfare depends on the degree of positive impact on the negative externality, the single producer's profit, and the degree of negative impact on consumer surplus.

3.3 Optimal Environmental Policy

We now find the optimal government decision regarding the environmental regulation in step 1 when there is no dynamic learning effect. The government chooses to adopt the environmental regulation, $\phi = 1$, if social welfare is larger under the regulation than under no regulation, i.e., $\Delta T_M(c) \equiv T(p_M(1, c), 1, c) - T(p_M(0, c), 0, c) > 0$, where $\Delta T_M(c)$ represents the effect of the environmental regulation on instantaneous social welfare. We assume that if the marginal cost of the eco-product is small enough so that the eco-product is diffused enough in the market, the environmental regulation cannot be justified from the standpoint of social efficiency, i.e., $\Delta T_M(0) < 0$. Then, we obtain the following result.

Proposition 3 (Environmental Policy) *For any ξ and ε , there exists a unique value $c_M \equiv c_M(\xi, \varepsilon) > 0$ such that the optimal policy is (1) to impose the environmental regulation on the conventional product if $c > c_M$; (2) not to impose the regulation if $c < c_M$.*

This result has an important implication for environmental policy. Proposition 2 states that the environmental regulation reduces consumer surplus while it improves the profit for the single producer and

reduces the negative externalities from the conventional product. Proposition 3 suggests a guideline for government policy. If the marginal cost of the eco-product is relatively high so that $c > c_M(\xi, \varepsilon)$, then the government should adopt the regulation at the expense of consumer surplus. In this case, the reduction of consumer surplus is covered through the increase in the profit for the single producer and the decrease in the negative externalities. On the other hand, if the marginal cost is relatively small so that $c < c_M(\xi, \varepsilon)$, then the government should not adopt the regulation. In this case, the negative impact of the regulation on consumer surplus cannot be offset by the positive impact on the profit for the single producer and the negative externalities.

The logic behind this result is carefully explained as follows. Let $\Phi(\xi, c) \equiv \Delta S_M(\xi, c) + \Delta \pi_M(\xi, c)$ denote the impact of the environmental regulation on social welfare excluding the impact on the negative externalities, where $\Delta S_M(\xi, c) \equiv S_M(1, c) - S_M(0, c)$ and $\Delta \pi_M(\xi, c) \equiv \pi_M(1, c) - \pi_M(0, c)$ represent the impact of the regulation on consumer surplus and the profit for the single producer, respectively. Notice that $\Delta S_M(\xi, c) < 0$ and $\Delta \pi_M(\xi, c) > 0$ by Proposition 2, and Φ does not depend on ε since ε affects only the negative externalities. Then, the effect of the regulation on social welfare can be rewritten as $\Delta T_M(c : \xi, \varepsilon) = \Phi(\xi, c) + \Gamma(\varepsilon y_M(0, c))$. This implies that adopting the regulation is optimal if $\Phi(\xi, c) > -\Gamma(\varepsilon y_M(0, c))$.

We first consider the effect of the change in c on Φ . Since it is assumed that $\Delta T_M(0 : \xi, \varepsilon) < 0$, it must hold that $\Phi(\xi, 0) < -\Gamma(\varepsilon y_M(0, 0)) < 0$. This assumption states that if c is small enough and close to zero, the impact of the regulation on social welfare excluding the negative externalities must be negative, i.e., the negative impact on consumer surplus dominates the positive impact on the profit for the single producer. In addition to this, since $\frac{\partial \Phi}{\partial c} = -\frac{3\gamma\xi}{4(1-\gamma^2)} < 0$, Φ is decreasing in c , i.e., the negative impact of the regulation on social welfare excluding the negative externalities becomes larger as c increases. The increment of the negative impact is constant irrespective of the level of c since $\frac{\partial \Phi}{\partial c}$ is independent of c . In Figure 1, line AA shows the graph of Φ , which is always negative and down-sloped.

Second, we consider the effect of the change in c on $\Gamma(\varepsilon y_M(0, c))$, which represents the negative externalities under no regulation. There are no negative externalities under the regulation. According to Proposition 1, the output of the conventional product, $y_M(0, c)$, is linearly increasing in c . Since Γ is strictly increasing and strictly convex in pollution εy_M , the negative externalities, $\Gamma(\varepsilon y_M)$, is strictly increasing and strictly convex in c . This captures a situation in which a higher c increases the production of the con-

ventional product and in turn the corresponding pollution and negative externalities. In Figure 1, curve BB represents the graph of $-\Gamma(\varepsilon y_M(0, c))$, which is always negative, down-sloped and strictly concave. Furthermore, the graphs of $\Phi(\xi, c)$ and $-\Gamma(\varepsilon y_M(0, c))$ have the single-crossing property in the sense that there exists a unique value $c_M > 0$ such that $\Phi(\xi, c) > -\Gamma(\varepsilon y_M(0, c))$ if $c > c_M$ and $\Phi(\xi, c) < -\Gamma(\varepsilon y_M(0, c))$ if $c < c_M$. This result depends on the crucial assumption that Γ is strictly convex.

When the marginal cost of the eco-product, c , is large enough, the negative externalities under no regulation are relatively large due to relatively large demand for the conventional product. In this case, the merit of the regulation is relatively large since the regulation avoids such negative externalities. Thus, adopting the regulation can be justified even though consumer surplus is reduced as a result. In contrast, when c is small enough, the negative externalities under no regulation is relatively small due to relatively small demand for the conventional product. In this case, the merit of the regulation is relatively small since the negative externalities that can be avoided are relatively small. Thus, the regulation cannot be justified even though it increases the profit for the single producer.

We next examine the comparative static of the critical value, $c_M(\xi, \varepsilon)$, related to ξ and ε . The critical value is important for policy makers to decide whether or not the environmental policy should be adopted. At the critical value, it must hold that $\Delta T_M(c_M(\xi, \varepsilon)) = 0$. Thus, we obtain the following results.

Proposition 4 (Critical Value of Marginal Cost) *The critical value $c_M(\xi, \varepsilon)$ is decreasing in the pollution level per unit of the output of the conventional product, ε . In contrast, the critical value is increasing (decreasing) in ξ if $\Phi(\xi, c)$ is decreasing (increasing) in the effect of the environmental regulation on the marginal cost of the conventional product, ξ .*

A rise in the pollution level ε increases the negative externalities under no regulation, i.e., the downward shift in curve BB of the graph of $-\Gamma(\varepsilon y_M(0, c))$ in Figure 1. In contrast, it does not affect the difference in social welfare excluding the negative externalities between when the regulation is present and when it is absent, i.e., no change in line AA of $\Phi(\xi, c)$. Thus, the critical value c_M decreases with a rise in ε . In other words, since an increase in the negative externalities associated with a rise in ε under no regulation makes the regulation more effective, the region of c , in which adopting the regulation is optimal ($c > c_M$), becomes larger as ε increases.

On the other hand, the impact of a change in the effect of the environmental regulation on the marginal cost of the conventional product, ξ , is in general ambiguous. The second part of Proposition 4 states that

the direction of the impact depends on the effect of a change in ξ on Φ , the difference in social welfare excluding the negative externalities between with and without the regulation. There are two channels through which ξ affects Φ : the first is that a rise (decline) in ξ increases (decreases) the profit for the single producer of the eco-product through a rise (decline) of the price of the conventional product; and the second is that a rise (decline) in ξ decreases (increases) consumer surplus in general. Thus, the impact of a change in ξ on Φ depends on which channel dominates the other. First, suppose that the first channel dominates the second, i.e., $\frac{\partial \Phi}{\partial \xi} > 0$. In this case, line AA of the graph of $\Phi(\xi, c)$ in Figure 1 shifts upward, and hence the critical value c_M declines. On the other hand, suppose that the second channel dominates the first, i.e., $\frac{\partial \Phi}{\partial \xi} < 0$. Contrary to the previous case, line AA of the graph of $\Phi(\xi, c)$ shifts downward, and hence the critical value c_M rises.

4 Dynamic Pricing Model under Learning-by-Doing

In the previous section, we examined the price of the eco-product and its effect on the economy under the assumption that there is no dynamic learning effect. It is now assumed that there is a positive dynamic learning effect, i.e., $\lambda > 0$. This section first studies the effect of the dynamic learning effect and characterizes the optimal price schedule over time decided by the single producer of the eco-product at step 2. We then analyze the governmental decision problem at step 1 in which the government can decide the environmental regulation $\phi \in \{0, 1\}$ at time $t = 0$ without the possibility of changing the policy after the decision. Finally, we show some numerical examples for a better understanding of our results.

4.1 Single Producer's Decision Problem

The single producer's problem is to maximize the present value of her profits, while considering the environmental regulation and the learning effects induced by current production through her pricing decisions. This problem can be stated as $\max_p \int_0^\infty \pi(p, \phi, c) e^{-rt} dt$ subject to $\dot{c} = -\lambda[x^*(p, \phi) - \sigma(c)]$ and $c(0) = c^0$, where $r > 0$ represents the discount rate. The value of $x^*(p, \phi)$ is the demand for the eco-product derived in equation (1). Applying the dynamic optimal control with the Hamiltonian, $H^F = \pi(p, \phi, c) - \mu_f \lambda [x^*(p, \phi) - \sigma(c)]$, the equilibrium of our model solves the following set of generalized Hamilton-Jacobi conditions: $x^*(p, \phi) + (p - c - \lambda \mu_f) x_p^*(p, \phi) = 0$ and $\dot{\mu}_f = (r + \lambda \delta) \mu_f + x^*(p, \phi)$. This

problem yields a linear differential system:

$$\begin{bmatrix} \dot{p} \\ \dot{c} \end{bmatrix} = M \begin{bmatrix} p \\ c \end{bmatrix} + \begin{bmatrix} \frac{\lambda\eta - (r + \lambda\delta)[1 - \gamma(1 - \tau(\phi))]}{2} \\ \frac{(1 - \gamma^2)\lambda\eta - \lambda[1 - \gamma(1 - \tau(\phi))]}{1 - \gamma^2} \end{bmatrix} \quad \text{where } M = \begin{bmatrix} r + \lambda\delta & -\frac{r + 2\lambda\delta}{2} \\ \frac{\lambda}{1 - \gamma^2} & -\lambda\delta \end{bmatrix}. \quad (4)$$

The matrix, M , is independent of whether or not to implement the regulation.

4.2 Steady State

This subsection examines the steady state, which is derived from the system (4), for any $\phi \in \{0, 1\}$. A pair of the marginal cost and the price of the eco-product, $(\bar{c}(\phi), \bar{p}(\phi))$, at which $\dot{c} = \dot{p} = 0$ in system (4), is called a steady state (see, e.g., Kamien and Schwartz (1991)). Since the system (4) is a linear differential system, we deduce the following result.

Proposition 5 (Existence and Uniqueness of Steady State) *Suppose that $\lambda > 0$. For any $\phi \in \{0, 1\}$, there exists a unique steady state $(\bar{c}(\phi), \bar{p}(\phi))$ in system (4).*

A steady state is stable if c and p converge to \bar{c} and \bar{p} , respectively (see, e.g., Kamien and Schwartz (1991)). Using matrix M , we obtain the following result related to the stability of the steady state.

Proposition 6 (Stability of Steady State) *Suppose that $\lambda > 0$. Then, the steady state, which is uniquely determined, is stable if $\det(M) < 0$ or $\gamma < \hat{\gamma}$, where $\hat{\gamma} \equiv (1 - \frac{r + 2\lambda\delta}{2\delta(r + \lambda\delta)})^{1/2}$.*

This result simply states that when the degree of product differentiation between the eco-product and the conventional product is small enough so that $\gamma < \hat{\gamma}$, the steady state that is uniquely determined in the system represented by the linear differential equations (4) is a saddlepoint and is stable. If $\gamma > \hat{\gamma}$, then the state variable never converges to the steady state but diverges. In the rest of the paper, we assume that $\gamma \in [0, \hat{\gamma})$.

We then characterize the steady state that is uniquely determined in the linear system (4). It is assumed that any variable is interior in the equilibrium path in the sense that the production and the consumption of the eco-product and the conventional product are positive. For any government policy $\phi \in \{0, 1\}$, by applying Cramer's Rule to system (4) with $\dot{p} = \dot{c} = 0$, we obtain the price and the marginal cost of the eco-product in the steady state, $(\bar{c}(\phi), \bar{p}(\phi))$. Using $(\bar{c}(\phi), \bar{p}(\phi))$, we derive the corresponding outputs of the eco-product, the conventional product, consumer surplus, the profit for the single producer, negative

externalities, the market size of the industry, and the market share of the eco-product in the steady state: $\bar{x}(\phi) \equiv x^*(\bar{p}(\phi), \phi)$, $\bar{y}(\phi) \equiv y^*(\bar{p}(\phi), \phi)$, $\bar{S}(\phi) \equiv S(\bar{p}(\phi), \phi)$, $\bar{\pi}(\phi) \equiv \pi(\bar{p}(\phi), \phi, \bar{c}(\phi))$, $\bar{E}(\phi) \equiv E(\bar{p}(\phi), \phi)$, $\bar{K}(\phi) \equiv \bar{x}(\phi) + \bar{y}(\phi)$, and $\bar{X}(\phi) \equiv \frac{\bar{x}(\phi)}{\bar{K}(\phi)}$. Notice that all instantaneous variables above are a function of the government's binary choice variable $\phi \in \{0, 1\}$. Based on the above equations, we focus on two effects on the steady state: the dynamic learning effect; and the effect of the environmental regulation.

4.2.1 Dynamic Learning Effect

The dynamic learning effect influences the equilibrium path and the steady state through the state equation $\dot{c} = -\lambda[x^*(p, \phi) - \sigma(c)]$. Recall that the degree of dynamic learning effect, λ , is an important parameter since the learning effect is the only source of technological progress in this study. To understand the effect of dynamic learning on the steady state, we first compare the variables in the steady state under positive learning effect ($\lambda > 0$) with the corresponding variables under no learning effect ($\lambda = 0$) associated with the initial marginal cost c_0 .

By simple calculations, we derive the optimal price of the eco-product in the steady state under a positive dynamic learning effect:

$$\bar{p}(\phi) = p_M(\phi, \bar{c}(\phi)) - \frac{\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)}, \quad (5)$$

where $p_M(\phi, c)$ is the optimal price for the single producer when there is no dynamic learning effect and the marginal cost equals c . For now it is assumed that the initial level of the marginal cost is high enough so that $c_0 > \bar{c}(\phi)$, and hence it must hold that $p_M(\phi, c_0) > p_M(\phi, \bar{c}(\phi))$ by Proposition 1. By equation (5), the impact of the dynamic learning effect on the price of the eco-product is described by:

$$\bar{p}(\phi) - p_M(\phi, c_0) = [p_M(\phi, \bar{c}(\phi)) - p_M(\phi, c_0)] - \frac{\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)} < 0. \quad (6)$$

The impact can be divided into two parts. The first part, $p_M(\phi, \bar{c}(\phi)) - p_M(\phi, c_0)$, which is negative, may be considered the cost-reduction effect in the sense that a decline in the marginal cost through the dynamic learning effect reduces the optimal price of the eco-product in the static case. The second part, $\bar{p}(\phi) - p_M(\phi, \bar{c}(\phi)) = -\frac{\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)}$, which is also negative, may be considered as the dynamic linkage effect. This sub-effect implies that the single producer sets a lower price in the steady state compared to the price

when there is no dynamic leaning effect and the marginal cost equals the steady state level of the marginal cost under a positive dynamic learning effect. As a result, the net impact of the dynamic learning effect on the price of the eco-product in the steady state is negative.

Similar to the discussion above, using equation (5), we obtain the outputs of the eco-product and the conventional product in the steady state:

$$\bar{x}(\phi) = x_M(\phi, \bar{c}(\phi)) + \frac{\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)(1 - \gamma^2)}; \quad \bar{y}(\phi) = y_M(\phi, \bar{c}(\phi)) - \frac{r\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)(1 - \gamma^2)}, \quad (7)$$

where $x_M(\phi, c)$ and $y_M(\phi, c)$ are respectively the outputs of the eco-product and the conventional product when there is no dynamic learning effect and the marginal cost equals c . Equations (7) yield the impact of the dynamic learning effect on the outputs of the eco-product and the conventional product:

$$\bar{x}(\phi) - x_M(\phi, c_0) = [x_M(\phi, \bar{c}(\phi)) - x_M(\phi, c_0)] + \frac{\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)(1 - \gamma^2)} > 0; \quad (8)$$

$$\bar{y}(\phi) - y_M(\phi, c_0) = [y_M(\phi, \bar{c}(\phi)) - y_M(\phi, c_0)] - \frac{r\lambda\sigma(\bar{c}(\phi))}{2(r + \lambda\delta)(1 - \gamma^2)} < 0, \quad (9)$$

which can be also divided into two sub-effects, the cost-reduction effect and the dynamic linkage effect. Since $\bar{c}(\phi) < c_0$ implies that $x_M(\phi, \bar{c}(\phi)) > x_M(\phi, c_0)$ and $y_M(\phi, \bar{c}(\phi)) < y_M(\phi, c_0)$ by Proposition 1, the cost-reduction effect is positive for the output of the eco-product, while it is negative for the output of the conventional product. Moreover, the dynamic linkage effect is positive for the output of the eco-product, while it is negative for the output of the conventional product. As a result, the net impact is positive for the output of the eco-product, while it is negative for the output of the conventional product. The dynamic learning effect causes the single producer of the eco-product to set a lower price in the steady state compared to the price of the eco-product when there is no dynamic learning effect with the initial marginal cost c_0 . This in turn achieves a larger level of production and consumption of the eco-product and smaller level of production and consumption of the conventional product in the steady state due to their substitutability.

The impact of the dynamic learning effect on consumer surplus is given by:

$$\bar{S}(\phi) - S_M(\phi, c_0) = [S_M(\phi, \bar{c}(\phi)) - S_M(\phi, c_0)] + [\bar{S}(\phi) - S_M(\phi, \bar{c}(\phi))], \quad (10)$$

where $S_M(\phi, \bar{c}(\phi)) - S_M(\phi, c_0)$ represents the cost-reduction effect, and $\bar{S}(\phi) - S_M(\phi, \bar{c}(\phi))$ represents the dynamic linkage effect. Since $\bar{p}(\phi) < p_M(\phi, \bar{c}(\phi))$ by equation (5) and $S(p, \phi)$ is decreasing in p due to the envelope theorem, we obtain $\bar{S}(\phi) = S(\bar{p}(\phi), \phi) > S(p_M(\phi, \bar{c}(\phi)), \phi) = S_M(\phi, \bar{c}(\phi))$, which implies that the dynamic linkage effect in equation (10) is positive. Moreover, since $S_M(\phi, c)$ is decreasing in c by Proposition 1, we obtain $S_M(\phi, \bar{c}(\phi)) > S_M(\phi, c_0)$ under the assumption of $\bar{c}(\phi) < c_0$. Thus, the cost-reduction effect is also positive. As a result, the net impact is positive for consumer surplus. Taking into account the fact that the negative externalities are reduced in the steady state through the reduction in production of the conventional product in equation (9), the dynamic learning effect allows consumers to obtain benefits from not only the reduction of the negative externalities but also the rise in consumer surplus.

Furthermore, using equations (7) we derive the market size of the industry and the market share of the eco-product in the steady state, $\bar{K}(\phi)$ and $\bar{X}(\phi)$. The impact of the dynamic learning effect on the market size of the industry and the market share of the eco-product are respectively described by:

$$\begin{aligned}\bar{K}(\phi) - K_M(\phi, c_0) &= [K_M(\phi, \bar{c}(\phi)) - K_M(\phi, c_0)] + \frac{\lambda\sigma(\bar{c}(\phi))(1-r)}{2(r+\lambda\delta)(1-\gamma^2)}; \\ \bar{X}(\phi) - X_M(\phi, c_0) &= [X_M(\phi, \bar{c}(\phi)) - X_M(\phi, c_0)] + \frac{Z[1 - X_M(\phi, \bar{c}(\phi))(1-r)]}{K_M(\phi, \bar{c}(\phi)) + Z(1-r)},\end{aligned}$$

where $Z \equiv \frac{\lambda\sigma(\bar{c}(\phi))}{2(r+\lambda\delta)(1-\gamma^2)} > 0$. These impacts can be also divided into the cost-reduction effect and the dynamic linkage effect. Since $\bar{c}(\phi) < c_0$ implies that $K_M(\phi, \bar{c}(\phi)) > K_M(\phi, c_0)$ and $X_M(\phi, \bar{c}(\phi)) < X_M(\phi, c_0)$ by Proposition 1, the cost-reduction effect is positive for both the market size of the industry and the market share of the eco-product. Moreover, noticing that $1 - X_M(\phi, \bar{c}(\phi))(1-r)$ due to $X_M(\phi, \bar{c}(\phi)) \in (0, 1)$, the dynamic linkage effect is also positive for both. As a result, the net impact is positive for both. This has an important implication about the relation between the dynamic learning effect and the promotion of the eco-product in the steady state. The dynamic learning effect substitutes the conventional product for the eco-product without shrinking the total market size of the industry.

It should be also noted that the impact of the dynamic learning effect on the profit for the single producer in the steady state is ambiguous in general. To see this, we consider the impact of dynamic

learning effect on the profit for the single producer, which is given by:

$$\bar{\pi}(\phi) - \pi_M(\phi, c_0) = [\pi_M(\phi, \bar{c}(\phi)) - \pi_M(\phi, c_0)] - \frac{\lambda^2[\sigma(\bar{c}(\phi))]^2}{4(r + \lambda\delta)^2(1 - \gamma^2)}, \quad (11)$$

where the first term $\pi_M(\phi, \bar{c}(\phi)) - \pi_M(\phi, c_0)$ is the cost-reduction effect, and the second term $\bar{\pi}(\phi) - \pi_M(\phi, \bar{c}(\phi)) = -\frac{\lambda^2[\sigma(\bar{c}(\phi))]^2}{4(r + \lambda\delta)^2(1 - \gamma^2)}$ is the dynamic linkage effect. Since $\pi_M(\phi, c)$ is decreasing in c with $\bar{c}(\phi) < c_0$, the cost-reduction effect is positive.¹¹ In contrast, since $\bar{\pi}(\phi) - \pi_M(\phi, \bar{c}(\phi)) < 0$, the dynamic linkage effect is negative. Thus, the direction of the dynamic learning effect on the profit for the single producer in the steady state depends on which sub-effect dominates the other. From the above discussion, we summarize the impact of dynamic learning effect on the steady state as follows:

Proposition 7 (Impact of Dynamic Learning Effect on Steady State) *Suppose that $\lambda > 0$ with $\sigma(\bar{c}(\phi)) = \eta - \delta\bar{c}(\phi) > 0$ and $c_0 > \bar{c}(\phi)$. For any $\phi \in \{0, 1\}$, the steady state must satisfy that, compared to the corresponding variable associated with the initial level of the marginal cost c_0 under no dynamic learning effect, (1) the price of the eco-product is lower; (2) the output of the eco-product is larger; (3) the output of the eco-product and the negative externalities are smaller; (4) consumer surplus is larger; (5) the market size of the industry and the market share of the eco-product are larger.*

We next examine the effect of a change in the degree of dynamic learning effect, λ , on the steady state, i.e., the comparative static of λ on the steady state. By the state equation, it must hold that $\bar{x}(\phi) = \sigma(\bar{c}(\phi))$ in the steady state, which yields

$$\sigma(\bar{c}(\phi)) = \frac{(r + \lambda\delta)[1 - \bar{c}(\phi) - \gamma(1 - \tau(\phi))]}{2(r + \lambda\delta)(1 - \gamma^2) - \lambda}. \quad (12)$$

Differentiating this equation with respect to λ , we deduce the following results of the comparative statics of λ :

Proposition 8 (Comparative Static) *Suppose that $\lambda > 0$ and $\gamma < \hat{\gamma}$. Then, the marginal cost of the eco-product in the steady state is decreasing in the degree of dynamic learning effect, λ . Furthermore, in the steady state, an increase in λ (1) decreases the optimal eco-product price; (2) increases the output of the eco-product; (3) decreases the output of the conventional product and the negative externalities; (4)*

¹¹For the proof that $\pi_M(\phi, c)$ is decreasing in c , see the proof of Proposition 1.

increases consumer surplus; and (5) increases the market size of the industry as well as the market share of the eco-product.

The first part of this proposition simply says that a larger degree of the dynamic learning effect induces a reduction in the marginal cost of the eco-product in the steady state. The second part shows the impact of a rise in the degree of dynamic learning effect on other variables. A rise in the degree of dynamic learning effect promotes the eco-product. In addition, its positive impact on social welfare in the steady state is to increase consumer surplus and to reduce the negative externalities, while its impact on the profit for the single producer in the steady state is ambiguous in general.

4.2.2 Environmental Regulation

We examine the effect of the environmental regulation, $\phi \in \{0, 1\}$, on the steady state in this economy. Solving equation (12) yields the marginal cost of the eco-product in the steady state, $\bar{c}(\phi)$. Then, we deduce the following results related to the marginal cost of the eco-product in the steady state.

Proposition 9 (Environmental Regulation and Marginal Cost in Steady State) *Suppose that $\lambda > 0$ and $\gamma < \hat{\gamma}$. Then, the environmental regulation reduces the marginal cost of the eco-product in the steady state, i.e., $\bar{c}(0) > \bar{c}(1)$.*

The environmental regulation improves the production technology of the eco-product in the steady state. This may be consistent with the claim in Porter (1991) and Porter and van der Linde (1995) that tightening the standards would trigger innovation within firms (in this case, not product innovation but process innovation) in the long run. This result crucially depends on the dynamics of the marginal cost of the eco-product that relates to the dynamic learning effect.

Based on the result in Proposition 9, we now examine the impact of the environmental regulation on the price of the eco-product in the steady state. By equation (5), the impact on the price of the eco-product in the steady state is rewritten as:

$$\bar{p}(1) - \bar{p}(0) = \frac{\gamma}{2}(\tau(1) - \tau(0)) + \frac{1}{2} \left[1 + \frac{\lambda\delta}{r + \lambda\delta} \right] (\bar{c}(1) - \bar{c}(0)), \quad (13)$$

which implies that the impact on the price of the eco-product can be divided into two sub-effects. The first, $\frac{\gamma}{2}(\tau(1) - \tau(0)) = \frac{\gamma\xi}{2}$, may be considered as the direct effect of the environmental regulation in the

sense that this term is directly affected by the rise in the price or the marginal cost of the conventional product under the regulation. The second sub-effect, $\frac{1}{2}[1 + \frac{\lambda\delta}{r+\lambda\delta}](\bar{c}(1) - \bar{c}(0))$, may be considered as the indirect effect of the environmental regulation in the sense that this term is affected by the decline in the marginal cost of the eco-product, which is caused by enhancing the dynamic learning effect through the regulation, as shown in Proposition 9.

Since the first sub-effect is positive while the second sub-effect is negative, the impact of the environmental regulation on the price of the eco-product in the steady state is ambiguous in general, and which sub-effect dominates the other determines whether the price of the eco-product rises or declines in the long run. If technological improvement through the dynamic learning effect is large enough so that the second sub-effect dominates the first, the price of the eco-product declines. This result is in contrast to the result in Proposition 2 that the environmental regulation always raises the price of the eco-product in the static setting. The crucial difference between the static and the dynamic settings is that the second sub-effect associated with technological improvement through the dynamic learning effect exists in the dynamic setting while it does not exist in the static setting.

Concerning the impact of the environmental regulation on the outputs of the eco-product and the conventional product in the steady state, we use equations (1) and (13) to obtain:

$$\bar{x}(1) - \bar{x}(0) = \frac{\gamma}{2(1 - \gamma^2)}(\tau(1) - \tau(0)) - \frac{1}{2(1 - \gamma^2)} \left[1 + \frac{\lambda\delta}{r + \lambda\delta} \right] (\bar{c}(1) - \bar{c}(0)); \quad (14)$$

$$\bar{y}(1) - \bar{y}(0) = -\frac{2 - \gamma^2}{2(1 - \gamma^2)}(\tau(1) - \tau(0)) + \frac{\gamma}{2(1 - \gamma^2)} \left[1 + \frac{\lambda\delta}{r + \lambda\delta} \right] (\bar{c}(1) - \bar{c}(0)). \quad (15)$$

Since $\tau(1) - \tau(0) > 0$ and $\bar{c}(1) - \bar{c}(0) < 0$ by Proposition 9, the environmental regulation increases the output of the eco-product but decreases the output of the conventional product in the steady state irrespective of the impact on the price of the eco-product, i.e., $\bar{x}(1) > \bar{x}(0)$ and $\bar{y}(1) < \bar{y}(0)$, as in the result of the static setting in Proposition 2. These results in turn imply that the environmental regulation increases the market share of the eco-product in the steady state, i.e., $\bar{X}(1) > \bar{X}(0)$. In addition to this, using equations (14) and (15), we derive the market size of the industry in the steady state:

$$\bar{K}(1) - \bar{K}(0) = -\frac{2 + \gamma}{2(1 + \gamma)}(\tau(1) - \tau(0)) - \frac{1}{2(1 + \gamma)} \left[1 + \frac{\lambda\delta}{r + \lambda\delta} \right] (\bar{c}(1) - \bar{c}(0)), \quad (16)$$

which implies that the impact on the market size of the industry in the steady state can be also divided into two sub-effects, the direct effect and the indirect effect, as in the discussion of the impact on the price of the eco-product in the steady state. Since the first sub-effect is positive while the second sub-effect is negative, the impact on the market size in the steady state is ambiguous in general. When the technological improvement through the dynamic learning effect is large enough so that the second sub-effect dominates the first, the environmental regulation raises the market size of the industry in the steady state, i.e., $\bar{K}(1) > \bar{K}(0)$.

These results have an important implication for the impact of the environmental regulation on the promotion of the eco-product in the long run. The environmental regulation promotes the eco-product through substituting the conventional product as in the result of the static setting in Proposition 2. However, in contrast to the result that the environmental regulation always shrinks the market size of the industry in the static setting, it may increase the market size of the industry in the steady state if the technological improvement through the dynamic learning effect is large enough.

Moreover, with relatively large technological improvement, the environmental regulation may have the similar effect to a rise in the degree of the dynamic learning effect in that it promotes the eco-product in the long run without shrinking the market size of the industry, as shown in Proposition 8. In this sense, the environmental regulation may be partially equivalent to a rise in the degree of dynamic learning effect. Then, we summarize the above discussions about the impact of the environmental regulation on the variables related to the promotion of the eco-product in the steady state as follows:

Proposition 10 (Environmental Regulation and Promotion of Eco-Product) *Suppose that $\lambda > 0$ and $\gamma < \hat{\gamma}$. Then, in the steady state, the environmental regulation (1) increases the output of the eco-product; (2) decreases the output of the conventional product; and (3) increases the market share of the eco-product. Furthermore, if the impact of the environmental regulation on the marginal cost of the eco-product is large enough, then (4) the price of the eco-product may decline, and (5) the market size of the industry may increase.*

We furthermore observe the impact of the environmental regulation on social welfare that is composed of the negative externalities, consumer surplus and the profit for the single producer in the steady state. By Proposition 10, the environmental regulation reduces the output of the conventional product in the steady state, which in turn reduces the negative externalities, i.e., $\bar{E}(1) < \bar{E}(0)$. However, in general, the

impact of the environmental regulation on consumer surplus and the profit for the single producer in the steady state are ambiguous. Hence, the impact on social welfare in the steady state is also ambiguous. Some numerical examples will be shown and some related implications will be also discussed in a later section.

4.3 Equilibrium Path

The previous subsection discussed the characterization of the steady state. This subsection examines the equilibrium path to the steady state. The equilibrium path is fully decided by the single producer of the eco-product, taking the marginal cost at the initial time, c_0 , as given. Propositions 5 and 6 have already shown that the system has a unique steady state that is stable if $\gamma < \hat{\gamma}$. Under the condition of $\gamma < \hat{\gamma}$, for any government policy $\phi \in \{0, 1\}$, the equilibrium path of state variable c in the system (4) is described by $\hat{c}_t(c_0, \phi) = [c_0 - \bar{c}(\phi)]e^{kt} + \bar{c}(\phi)$, where $k = \frac{r}{2} - \frac{r+2\lambda\delta}{2} \left[1 - \frac{2\lambda}{(1-\gamma^2)(r+2\lambda\delta)}\right]^{\frac{1}{2}} \in (-\lambda\delta, 0)$ since $\gamma < \hat{\gamma}$.¹² Then, for a given state variable $\hat{c} = \hat{c}_t(c_0, \phi)$, the optimal price of the eco-product decided by the single producer is:

$$\hat{p}(\phi, \hat{c}) = \alpha_1 \hat{c} + \alpha_2(\phi), \quad (17)$$

where $\alpha_1 = \frac{(1-\gamma^2)(k+\lambda\delta)}{\lambda} > 0$ and $\alpha_2(\phi) = -\frac{k(1-\gamma^2)}{\lambda} \bar{c}(\phi) - \eta(1-\gamma^2) + 1 - \gamma(1-\tau(\phi))$. Since k is independent of ϕ , α_1 is also independent of ϕ , but $\alpha_2(\phi)$ is dependent of ϕ . Using (17), given ϕ and \hat{c} , the equilibrium levels of the outputs of the eco-product and the conventional product, consumer surplus, the profit for the single producer, the negative externalities, the market size of the industry, and the share of the eco-product can be respectively represented by $\hat{x}(\phi, \hat{c}) \equiv x^*(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{y}(\phi, \hat{c}) \equiv y^*(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{S}(\phi, \hat{c}) \equiv S(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{\pi}(\phi, \hat{c}) \equiv \pi(\hat{p}(\phi, \hat{c}), \phi, \hat{c})$, $\hat{E}(\phi, \hat{c}) \equiv E(\hat{p}(\phi, \hat{c}), \phi)$, $\hat{K}(\phi, \hat{c}) \equiv K(\hat{p}(\phi, \hat{c}), \phi)$, and $\hat{X}(\phi, \hat{c}) \equiv X(\hat{p}(\phi, \hat{c}), \phi)$. Then, we deduce the following results related to the equilibrium path to the steady state.

Proposition 11 *Suppose that $\lambda > 0$ and $\gamma < \hat{\gamma}$. For given $\phi \in \{0, 1\}$, if $c_0 > \bar{c}(\phi)$, the marginal cost of the eco-product is monotone decreasing over time. In this case, the output of the eco-product, consumer surplus, the market size of the industry and the market share of the eco-product are monotone increasing over time, while the price of the eco-product, the output of the conventional product and the negative externalities*

¹²See the proof of Proposition 6 in the Appendix.

are monotone decreasing over time.

In an infinite horizon autonomous problem with just one state variable, if there is an optimal path to a steady state, the state variable is monotonic over time, and the steady state must be a saddlepoint. The result in this proposition states that if the initial marginal cost of the eco-product is high enough so that $c_0 > \bar{c}$, then the marginal cost, which is the state variable, is monotonically decreasing over time, and hence other variables are also monotonic over time. Figure 2 indicates the general direction of movement that (c, p) would take from any location. c is momentarily stationary along a path as the $\dot{c} = 0$ locus is crossed and k is stationary as the $\dot{k} = 0$ locus is crossed. The condition $\gamma < \hat{\gamma}$ guarantees that there is a steady state level of (\bar{c}, \bar{p}) that is sustainable forever.

4.4 Government's Optimal Decision

Until the previous subsection, we examined an economy with a dynamic learning effect in step 2, taking the government policy ϕ as given. This subsection discusses the optimal governmental decision problem related to the environmental regulation in step 1. Recall that the government has to decide whether or not to adopt the environmental regulation on the conventional product only at the initial period, and it cannot change its policy forever. It is also assumed that the effects of the environmental regulation on the marginal cost of the conventional product and the pollution level per unit of the output of the conventional product, which are respectively characterized by the parameters, ξ and ε , have been already determined exogenously.

The government's problem is to maximize the present value of social welfare over time, consisting of consumer surplus, the profit for the single producer and the negative externalities, taking into account the pricing decision made by the single producer of the eco-product. Since instantaneous social welfare is described by $\hat{T}(\phi, c) = \hat{S}(\phi, c) + \hat{\pi}(\phi, c) - \hat{E}(\phi, c)$ for any state variable c , the government's decision problem in time $t = 0$ can be restated as $\max_{\phi \in \{0,1\}} W(\phi, c_0) \equiv \int_0^\infty \hat{T}(\phi, c_t(c_0, \phi)) e^{-rt} dt$, where $c_t(c_0, \phi)$ is the equilibrium path taking ϕ and c_0 as given. $W(\phi, c_0)$ is the present value of social welfare taking ϕ and c_0 as given. By a simple calculation, we obtain $W(\phi, c_0) = \hat{\alpha}_0(\phi) + \hat{\alpha}_1(\phi)c_0 + \hat{\alpha}_2(\phi)c_0^2$, where $\hat{\alpha}_l(\phi)$ is constant for $l \in \{0, 1, 2\}$ and $\phi \in \{0, 1\}$. Then, the government's optimal decision is to adopt the environmental

regulation if $W(1, c_0) > W(0, c_0)$ or

$$W(1, c_0) - W(0, c_0) = \int_0^\infty [\hat{T}(1, c_t(c_0, 1)) - \hat{T}(0, c_t(c_0, 0))]e^{-rt} dt > 0, \quad (18)$$

and not to adopt the environmental regulation if $W(1) < W(0)$, where $\hat{T}(1, c_t(c_0, 1)) - \hat{T}(0, c_t(c_0, 0))$ is the difference in instantaneous social welfare between under the regulation and under no regulation.¹³

It should be noted that the present paper considers only a situation where there are two feasible choices by the government: to adopt the environmental regulation and not to adopt it. In reality, however, there are more than two feasible choices. When the government has a finite or an infinite number of feasible policies, $\{\phi_n\}$, instead of two feasible choices, it can find the optimal policy ϕ_{n^*} such that $W(\phi_{n^*}, c_0) = \max_n W(\phi_n, c_0)$. For our primary purpose in discussing the impacts of the dynamic learning effect and the environmental regulation, we believe that analyzing our simple setting with two feasible choices is valuable. It will be proposed that a numerical solution may be an adequate guide for policy makers to choose the optimal policy at a later part.

5 Numerical Examples

In the section, we explore some numerical examples of both a static model and a dynamic model to illustrate the results and their economic implications. The focus is on how the learning effect and the environmental regulation can affect the outcomes. For the purpose of comparison, we first show a numerical illustration of a static model and secondly present the numerical solution of a dynamic pricing model with learning-by-doing.

5.1 Static Model

For our base case of a static model, we assume that instantaneous negative externalities are represented by $E(p, \phi) = \frac{\epsilon y^*(p, \phi)^2}{M - y^*(p, \phi)}$, where M sets the upper bound to which extent a society can accept the negative externalities generated from conventional products. Throughout this section, we employ $M = 0.61$.¹⁴ Notice

¹³Since the government's decision problem is just to choose the policy $\phi \in \{0, 1\}$ in the initial time such that the policy attains the maximum level of the present value of the integral of social welfare over time, and since the government has to commit its policy forever, it is not the optimal stopping problem, where the government chooses when the regulation should be stopped. Our model in this study may be extended to such optimal stopping problem.

¹⁴The parameter set introduced in what follows are selected so that $K = 0.61$ is compatible with reasonable economic conditions.

that when $y^*(p, \phi)$ approaches M , then the negative externalities go to infinity. For the other parameters in a static model, we choose a coefficient of negative externalities, $\varepsilon = 0.01$, degree of substitution between an eco-product and a conventional product, $\gamma = 0.25$, the marginal cost of conventional products, $\underline{\tau} = 0.4$, and an additional marginal cost of the conventional product imposed by the environmental regulation, $\xi = 0.1$.

Figures 3 and 4 show the market size and the share of the eco-product in an equilibrium of a static model as a function of the marginal cost of eco-products, c . Consistent with Proposition 1 and 2, technological progress, or a decline in c , increases the market size of the industry and the market share of the eco-product, and the environmental regulation shrinks the market size of the industry but increases the market share of the eco-product. Furthermore, Figure 5 characterizes the critical marginal cost, c_M , that determines whether or not the environmental regulation should be implemented, as in Proposition 3. Following the discussion of Proposition 3, the graph shows the two curves of $\Phi(\xi, c)$ and $\Gamma(\varepsilon y_M)$ as a function of the marginal cost. In this example, there exists an interior unique critical marginal cost, $c_M = 0.4457$, such that the optimal policy is to adopt the environmental regulation if $c > c_M$ and not to adopt it otherwise.

5.2 Dynamic Model

The dynamic model needs additional parameters: $(\eta, \delta) = (0.9, 1.15)$ is set such that $\dot{c} = -\lambda[x^* - (0.9 - 1.15c)]$. In addition we choose a discount factor of $r = 0.05$, an initial marginal cost of $c_0 = 0.54$ and the two values of learning parameters $\lambda = \{\lambda_H, \lambda_L\} = \{2, 0.65\}$. Throughout this section, we employ four scenarios that are respectively represented by the notations of $\{(R, H), (NR, H), (R, L), (NR, L)\}$ where R and NR represent ‘Regulation’ and ‘No regulation,’ respectively, while H and L correspond to the cases of a high learning effect λ_H and a low learning effect λ_L , respectively. For example, the notation (NR, H) corresponds to the equilibrium outcome when the economy has a high learning effect with the regulation, keeping the other parameters fixed.

The above four possible scenarios are assumed for clarity and are sufficient to exhaustively present our numerical illustration and the economic implications in a dynamic setting. Figures 6 to 11 show the four trajectories of the equilibrium outcome, each of which represents one scenario, and each figure corresponds to the marginal cost, the eco-product price, the market share of the eco-product, the market size of the industry, consumer surplus, and social welfare, as shown in the captions.

We first focus on the learning effect. The two trajectories of (NR, L) and (NR, H) in Figures 6 to 11 simply reveal general qualitative outcomes. When the learning effect is high, a single producer has an incentive to set a lower price than in the case of a low learning effect, given the same initial marginal cost c_0 . Although $c_0 = 0.54$ is employed in this example, the single producer's pricing scheme generally holds. The equilibrium path of c and p are illustrated in Figures 6 and 7. Furthermore, a rise in the learning effect increases the market share of the eco-product as well as the market size of the industry (see (NR, L) and (NR, H) in Figures 8 and 9). In terms of social welfare, a high learning effect in general brings about favorable outcomes for each component of social welfare, that is, consumer surplus, the single producer's profit, and the negative externalities (see (NR, L) and (NR, H) in Figures 10 and 11).¹⁵

We next examine the impact of the environmental regulation. The regulation raises the marginal cost of the conventional product to a certain degree. In this example, we set the increase as $\xi = 0.1$. As shown in a static model, the regulation has a potential adverse effect on consumer surplus. The difference between a static model and a dynamic model is that the regulation in a dynamic model may offset such a negative impact with the potential future gain through a decline in the price of the eco-product when the learning effect is sufficiently high. The following discussion will illustrate this point.

Two trajectories of (NR, H) and (R, H) reveal some general results when the regulation is adopted. Figures 6 and 7 show that the regulation reduces the marginal cost and the price of the eco-product (see the trajectories of (NR, H) and (R, H)). In addition, Figures 8 and 9 illustrate that the regulation increases the market share of the eco-product and the market size of the industry (at the steady state) due to the fact that it causes the price of the eco-product to decline in the long run (see the trajectories of (NR, H) and (R, H)). Up to this point, the regulation basically has a similar effect as in a "high learning effect case." Therefore, we would say that the regulation may be considered partially equivalent to a rise in the degree of the learning effect.

So far we have explained the positive side of the regulation. However, as a negative impact, the regulation induces social loss during some early periods. The trajectories of (R, H) and (NR, H) in Figures 9 and 10 reveal that the market size of the industry shrinks, and consumer surplus is reduced during some early periods. The loss of consumer surplus is mainly caused by a rise in the price of the conventional product associated with the regulation. In this example, we show the case in which the learning effect

¹⁵For the case of the other two trajectories (R, H) and (R, L) , the same qualitative results hold. Therefore, we have not presented that case.

is high enough so that the relation becomes converse in the long run, i.e., consumer surplus in (R, H) is higher than that in (NR, H) at the steady state. In contrast, examination of the trajectories of (R, L) and (NR, L) in Figures 9 and 10 reveals that when the learning effect is low, the market size of the industry and consumer surplus in (R, L) are kept lower than those in (NR, L) throughout the equilibrium path. This implies that with a low learning effect, the regulation cannot totally offset the initial loss of consumer surplus even in the long run. In summary, whether or not the regulation improves consumer surplus depends on the degree of the learning effect. These results are basically consistent with the theoretical results shown in the previous section.

The difference in the effect of the regulation between high and low learning cases leads to some interesting outcomes. When the learning effect is sufficiently high, a Pareto improvement outcome may be achieved by the regulation in the steady state, i.e., every component of social welfare, consumer surplus, the single producer's profit as well as the negative externalities may be all better off in the end. The trajectories of (R, H) and (NR, H) in Figures 10 and 11 show the improvement of consumer surplus and social welfare in the long run.

Finally, we present the numerical outcome of whether social planners should adopt the regulation with an initial marginal cost of $c_0 = 0.54$, following the criteria (18) in the previous section. When $\lambda = \lambda_H = 2.0$, the regulation increases social welfare over time by $W(1, c_0) - W(0, c_0) \approx 5.964 - 4.9758 = 0.9882$. In this case, the regulation not only brings about an increase in social welfare, but also a Pareto improvement. However, if the government happens to ignore the learning effect, i.e., $\lambda = 0$, then the result of a static model implies that the policy decision becomes converse. That is, since $c_0 = 0.54 > c_M = 0.4557$, the regulation cannot be justified. This numerical result of policy decisions illustrates the importance of recognizing the learning effect in deciding to adopt the regulation.

6 Conclusions

This paper has analyzed a wide set of economic implications in a situation where an eco-product, which is invented and introduced by a single producer into a market, must compete with a pre-existing product supplied under perfect competition. Both a static model and a dynamic model have been examined from the standpoint of the social planner who has the option of tightening environmental regulation to improve social welfare. An essential feature of a dynamic model is learning-by-doing in eco-product planning, and

it is shown that the equilibrium outcomes in the presence of learning are more favorable to social welfare. The optimal government policy of whether or not to adopt the regulation depends on the marginal cost of the eco-product in a static setting, while it is highly dependent upon the degree of the learning effects in a dynamic setting. We numerically illustrated how the degree of the learning effect affects the equilibrium path as well as the effectiveness of the environmental regulation.

The results potentially include important policy implications and can offer some justification for supporting tighter environmental regulation on conventional products. First, the regulation induces more technological progress and partially plays the same role as a rise in the degree of learning effects. Second, the regulation not only improves social welfare by mitigating negative externalities in the long run, but also may induce a Pareto improvement by expanding the market size of the industry and promoting the market share of the eco-product. Of course, there are certain conditions to guarantee that the regulation makes a society better off. However, in general the greater the learning potentials in the eco-product planning are, the more important government intervention is. This is simply due to the fact that as the learning effect becomes larger, the regulation is more effective as illustrated in our numerical examples.

There are a number of additional topics that can be addressed in the future. One potential issue is how incentive schemes such as subsidy/tax imposed by a government affects the promotion of eco-products. It would also be interesting to introduce learning spillovers from eco-product planning on the production technologies of conventional products. Furthermore, research and development activities can also be an important factor affecting the progress of eco-product technology. This paper is, to our best knowledge, a first attempt at analyzing the role of learning-by-doing in eco-product planning and its relation with environmental regulations. We are hopeful that the results of our research clarify the great potential of tightening environmental regulations to promote eco-products and stimulate further research questions.

7 Appendix

We first derive some important variables in case of static pricing of the eco-product. Using (3), we obtain $x_M(\phi, c) = \frac{1-c-\gamma(1-\tau(\phi))}{2(1-\gamma^2)} > 0$, $y_M(\phi, c) = \frac{(1-\tau(\phi))(2-\gamma^2)-\gamma(1-c)}{2(1-\gamma^2)} > 0$, $\pi_M(\phi, c) = \frac{[1-c-\gamma(1-\tau(\phi))]^2}{4(1-\gamma^2)}$, $K_M(\phi, c) = \frac{(1-c)+(1-\tau(\phi))(2+\gamma)}{2(1+\gamma)}$, and $X_M(\phi, c) = \frac{1-c-(1-\tau(\phi))\gamma}{(1-\gamma)[(1-c)+(1-\tau(\phi))(2+\gamma)]}$. Moreover, using (3), we obtain $E_M(\phi, c) = \Gamma(\varepsilon y_M(0, c))$ if $\phi = 0$ and $E_M(\phi, c) = 0$ if $\phi = 1$.

Proof of Proposition 1 Differentiating p_M , x_M and y_M with respect to c , we obtain $\frac{\partial p_M}{\partial c} = \frac{1}{2} > 0$; $\frac{\partial x_M}{\partial c} = -\frac{1}{2(1-\gamma^2)} < 0$; and $\frac{\partial y_M}{\partial c} = \frac{\gamma}{2(1-\gamma^2)} > 0$, which are the desired results in (1), (2) and (3) in this Proposition. Noticing that $\frac{\partial S}{\partial p} = (u_x - p)\frac{\partial x^*}{\partial p} + (u_y - \tau(\phi))\frac{\partial y^*}{\partial p} - x^* = -x^* < 0$ and that $x_M(\phi, c) > 0$ or $1 - c - \gamma(1 - \tau(\phi)) > 0$, we obtain $\frac{\partial S_M}{\partial c} = \frac{\partial S}{\partial p} \frac{\partial p_M}{\partial c} = -x_M(\phi, c)\frac{\partial p_M}{\partial c} < 0$ and $\frac{\partial \pi_M}{\partial c} = -\frac{1-c-\gamma(1-\tau(\phi))}{2(1-\gamma^2)} < 0$, which are the desired results in (4). Finally, differentiating K_M and X_M with respect to c yields $\frac{\partial K_M}{\partial c} = -\frac{1}{2(1+\gamma)} < 0$ and $\frac{\partial X_M}{\partial c} = -\frac{2(1+\gamma)(1-\tau(\phi))}{(1-\gamma)[(1-c)+(1-\tau(\phi))(2+\gamma)]^2} < 0$, which is the desired result in (5). \square

Proof of Proposition 2 Let $\tilde{p}(\xi, c) \equiv p_M(1, c : \xi)$, $\tilde{x}(\xi, c) \equiv x_M(1, c : \xi)$, $\tilde{y}(\xi, c) \equiv y_M(1, c : \xi)$, $\tilde{S}(\xi, c) \equiv S_M(1, c : \xi)$, $\tilde{\pi}(\xi, c) \equiv \pi_M(1, c : \xi)$, $\tilde{K}(\xi, c) \equiv K_M(1, c : \xi)$, and $\tilde{X}(\xi, c) \equiv X_M(1, c : \xi)$. Notice that $p_M(0, c) = \tilde{p}(0, c)$, $x_M(0, c) = \tilde{x}(0, c)$, $y_M(0, c) = \tilde{y}(0, c)$, $S_M(0, c) = \tilde{S}(0, c)$, $\pi_M(0, c) = \tilde{\pi}(0, c)$, $K_M(0, c) = \tilde{K}(0, c)$, and $X_M(0, c) = \tilde{X}(0, c)$. Since $\frac{\partial \tilde{p}}{\partial \xi} = \frac{\gamma}{2} > 0$, it must hold that $p_M(0, c) = \tilde{p}(0, c) < \tilde{p}(\xi, c) = p_M(1, c : \xi)$ for any $\xi > 0$, which is the desired result in (1). Similarly, since $\frac{\partial \tilde{x}}{\partial \xi} = \frac{\gamma}{2(1-\gamma^2)} > 0$ and $\frac{\partial \tilde{y}}{\partial \xi} = -\frac{2-\gamma^2}{2(1-\gamma^2)} < 0$, it must hold that $x_M(0, c) = \tilde{x}(0, c) < \tilde{x}(\xi, c) = x_M(1, c : \xi)$ and $y_M(0, c) = \tilde{y}(0, c) > \tilde{y}(\xi, c) = y_M(1, c : \xi)$ for any $\xi > 0$, which are the desired results in (2) and (3). Moreover, notice that $\frac{\partial \tilde{S}}{\partial \xi} = [u_x - \tilde{p}]\frac{\partial \tilde{x}}{\partial \xi} + [u_y - (\tau + \xi)]\frac{\partial \tilde{y}}{\partial \xi} - \tilde{x}\frac{\partial \tilde{p}}{\partial \xi} - \tilde{y} = -\tilde{x}\frac{\partial \tilde{p}}{\partial \xi} - \tilde{y} < 0$. Thus, it must hold that $S_M(0, c) = \tilde{S}(0, c) > \tilde{S}(\xi, c) = S_M(1, c : \xi)$ for any $\xi > 0$, which is the desired result in the first part of (4). Since $\frac{\partial \tilde{\pi}}{\partial \xi} = \frac{\gamma[1-c-\gamma(1-\tau-\xi)]}{2(1-\gamma^2)} > 0$, it must hold that $\pi_M(0, c) = \tilde{\pi}(0, c) < \tilde{\pi}(\xi, c) = \pi_M(1, c : \xi)$ for any $\xi > 0$, which is the desired result in the second part of (4). Finally, since $\frac{\partial \tilde{K}}{\partial \xi} = -\frac{2+\gamma}{2(1+\gamma)} < 0$ and $\frac{\partial \tilde{X}}{\partial \xi} = \frac{2(1+\gamma)(1-c)}{(1-\gamma)[(1-c)+(1-\tau-\xi)(2+\gamma)]^2} > 0$, it must hold that $K_M(0, c) = \tilde{K}(0, c) > \tilde{K}(\xi, c) = K_M(1, c : \xi)$ and $X_M(0, c) = \tilde{X}(0, c) < \tilde{X}(\xi, c) = X_M(1, c : \xi)$ for any $\xi > 0$, which are the desired results in (5). \square

Proof of Proposition 3 Noticing that $\Delta T_M(c) = T(p_M(1, c), 1, c) - T(p_M(0, c), 0, c) = \Gamma(\varepsilon y_M(0, c)) + \Phi(\xi, c)$, where $\Gamma(\varepsilon y_M(0, c)) = \Gamma(\varepsilon \frac{(1-\tau)(2-\gamma^2)-\gamma(1-c)}{2(1-\gamma^2)})$ and $\Phi(\xi, c) = (S_M(1, c) + \pi_M(1, c)) - (S_M(0, c) + \pi_M(0, c))$. For any ε , $\Gamma(\varepsilon y_M(0, c))$ is strictly increasing and strictly convex in c with $\lim_{c \rightarrow \infty} \Gamma(\varepsilon y_M) = \infty$ since $\frac{\partial \Gamma}{\partial c} = \Gamma'(\varepsilon y_M)\varepsilon \frac{\partial y_M}{\partial c} > 0$ and $\frac{\partial^2 \Gamma}{\partial c^2} = \varepsilon[\Gamma''(\varepsilon y_M)\varepsilon \left(\frac{\partial y_M}{\partial c}\right)^2 + \Gamma'(\varepsilon y_M)\frac{\partial^2 y_M}{\partial c^2}] > 0$. Moreover, $\Phi(\xi, c)$ is strictly decreasing in c with linearity since $\frac{\partial \Phi(\xi, c)}{\partial c} = -\frac{3\gamma\xi}{4(1-\gamma^2)} < 0$. Taking into account that $\Delta T_M(0) < 0$ or $\Phi(\xi, 0) < -\Gamma(\varepsilon y_M(0, 0))$ and that Γ is increasing and strictly convex with $\Gamma(\varepsilon) = \infty$, it must hold that Γ and Φ have the single-crossing property related to c (see Figure 1). Thus, there exists a unique critical value $c_M > 0$ such that $\Phi(\xi, c) > -\Gamma(\varepsilon y_M(0, c))$ or $T(p_M(1, c), 1, c) > T(p_M(0, c), 0, c)$ if $c > c_M$ and $\Phi(\xi, c) < -\Gamma(\varepsilon y_M(0, c))$ or $T(p_M(1, c), 1, c) < T(p_M(0, c), 0, c)$ if $c < c_M$. \square

Proof of Proposition 4 Notice that $\Delta T_M(c) = \Phi(\xi, c) + \Gamma(\varepsilon y_M(0, c)) = 0$ is increasing in c in the neighborhood of $c = c_M(\xi, \varepsilon)$. This implies that at $c = c_M(\xi, \varepsilon)$, $\Gamma'(\varepsilon y_M)\varepsilon \frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c} > 0$. Differentiating $\Delta T_M(c_M(\xi, \varepsilon)) = \Phi(\xi, c_M(\xi, \varepsilon)) + \Gamma(\varepsilon y_M(0, c_M(\xi, \varepsilon))) = 0$ with respect to ε , we obtain $\frac{\partial c_M(\xi, \varepsilon)}{\partial \varepsilon} = -\Gamma'(\varepsilon y_M)y_M[\Gamma'(\varepsilon y_M)\varepsilon \frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c}]^{-1} < 0$. since $\Gamma'(\varepsilon y_M)\varepsilon \frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c} > 0$ and $\Gamma' > 0$. Thus, $c_M(\xi, \varepsilon)$ is decreasing in ε . Furthermore, differentiating $\Delta T_M(c_M(\xi, \varepsilon)) = 0$ with respect to ξ yields that $\frac{\partial c_M(\xi, \varepsilon)}{\partial \xi} = -\frac{\partial \Phi}{\partial \xi} \left[\Gamma'(\varepsilon y_M)\varepsilon \frac{\partial y_M}{\partial c} + \frac{\partial \Phi}{\partial c} \right]^{-1} \geq 0 \Leftrightarrow \frac{\partial \Phi}{\partial \xi} \leq 0$. Thus, the sign of $\frac{\partial c_M}{\partial \xi}$ depends on the one of $\frac{\partial \Phi}{\partial \xi}$. \square

Proof of Proposition 5 Since the system (4) is a linear differential system, the desired result is directly obtained. \square

Proof of Proposition 6 The proof follows Kamien and Schwartz (1991). The characteristic equation associated with (4) is $k^2 - rk - \lambda\delta(r + \lambda\delta) + \frac{(r+2\lambda\delta)\lambda}{2(1-\gamma^2)} = 0$ with roots $k_1, k_2 = \frac{r}{2} \pm \frac{1}{2}[(r+2\lambda\delta)^2 - \frac{2\lambda(r+2\lambda\delta)}{1-\gamma^2}]^{1/2} \equiv \frac{r}{2} \pm \frac{1}{2}D^{1/2}$. Then, the solution has the form of $c(t) = Ae^{k_1t} + Be^{k_2t}$ for k_1, k_2 real and distinct, or $c(t) = (A + Bt)e^{rt/2}$ if $k_1 = k_2 = r/2$, or $c(t) = e^{rt/2}(A \cos pt + B \sin pt)$, where $p = (-D)^{1/2}$, if p is real. If the roots are real, the larger root must be positive. The smaller root may be either positive or negative. It will be negative if $r < D^{1/2}$; that is if $\delta(r + \lambda\delta) > \frac{r+2\lambda\delta}{2(1-\gamma^2)}$ or $\gamma^2 < 1 - \frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)} \equiv \hat{\gamma}^2$, the roots are real and of opposite signs. Let $k_1 > 0 > k_2$ if $\gamma^2 < \hat{\gamma}^2$ holds. Then, $c(t)$ will converge to zero provided we take $A = 0$. Thus, if $\gamma^2 < \hat{\gamma}^2$ holds, then the roots must be real and the steady state must satisfy the conditions to be a saddlepoint. On the other hand, the roots will be real and nonnegative if $D \geq 0$ and $\gamma^2 < \hat{\gamma}^2$ fails. As long as the roots are both nonnegative, $c(t)$ cannot converge to the steady state. It will move away from it, unless the initial position happens to be the steady state. Finally, the roots will be complex if $D < 0$. Note that since the real part of the complex roots is positive ($= r/2$), the path moves away from the steady state so that the steady state is unstable. In sum, a solution to the system (4) can converge only if $\gamma^2 < \hat{\gamma}^2$ holds. In other cases, all paths diverge. \square

Proof of Proposition 7 From the discussions in the text, we can deduce all desired results under the assumption that $\sigma(\bar{c}(\phi)) = \eta - \delta\bar{c}(\phi) > 0$ and $c_0 > \bar{c}(\phi)$. \square

Proof of Proposition 8 By (12), we obtain $x_M(\phi, \bar{c}(\phi)) = \sigma(\bar{c}(\phi))\left[1 - \frac{\lambda}{2(r+\lambda\delta)(1-\gamma^2)}\right]$. We first show that $\frac{\partial \bar{c}}{\partial \lambda} < 0$. Differentiating the above equation with respect to λ yields $\frac{\partial \bar{c}(\phi)}{\partial \lambda} = -\frac{r\sigma(\bar{c})}{2(r+\lambda\delta)^2(1-\gamma^2)}\left[\frac{\partial x_M}{\partial c} - \frac{\partial \sigma}{\partial c}\left(1 - \frac{\lambda}{2(r+\lambda\delta)(1-\gamma^2)}\right)\right]^{-1}$. Notice that since $\gamma < \hat{\gamma}$, we obtain $\frac{\partial x_M}{\partial c} - \frac{\partial \sigma}{\partial c}\left(1 - \frac{\lambda}{2(r+\lambda\delta)(1-\gamma^2)}\right) = -\frac{\delta}{1-\gamma^2}[\gamma^2 - (1 -$

$\frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)}] > 0$. Thus, we obtain $\frac{\partial \bar{c}}{\partial \lambda} < 0$. We then show that $\frac{\partial \bar{p}}{\partial \lambda} < 0$. Differentiating equation (5) with respect to λ yields $\frac{\partial \bar{p}(\phi)}{\partial \lambda} = \frac{\partial p_M}{\partial c} \frac{\partial \bar{c}}{\partial \lambda} - \frac{1}{2} \left[\frac{r\sigma(\bar{c}(\phi))}{(r+\lambda\delta)^2} - \frac{\lambda\delta}{r+\lambda\delta} \frac{\partial \bar{c}}{\partial \lambda} \right]$. Since $\frac{\partial p_M}{\partial c} > 0$ by Proposition 1 and $\frac{\partial \bar{c}}{\partial \lambda} < 0$, it must hold that $\frac{\partial \bar{p}(\phi)}{\partial \lambda} < 0$. We then show that $\frac{\partial \bar{x}}{\partial \lambda} > 0$, $\frac{\partial \bar{y}}{\partial \lambda} < 0$ and $\frac{\partial \bar{E}}{\partial \lambda} \leq 0$ with equality if $\phi = 1$. By (1), the outputs of the eco-product and the conventional product in the steady state are given by $\bar{x}(\phi) = x^*(\bar{p}(\phi), \phi)$ and $\bar{y}(\phi) = y^*(\bar{p}(\phi), \phi)$, which implies that $\frac{\partial \bar{x}(\phi)}{\partial \lambda} = \frac{\partial x^*}{\partial p} \frac{\partial \bar{p}(\phi)}{\partial \lambda} = -\frac{1}{1-\gamma^2} \frac{\partial \bar{p}(\phi)}{\partial \lambda} > 0$ and $\frac{\partial \bar{y}(\phi)}{\partial \lambda} = \frac{\partial y^*}{\partial p} \frac{\partial \bar{p}(\phi)}{\partial \lambda} = \frac{\gamma}{1-\gamma^2} \frac{\partial \bar{p}(\phi)}{\partial \lambda} < 0$. Since $\frac{\partial \bar{y}}{\partial \lambda} < 0$, we obtain $\frac{\partial \bar{E}(\phi)}{\partial \lambda} < 0$. We next show that $\frac{\partial \bar{S}(\phi)}{\partial \lambda} > 0$. Using $\bar{S}(\phi) = S(\bar{p}(\phi), \phi)$, $\frac{\partial \bar{p}}{\partial \lambda} < 0$ and $\frac{\partial S}{\partial p} = -\bar{x}(\phi)$, we obtain $\frac{\partial \bar{S}(\phi)}{\partial \lambda} = \frac{\partial S}{\partial p} \frac{\partial \bar{p}}{\partial \lambda} = -\bar{x}(\phi) \frac{\partial \bar{p}}{\partial \lambda} > 0$. We then show that $\frac{\partial \bar{K}(\phi)}{\partial \lambda} > 0$. By (1), it must hold that $\gamma \bar{x}(\phi) + \bar{y}(\phi) = 1 - \tau(\phi)$. Then, we obtain $\bar{K}(\phi) = \bar{x}(\phi) + \bar{y}(\phi) = (1-\gamma)\bar{x}(\phi) + 1 - \tau(\phi)$. Since $\frac{\partial \bar{x}(\phi)}{\partial \lambda} > 0$, we obtain $\frac{\partial \bar{K}(\phi)}{\partial \lambda} = (1-\gamma) \frac{\partial \bar{x}(\phi)}{\partial \lambda} > 0$. We next show that $\frac{\partial \bar{X}(\phi)}{\partial \lambda} > 0$. Differentiating $\bar{X}(\phi) = \frac{\bar{x}(\phi)}{K(\phi)}$ with respect to λ yields $\frac{\partial \bar{X}(\phi)}{\partial \lambda} = \frac{1}{K^2} [\bar{K} \frac{\partial \bar{x}(\phi)}{\partial \lambda} - \bar{x} \frac{\partial \bar{K}(\phi)}{\partial \lambda}] = \frac{1-\tau(\phi)}{K^2} \frac{\partial \bar{x}(\phi)}{\partial \lambda} > 0$, since $\bar{K}(\phi) = (1-\gamma)\bar{x}(\phi) + 1 - \tau(\phi)$ and $\frac{\partial \bar{K}(\phi)}{\partial \lambda} = (1-\gamma) \frac{\partial \bar{x}(\phi)}{\partial \lambda}$. \square

Proof of Proposition 9 Equation (12) can be rewritten by $\eta - \delta \bar{c} = \frac{(r+\lambda\delta)[1-\bar{c}-\gamma+\gamma(\tau+\xi)]}{2(r+\lambda\delta)(1-\gamma^2)-\lambda}$. Note that \bar{c} is a function of ξ . Also note that $\xi = 0$ corresponds to the case in which $\phi = 0$, and $\xi > 0$ corresponds to the case in which $\phi = 1$. It is enough to show that \bar{c} is decreasing in ξ to prove that $\bar{c}(0) > \bar{c}(1)$. Differentiating this with respect to ξ yields

$$\left[\frac{r+\lambda\delta}{2(r+\lambda\delta)(1-\gamma^2)-\lambda} - \delta \right] \frac{\partial \bar{c}}{\partial \xi} = \frac{(r+\lambda\delta)\gamma}{2(r+\lambda\delta)(1-\gamma^2)-\lambda}.$$

Since the right-hand side is positive and

$$\left[\frac{r+\lambda\delta}{2(r+\lambda\delta)(1-\gamma^2)-\lambda} - \delta \right] < 0 \Leftrightarrow \gamma < \hat{\gamma} \equiv \left[1 - \frac{r+2\lambda\delta}{2\delta(r+\lambda\delta)} \right]^{1/2},$$

it must hold that $\frac{\partial \bar{c}}{\partial \xi} < 0$ if $\gamma < \hat{\gamma}$. Thus, \bar{c} is decreasing in ξ if $\gamma < \hat{\gamma}$. This implies that the environmental regulation reduces the marginal cost of the eco-product, \bar{c} , in the steady state, i.e., $\bar{c}(0) > \bar{c}(1)$. \square

Proof of Proposition 10 From the discussions in the text, we can deduce all desired results. \square

Proof of Proposition 11 It is easily shown that if $c > c_0$, the saddle path $p = \hat{p}(\phi, c)$ is below line $\dot{p} = 0$ in Figure 4. Thus, \hat{p} is monotone decreasing over time. Since $x^*(p, \phi)$ is decreasing in p and $y^*(p, \phi)$ is increasing in p with $\hat{x}(\phi, c) = x^*(\hat{p}(\phi, c), \phi)$ and $\hat{y}(\phi, c) = y^*(\hat{p}(\phi, c), \phi)$, \hat{x} is decreasing in \hat{p} and \hat{y} is increasing in \hat{p} . Since \hat{p} is monotone decreasing over time, \hat{x} is monotone increasing and

\hat{y} is monotone decreasing over time. Moreover, $S(p, \phi)$, $K(p, \phi)$ and $X(p, \phi)$ are decreasing in p since $\frac{\partial S}{\partial p} = -x^* < 0$, $\frac{\partial K}{\partial p} = \frac{-1}{1+\gamma} < 0$, and $\frac{\partial X}{\partial p} < 0$. Noticing that $\hat{S}(\phi, c) = S(\hat{p}(\phi, c), \phi)$, $\hat{K}(\phi, c) = K(\hat{p}(\phi, c), \phi)$, $\hat{X}(\phi, c) = X(\hat{p}(\phi, c), \phi)$ and that \hat{p} is monotone decreasing over time, \hat{S} , \hat{K} and \hat{X} are monotone increasing over time. \square

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Figure 1: Critical marginal cost, c_M , that determines whether environmental regulation is better or not

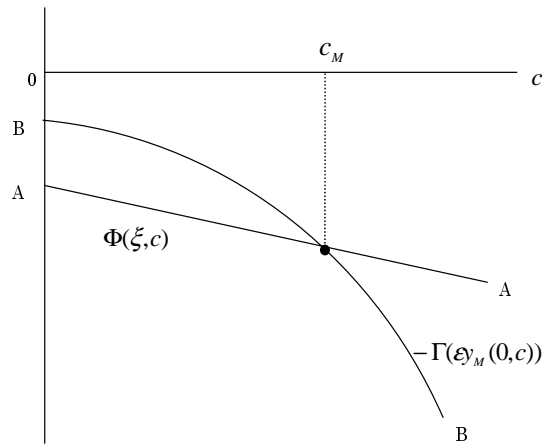


Figure 2: Phase diagram in marginal cost-price space

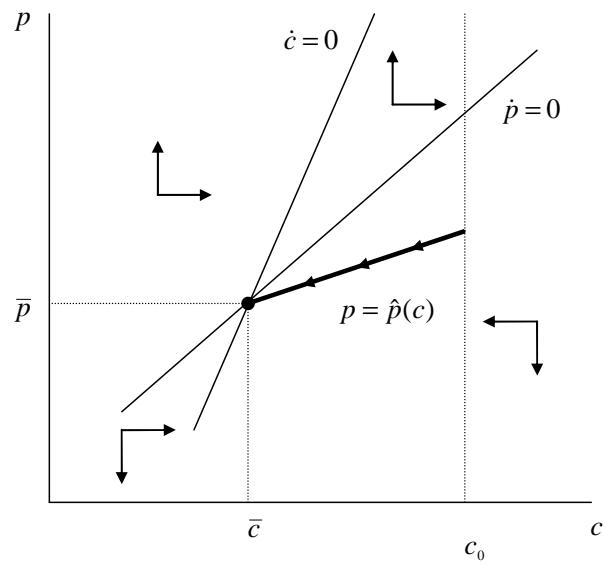


Figure 3: Market size as a function of marginal cost c in a static model

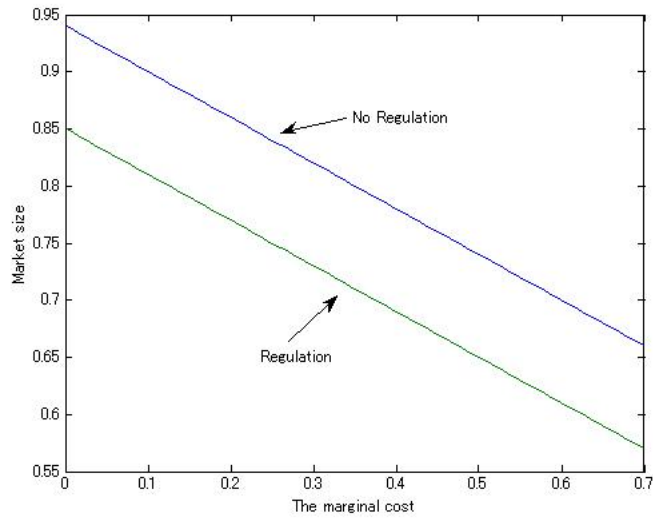


Figure 4: Market share of eco-products as a function of marginal cost c in a static model

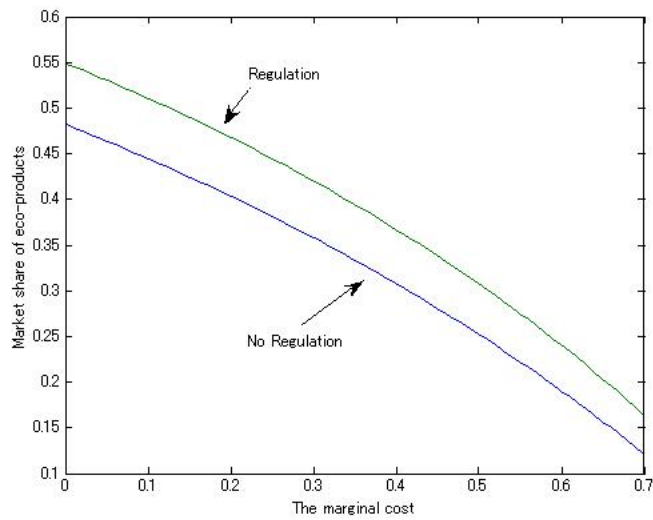


Figure 5: Critical marginal cost and change of the social welfare

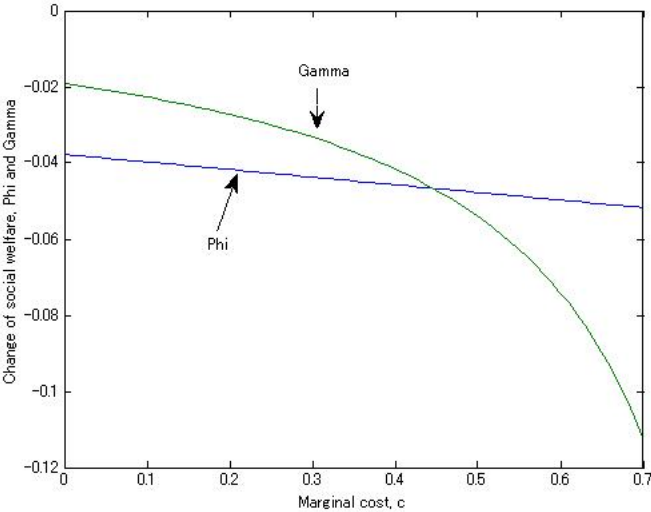


Figure 6: Equilibrium Path of Marginal Cost over Time

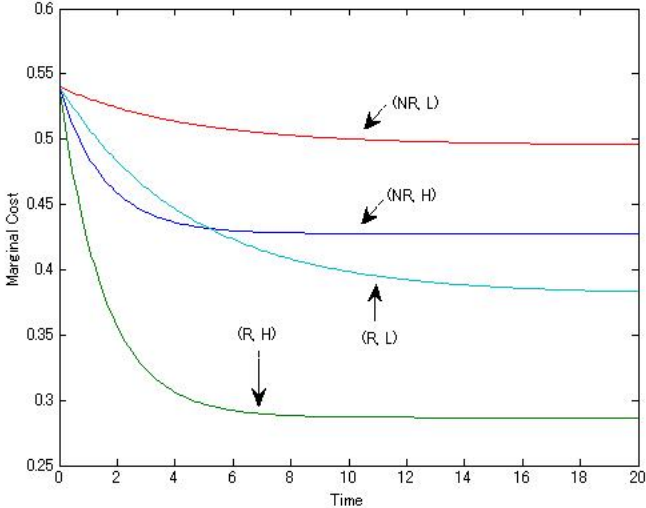


Figure 7: Equilibrium Path of Eco-product Price over Time

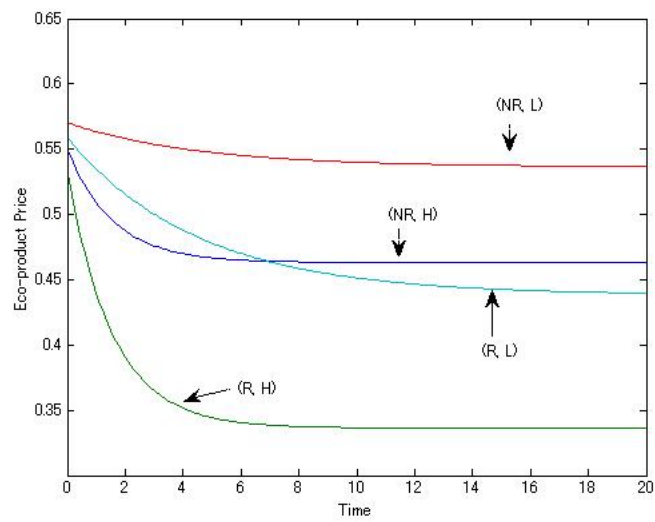


Figure 8: Equilibrium Path of Market Share over Time

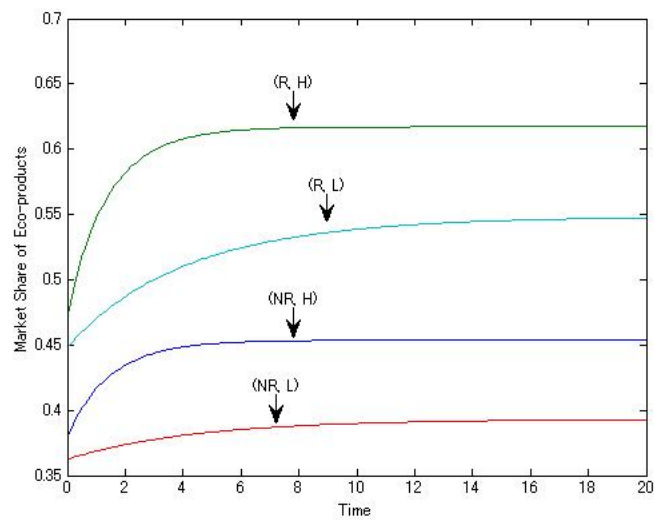


Figure 9: Equilibrium Path of Market Size over Time

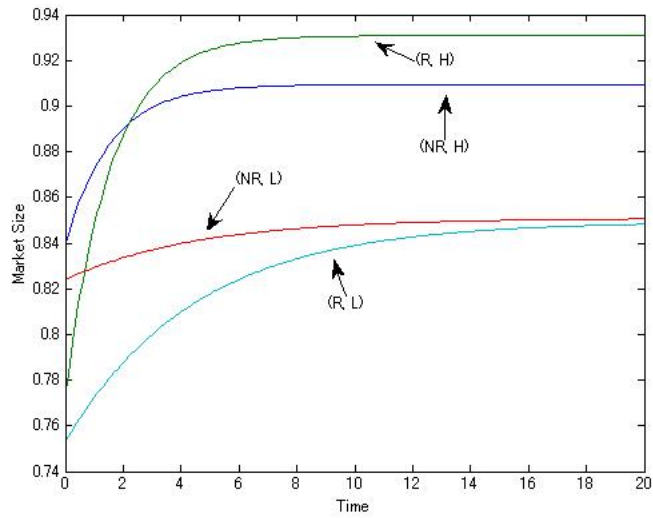


Figure 10: Equilibrium Path of Consumer Surplus over Time

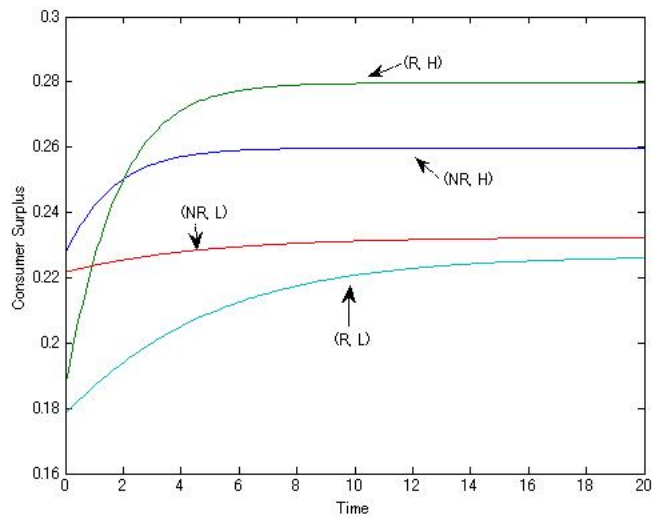


Figure 11: Equilibrium Path of Social Welfare over Time

