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Optimal Programs on Invasive Species Management under Growth Uncertainty and Measurement Error*

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Abstract

The management programs for invasive species have been proposed in many regions. The resulting outcome on success or failure seems to be significantly affected by the degrees of multiple uncertainties, such as growth uncertainty and measurement error, associated with management practices. This study first examines the optimal policy on invasive species management under growth uncertainty, and then incorporates measurement error into the model. We find various novel results and discuss related policy implications that emanate from the interplays between two sources of the uncertainty. The corresponding values of the optimal programs are also examined.

Key Words: bioeconomic model, invasive species management, growth uncertainty, measurement error, stochastic dynamic programming, value of optimal program

JEL Classification: Q57; Q58

* We are responsible for any remaining errors.

1 Introduction

The problems of controlling invasive species have been increasingly common, as every part of the world is intertwined each other in a globalized world and there is no way to perfectly prevent potential entries in such circumstances (see, e.g., Perrings, Williamson, and Dalmazzone (2000) and Pimentel, Zuniga, and Morrison (2005) for general discussions). What we can do best for this problem includes (i) to take countermeasure against pre-invasion and (ii) to manage an established invasive species as a consequence of post-invasion, many of which cause serious social damage on indigenous ecosystem and agriculture. The topic addressed in this paper is concerned with the latter: how to manage the established invasive species, especially focusing on analysis of the optimal strategies of removals on invasive species in the presence of uncertainty.

Rational decisions on invasive species management are uneasy. The government authorities must determine whether to aim at eradication as well as how to manage the invasive species in eternity if the option of eradication is abandoned or infeasible. If controlling costs are not taken into account and eradication is feasible, there is no question that eradication is the first best for a society. In reality, however, it is empirically shown that any policy aiming at eradication can be prohibitively expensive when catchability rapidly declines as the existing invasive species stock becomes less and less (see, e.g., Myers, Savoie, and van Randen (1998), Bomford and O'Brien (1995), and Simberloff (2002)).[1] This is the first issue related to "stock-dependent catchability," which plagues the decisions of management practices. Our previous work of Kotani, Kakinaka, and Matsuda (2006) focuses on this issue.

To make matters worse, there is another important factor that makes the decision more complex. Management practices are accompanied with various stochasticity, such as "growth uncertainty" and "measurement error." In the field of renewable resource economics, it is well-known that growth uncertainty does not generally affect the qualitative feature of the

[1] In these review papers, there is an anecdote that killing the first 99% of a target population can cost less than eliminating the last 1%.

optimal policy, especially in a fishery model, if the current stock is accurately known (Reed (1979)). However, several authors point out that in more realistic settings, the decision of management practices must be made in the informational absence of current states. Indeed, such factors could fundamentally affect the optimal programs of renewable resource management (see, e.g., Clark and Kirkwood (1986), Roughgarden and Smith (1996), and Loehle (2006)). Considering the context of invasive species management, such a rule of thumbs equivalently applies. Thus, it is valuable to analyze the optimal strategy as well as the values of the management program under various uncertainties. This is the second issue, related to "stochasticity," which we examine in the present paper.

Several previous researches examine the optimal control of invasive species in economic dynamic models in which the objective of a society is to minimize the long-run social cost. Olson and Roy (2002) theoretically develop a discrete-time dynamic model under a stochastic invasion growth and study the optimal policy of eradication. Eisewerth and Johnson (2002) develops a continuous-time optimal control model, and their focus is mainly on the long-run equilibrium outcomes without analysis on the decision of eradication. Moreover, Eisewerth and van Kooten (2002) make the assumption that the current stock is inaccurately known and apply the fuzzy membership function in the invasive species controls. However, all of the above works employ the assumption that the cost of removal operations is independent of the stock size and do not consider the stock-dependent issue so that their analysis might miss the point of when to eradicate.

Olson and Roy (2004) is a pioneering work that considers the issues of stock-dependent removal costs and derives the conditions under which eradication or non-eradication can be optimal in the deterministic setting. While their innovative model is built under very general settings, they do not explicitly examine the implications of stock-dependent catchability so that it might be difficult to connect their analytical results into real management practices. To do that, Kotani, Kakinaka, and Matsuda (2006) focus on analyzing the policy implications of stock-dependent catchability through deriving the conditions for various optimal policies in the deterministic setting. More specifically, our previous work shows that if the sensitivity of catchability is sufficiently high, eradication policy is never optimal and in effect the constant escapement policy with some interior level is optimal. In contrast, if the sensitivity of catchability is sufficiently small, eradication policy could be optimal and there may exist a threshold of the initial stock (called a Skiba point) which differentiates optimal actions between immediate eradication and giving-up without controls. If the sensitivity of catchability takes some intermediate values, more complex policies would be optimal.

Building upon Kotani, Kakinaka, and Matsuda (2006), the aim of this paper is to analyze a stochastic model of the invasive species management in which the decision of controls must be made when the pertinent stock is inaccurately known in addition to growth uncertainty. To the best of our knowledge, all previous analyses lack at least an issue of either "stock-dependent catchability" or "stochasticity." In addition, most of them do not carefully examine the impacts of uncertainty in both cases where (i) eradication and (ii) non-eradication are set as goals. Given this state of affairs, our analysis seeks to answer the following questions: (i) how should we adapt the optimal decision of eradication and non-eradication to uncertainties?; and (ii) how does the value of optimal management programs vary with uncertainties?

The specification of stock-dependent catchability is adopted following Reed (1979), Clark (1990), and Kotani, Kakinaka, and Matsuda (2006). The uncertainties we consider are (i) growth uncertainty and (ii) stock measurement error, following the specification of Sethi, Costello, Fisher, Hanemann, and Karp (2005), so that a Markovian process is assumed throughout the analysis. Although we admit that there are other uncertainties such as implementation uncertainty, we believe that the model setup in this paper is a valuable starting point.

This paper is organized as follows. In the next section, we elaborate on the basic elements of the model. The section is followed by the analysis of a stochastic model only with growth uncertainty and presents how growth uncertainty affects the optimal decision rule. In the next section, measurement error is introduced in the model. We show how the interaction between growth uncertainty and measurement error affects the optimal decision and discuss various intuitions of the results. In our analyses, we concentrate on two cases: (i) the non-eradication case where the optimal policy is in a class of interior constant escapement rules in the deterministic setting, and (ii) the eradication case where the optimal policy would be immediate eradication in the deterministic setting. We also attempt to provide an explanation of how the value of optimal management programs varies with the uncertainties. In the final section, we offer some discussions and conclusions.

2 The Model

We consider an infinite-period stochastic model of invasive species management, following a deterministic version of the dynamic model of Kotani, Kakinaka, and Matsuda (2006). Our model below is appropriate for setting up and solving a stochastic dynamic programming problem with Markovian transitions. As studied in Sethi, Costello, Fisher, Hanemann, and Karp (2005), we assume that there are two random variables capturing the uncertainties in each period t: growth uncertainty and stock measurement error, which are represented by Z_t^g and Z_t^m, respectively.[2] The random variable Z_t^g reflects uncontrollable environmental variability, while the random variable, Z_t^m, reflects potentially controllable error. These variables are independent of each other and of period t. We assume that Z_t^g and Z_t^m are respectively distributed over some finite intervals $[a^g, b^g]$ and $[a^m, b^m]$ with the mean of unity, where $0 < a^k < 1 < b^k < \infty$, according to a common distribution function Φ^k, for $k = g, m$. The society officials know the statistical distribution for each of these random variables.

The stock (population) of existing invasive species in period t is governed by the following

[2] Sethi, Costello, Fisher, Hanemann, and Karp (2005) examine three sources of uncertainty: (1) growth uncertainty; (2) stock measurement error; and (3) inaccurate implementation of action. In this paper, we pay attention to growth uncertainty and stock measurement error as sources of multiple uncertainty.

state equation:

$$x_t = Z_t^g F(s_{t-1}), \tag{1}$$

where $F(s_{t-1})$ is the expected or average reproduction function that gives the stock x_t as a function of the previous period escapement, s_{t-1}. We assume that F is differentiable and strictly concave with $F(0) = 0$ and $F'(0) \in (1, \infty)$ and that there exists an undisturbed level of the stock of invasive species, $\tau > 0$, with $\tau = F(\tau)$ such that $F(s) > s$ and $F'(s) \geq 0$ if $s \in (0, \tau)$. This specification implies that the deterministic stock-recruitment relationship as assumed in Clark (1990) holds in terms of conditional expectations, i.e., $E(x_t|s_{t-1} = s) = F(s)$. We further assume that $b^g F(0) > 1$ and $\lim_{x\to\infty} a^g F'(x) < 1$.

The stock measurement, m_t, in period t is given by the following:

$$m_t = Z_t^m x_t. \tag{2}$$

The society officials use only the current measurement when they form expectations. This assumption requires that the current measurement is the only state variable for the dynamic problem. We assume that the society officials ignore past measurements so that they condition their decision on a single state variable.[3] Furthermore, although actual data are not sufficient to distinguish between a multiplicative and an additive stochastic term, we believe that the chosen expression with multiplicative stochastic term is the most convenient since it does not allow for negative values of the stock. Notice that if all the two random variables are constant at unity, our dynamic problem becomes deterministic.

We assume that the social cost in each period consists of the social damage from the escapement of invasive species and the cost associated with the removal operation. The former cost in period t is given by $D(s_t)$, where D is increasing in s. The latter cost in

[3] Sethi, Costello, Fisher, Hanemann, and Karp (2005) justify this assumption based on modeling choice and practical considerations, although they admit that a decision rule could be dependent on past measurements history, which may include some information about the current stock.

period t is given by $C(w_t, x_t)$, where w_t is actual removal, and C is increasing in w_t with $C(w_t, x_t) \geq 0$ for any w_t and x_t. The removal cost in each period depends not only on the number of actual removal but also on the stock of existing invasive species in that period. Specifically, given the stock at the beginning of period t, x_t, and the total number of removals during period t, w_t, the total cost of removal operations during period t is described by:

$$C(w_t, x_t) = \int_{x_t - w_t}^{x_t} c(q)dq, \tag{3}$$

where $c(x)$ represents the unit cost that is a function of the current stock x. This specification implies that the feasibility of eradication depends on the functional form of the unit cost function $c(x)$. That is, for given stock x, the eradication is feasible if $C(x, x)$ is finite, and it is infeasible if $C(x, x)$ is infinity. From the above, the payoff for the society in period t is given by:

$$u(x_t, w_t) = -D(x_t - w_t) - C(w_t, x_t). \tag{4}$$

For our explanatory purpose to connect our arguments into catchability, we consider the unit cost function:

$$c(x) = kx^{-\theta}, \tag{5}$$

where $\theta > 0$ is the sensitivity of catchability with some constant $k > 0$. The implication of the parameter θ is discussed by Reed (1979), Clark (1990), and Moxnes (2003) in the context of harvesting management and by Kotani, Kakinaka, and Matsuda (2006) for invasive species management. The value of $xc(x)$ can be thought of as the cost associated with the realization of a given per capita rate of removal mortality when the stock is at level x. Reed (1979) states that in most harvesting problems, $xc(x)$ can be expected to stay constant (Schaefer function) or increase with an increase in stock size, although it is conceivable that it could decrease. The former case corresponds to the one of $\theta = 1$ with the linearity of $xc(x)$, and the latter case corresponds to the one of $\theta \in (0, 1)$ with the concavity of $xc(x)$. However, the nature of operational costs for controlling invasive species is often different from that for harvesting problems in that the sensitivity of catchability θ may be larger than unity (see, e.g., Clark (1990)).

To analyze the optimal policy under multiple uncertainty, we consider a society which extends over the following stages in each period. Given the escapement s_{t-1} in the previous period, the true stock x_t is randomly determined according to the reproduction process (1). The society officials do not know the true stock but obtain the measured stock m_t, following the stochastic process (2). Then, given m_t, the society officials decide the target removal y_t. If y_t is equal to or larger than x_t, they may be able to remove all existing stock. If y_t is less than x_t, they actually cut the target removal, but they never know whether to achieve the target level of the escapement. The actual removal and the true escapement are represented by $w_t = \min\{y_t, x_t\}$ and $s_t = x_t - w_t$, respectively. Then, the next period $t + 1$ proceeds.

The society official maximizes the expected present value of the payoffs (minimizes the present value of the payoff losses) by choosing a sequence of target removals $\{y_t\}_{t=0}^{\infty}$:

$$\begin{aligned}
&\max_{0 \le y_t} \mathbb{E}\left\{\sum_{t=0}^{\infty} \rho^t u(x_t, w_t)\right\} \\
\text{s.t.} \quad & x_t = Z_t^g F(s_{t-1}) \\
& m_t = Z_t^m x_t \\
& w_t = \min\{y_t, x_t\} \\
& s_t = x_t - w_t,
\end{aligned}$$

where $\rho \in (0, 1)$ is the discount factor and $\mathbb{E}$ is the expectation operator. The Bellman equation for this problem is:

$$v_t(m_t) = \max_{0 \le y_t} \mathbb{E}\{u(x_t, w_t) + \rho v_{t+1}(Z_{t+1}^m Z_{t+1}^g F(x_t - w_t))\}, \tag{6}$$

where $v_t(m_t)$ is the value function given the current stock measurement of existing invasive species, m_t. For a given stock measurement, a sequence of the optimal target removals is the one that maximizes the expected present value of payoff over time.[4]

Before discussing our stochastic problem, we should mention some results of the deterministic model in Kotani, Kakinaka, and Matsuda (2006). The optimal policy sequence can drastically change, depending on the sensitivity of catchability in response to a change in the stock size, as well as on the initial stock. If θ is relatively high, the constant escapement policy with some interior target level is optimal. In contrast, if θ is relatively low, immediate eradication could be optimal for any initial state.[5] In the intermediate range, more complex policies could be optimal. For comparison between the deterministic and stochastic models, this study pays attention to the following two cases: (1) the non-eradication case with a relatively large θ in which the constant escapement policy would be optimal in the deterministic setting; and (2) the eradication case with a relatively small θ in which immediate eradication policy could be optimal at least for any domain of stock level in the deterministic setting.

3 Growth Uncertainty

This section seeks to characterize how growth uncertainty in the reproduction function affects the optimal policy without measurement error, i.e., Z_t^m is constant at unity. The growth uncertainty can be interpreted as environmental variability that influences the reproduction of invasive species. To characterize the officials' optimal policy in the stochastic dynamic problem, we transform the payoff in period t during a decrease in the stock size from x_t to $s_t = x_t - w_t$ into the form of $u(x_t, w_t) = -[Q(x_t - w_t) - Q(x_t)] - D(x_t - w_t)$, where $Q(x) \equiv$

[4]See Sethi, Costello, Fisher, Hanemann, and Karp (2005) for concrete derivations of the conditional density that is required in the Bellman equation.

[5]Kotani, Kakinaka, and Matsuda (2006) present that in the deterministic model, there could exist a threshold of the initial stock which differentiates the optimal policy between immediate eradication and giving-up without any control if θ is sufficiently low. It is also shown that immediate eradication could be optimal for any initial state if θ is relatively low with some conditions. In the present paper, we focus on the latter case to clarify the impact of the uncertainty. As will be explained in a later part, the introduction of the uncertainty will cause the former case to emerge.

$\int_x^m c(w)dw \in [0, \infty]$ represents the operational cost of removing invasive species from the stock level m to some stock level x. Notice that $Q'(x) = -c(x) < 0$ and $Q''(x) = -c'(x) > 0$. Using $x_{t+1} = Z_t^g F(s_t)$, we rewrite the objective function as:

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \rho^t u(x_t, x_t - s_t)\right\} = Q(x_0) + \mathbb{E}\left\{\sum_{t=0}^{\infty} \rho^t \Gamma(s_t, Z_t^g)\right\}, \tag{7}$$

where $\Gamma(s_t, Z_t^g) \equiv -Q(s_t) - D(s_t) + \rho Q(Z_t^g F(s_t))$ represents the discounted growth in the immediate value. We denote the expected discounted growth in the immediate value by:

$$g(s) = \int_{a^g}^{b^g} \Gamma(s, z) d\Phi^g(z) = -Q(s) - D(s) + \rho \int_{a^g}^{b^g} Q(zF(s)) d\Phi^g(z). \tag{8}$$

In general, the shape of the graph $g(s)$ is highly dependent on its functional forms, $c(s)$, $F(s)$ and $D(s)$. The maximization of the expected total discounted payoff (or expected present value (EPV)) is to find a sequence $\{y_t\}$ to maximize (7) subject to the state equation $x_{t+1} = Z_t^g f(x_t - \min\{y_t, x_t\})$ and the initial stock x_0. Notice that the discounted growth in the immediate value in the deterministic case corresponding to the function (8) is given by:

$$h(s) = -Q(s) - D(s) + \rho Q(F(s)). \tag{9}$$

This is a special case of the stochastic setting in that $a^g = b^g = 1$ in equation (8). The deterministic model of Kotani, Kakinaka, and Matsuda (2006) shows that h is strictly convex if θ is sufficiently low, and h is strictly concave if θ is sufficiently high. The shape of h is crucial to determine the optimal policy in the deterministic case.

3.1 Non-Eradication Case

The examination in this subsection is on the non-eradication case where the sensitivity of catchability is relatively large so that the optimal policy is in a class of interior constant escapement rules in the deterministic setting. For our explanatory purpose to make comparison between the deterministic and stochastic settings, we focus on a case where both g and h are strictly concave and unimodal. This requires the condition that

$$g'(s) = \left[c(s) - \rho \int_{a^g}^{b^g} zc(zF(s))F'(s)d\Phi^g(z)\right] - D'(s)$$

is strictly decreasing in s over $[0, m]$, where $m = \max\{b^g F(s) | b^g F(s) \geq s, s \geq 0\}$.[6] Let σ denote the escapement level attaining the maximum of $g(s)$, which is called the short-sighted optimal escapement level. The unimodality condition requires σ is interior so that $g'(\sigma) = 0$ holds with $\sigma \in (0, m)$.

In general, the unimodality of g is not enough to guarantee that σ is the optimal level of escapement in the stochastic case. Similar to the discussions in the dynamic harvesting models of Reed (1979), in order for the constant escapement policy with the target level σ to be optimal, we need to make the additional assumption that σ is self-sustaining, i.e., $zF(\sigma) \geq \sigma$ for all z such that $\Phi^g(z) > 0$, or the stock in the next period is required to be always larger than the escapement in the current period.[7] The next subsection analytically studies the optimal policy when the short-sighted optimal escapement level σ is self-sustaining. Then, the following subsection examines the case where σ is not self-sustaining. Since it is difficult to derive analytical solutions in this case, we adopt numerical analysis for the characterization

[6] Notice that $c(s)$ is the marginal increase in current cost associated with a unit removal at the escapement level s, while $\rho \int_{a^g}^{b^g} zc(zF(s))F'(s)d\Phi^g(z)$ is the expected discounted present value of the marginal increase in sustained future removal cost resulting from a unit increase in the escapement. Thus, the value of $B(s) \equiv c(s) - \rho \int_{a^g}^{b^g} zc(zF(s))F'(s)d\Phi^g(z)$ could be regarded as the expected marginal benefit associated with the unit escapement in current period. If $\rho \int_{a^g}^{b^g} zc(zF(s))F'(s)d\Phi^g(z)$ is relatively large compared to $c(s)$, then it is more costly to remove the stock in the future so that the policymakers should involve removal action in the current period.

[7] Recall that Z_t^g is distributed over some finite interval $[a^g, b^g]$ with the mean of unity, where $0 < a^g < 1 < b^g < \infty$, according to a common distribution function Φ^g. It then follows that there exist finite stock levels, $m = \max\{b^g F(s) | b^g F(s) \geq s, s \geq 0\}$ and $r = \max\{a^g F(s) | a^g F(s) \geq s, s \geq 0\}$, such that with probability 1, the stock will eventually stay within the interval $[r, m]$ and on which it will attain a stationary probability distribution in the limit. Any escapement level s for which $\text{Prob}\{x_t \geq s | s_{t-1} = s\} = 1$ is called self-sustaining. This implies that any population level in the interval $[0, r]$ is self-sustaining. In the dynamic harvesting models, Reed (1979) shows that the constant escapement policy is optimal under some conditions, and the level σ at which $g(s)$ is maximized is lower-bound for the optimal level of escapement. He also states that the optimal escapement level is σ if σ is self-sustaining, and the optimal level is greater than σ if σ is not self-sustaining.

of the optimal policy.

3.1.1 Self-Sustaining Short-Sighted Escapement

This subsection attempts to compare the optimal escapement level in the stochastic model with that in the deterministic model, assuming that g and h are strictly concave and unimodal with their interior maximum of σ and $\bar{s}$, respectively, and that the self-sustainability of σ is satisfied. In this situation, $\bar{s}$ and σ are the optimal escapement level in the deterministic and the stochastic cases, respectively. Differentiating $g(s) - h(s)$ yields:

$$g'(s) - h'(s) = \rho F'(s) \left[c(F(s)) - \int_{a^g}^{b^g} zc(zF(s)) d\Phi^g(z) \right]. \tag{10}$$

Our specification implies that $(R1)$ $\sigma < \bar{s}$ if $xc(x)$ is strictly convex; $(R2)$ $\sigma = \bar{s}$ if $xc(x)$ is linear; and $(R3)$ $\sigma > \bar{s}$ if $xc(x)$ is strictly concave.[8] In terms of the sensitivity of catchability θ, (1) $\sigma < \bar{s}$ if $\theta > 1$; (2) $\sigma = \bar{s}$ if $\theta = 1$; and (3) $\sigma > \bar{s}$ if $\theta \in (0, 1)$. These results are consistent with those in the dynamic harvesting models of Reed (1979).

Reed (1979) states that in most harvesting problems, $xc(x)$ is likely to stay constant ($\theta = 1$) or to increase with an increase in stock size ($\theta \in (0, 1)$). Thus, he concludes that if $\theta \in (0, 1)$, then growth uncertainty increases the optimal escapement level under the condition that the concavity and the unimodality of g and h are met. However, on invasive species management, the sensitivity of catchability θ could be larger than unity in some situations. In addition, the deterministic model of Kotani, Kakinaka, and Matsuda (2006) shows that h tends to be convex if θ is relatively small, and h tends to be concave if θ is relatively large. This implies that for a relatively small θ, the optimal policy is not in a class of constant escapement policy so that the above argument cannot be applied. In contrast, for a relatively large θ, g and h are likely to be concave so that the above argument can be applied if the property of self-sustaining is satisfied. Thus, the result $(R3)$ seems implausible,

[8]Let the random variable Y be such that $Y = Z^g F(s)$. Then, it follows that $\int_{a^g}^{b^g} wc(wF(s)) d\Phi^g(w) = E[Yc(Y)]/F(s)$. Applying Jensen's inequality with $F'(s) > 0$, it must hold that the value of $g'(s) - h'(s)$ is negative, zero, or positive for all s if $sc(s)$ is strictly convex, linear, or strictly concave in s.

while the result ($R1$) seems plausible in our invasive species problems. As a result, if θ is relatively large with the property of self-sustainability, then the constant escapement policy is optimal, and growth uncertainty causes the optimal level of escapement to decrease.

Given the previous results, we now detail how the degree of growth uncertainty affects the optimal target level of the escapement on the condition that g and h are strictly concave holding their maximum interior and self-sustaining properties. For simplicity, we assume that Z_t^g is uniformly distributed over the interval $[1 - z_g, 1 + z_g]$. The parameter $z_g \in [0, 1)$ could be interpreted as the degree of growth uncertainty. The case of $z_g = 0$ corresponds to the one of the deterministic case. Notice that our specification implies that only the result ($R1$) is plausible. Notice also that when $\theta > 1$, the absolute value of $g'(s) - h'(s) < 0$ is increasing in z_g, and the difference between σ and $\bar{s}$ is increasing in z_g. Thus, if θ is large enough with the property of self-sustainability, then a higher degree of growth uncertainty has a larger impact on the optimal level of the escapement and causes the optimal target level of the escapement to decrease more. This could provide the possibility of a sharp contrast to the result in harvesting models of Reed (1979) in terms of the direction of the impact of growth uncertainty on the optimal escapement level. Reed (1979) emphasizes that in most realistic harvesting cases ($\theta \in (0, 1]$), the optimal escapement level is larger under growth uncertainty than that in the deterministic setting. In contrast, on invasive species management, the opposite impact might be plausible under the assumption that self-sustainability of σ is met since θ could be large enough.

3.1.2 Non-Self-Sustaining Short-Sighted Escapement

The discussion in the previous subsection is based on a crucial assumption that the short-sighted escapement level σ is self-sustaining so that it is the optimal level of the escapement. Since this assumption is not always guaranteed, this subsection examines the optimal policy when σ is not self-sustaining. In general, it is difficult to find the optimal policy analytically, and thus we illustrate the relation between growth uncertainty and the optimal policy through numerical analysis.[9]

For the sake of computation, we make the following two specific assumptions in terms of functional forms. First, the social damage from invasive species is represented by the linear quadratic form:

$$D(s) = a_1 s + \frac{a_2 s^2}{2}, \tag{11}$$

where s denotes the escapement with $a_1 \geq 0$ and $a_2 > 0$. The parameter a_2 represents the degree of convexity. Second, the reproduction process of invasive species follows the conventional logistic curve:

$$F(s) = rs\left(1 - \frac{s}{K}\right) + s, \tag{12}$$

where $r > 0$ is the intrinsic growth rate and $K > 0$ is the carrying capacity. The two functional forms satisfy the assumptions specified in the previous sections and are also employed by some other authors in the settings of invasive species management (see, e.g., Olson and Roy (2004) and Eisewerth and Johnson (2002)).[10]

The value function iteration algorithms introduced in Judd (1998) are adopted to approximate the value function $v(m)$ as well as optimal policy function $y^*(m)$ that are characterized by the Bellman equation (6).[11] This algorithm first involves the discretization of the state space, and then iterates on the Bellman equation with an initial guess for the value function. It is shown that by the contraction theorem, the Bellman equation does fix a unique value function, $v(m)$, and the iterative process converges to the true value function. Accordingly,

[9]The difficulty of analytical derivation on the optimal policy in this case is noted by Reed (1979).

[10]We evaluate the sensitivity of the qualitative results in response to changes in the model specifications and parameter sets: some alternative reproduction functions of invasive species stock, $F(s)$, (logistic and Ricker forms) and some alternative parameter sets of social damage function, $D(s)$, (a_1 and a_2 on linear quadratic social damage function). In all of these cases, we make the sensitivity analysis by changing the degrees of growth uncertainty as well as measurement error in the same way that we examine in this study. We generally find that the optimal policy and the corresponding value function exhibit the same pattern as the ones that will be presented in the later section.

[11]Matlab code is written for numerical solutions.

a particular optimal policy $y^*(m)$ is obtained.

For our baseline, we choose $k = 250$ for the unit cost function, $c(x)$; $a_1 = 1$ and $a_2 = 2$ for social damage function, $D(s)$; $r = 0.3$ and $K = 10$ for the reproduction of invasive species, $F(s)$; and $\rho = 0.95$ for a social discount rate. We also set $\theta = 1.1$ so that the sensitivity of catchability is relatively large. In this case, the optimal policy in the deterministic case is the constant escapement policy with the interior target level $\bar{s} = 3.9$. Given these values and assumptions, we attempt to find the optimal policy based on each of three different values of the degree of growth uncertainty; $z_g \in \{0.25, 0.50, 0.75\}$, under each of which the escapement level $\bar{s} = 3.9$ is not self-sustaining in the stochastic setting.

Figure 1 illustrates how the degree of growth uncertainty affects the optimal policy. First, it is clear that the constant escapement policy is optimal as in the deterministic case. This implies that the existence of growth uncertainty keeps the optimal policy to be in a class of the constant escapement rule. Second, a rise in the degree of growth uncertainty monotonically increases the optimal target level of the escapement. The intuition of this result could be explained as follows. When the degree of growth uncertainty is relatively large, the stock level in the next period is highly likely to be in a non-self-sustaining region in that the stock level tends to be reduced from the current period to the next period with some positive probability, even without any removal operations. This implies that a larger degree of growth uncertainty causes the current removal to be less attractive for the society officials. This is in sharp contrast to the result of the case where the optimal short-sighted escapement level, σ, is self-sustaining in that the degree of growth uncertainty decreases the optimal target level in the previous discussion.

The issues of self-sustaining and non-self-sustaining short-sighted escapement provide an interesting conjecture on the feature of the optimal target level of escapement related to the degree of growth uncertainty z_g. When z_g is small enough, σ may be self-sustaining so that σ is the optimal target level of escapement, which is decreasing in z_g. In contrast, when z_g is large enough, σ is unlikely to be self-sustaining so that it is not the optimal escapement level. In this case, the optimal target level of escapement is increasing in z_g. These results imply the possibility that the optimal target level of escapement could follow a U-shaped track: as z_g rises, the optimal target level of escapement decreases and then increases.

3.2 Eradication Case

This subsection examines how growth uncertainty affects the optimal policy when the corresponding deterministic model could yield the optimal policy of immediate eradication for any initial stock. This case may be corresponding to the one that social damage out of invasive species is sufficiently large and the sensitivity of catchability is sufficiently small. In this case, it is also difficult to find the optimal decision analytically, and thus we will approach this case through numerical analysis, as in the previous discussion. As a benchmark, we newly set $\theta = 0.5$, keeping other parameters unchanged. The optimal policy of the corresponding deterministic model yields immediate eradication for all level of the stock. Examining this case generates a set of implications associated with the degree of growth uncertainty.

Similar to the previous discussion, we examine three different values of the degree of growth uncertainty; $z_g \in \{0.25, 0.50, 0.75\}$. The results are summarized in Figure 2, which suggests that a rise in the degree of growth uncertainty affects the optimal policy. The introduction of growth uncertainty yields a Skiba point of the initial stock, which separates the optimal actions between immediate eradication and giving-up without any control. Moreover, as the degree of growth uncertainty rises, the Skiba point is monotonically getting smaller and approaching to zero. We also confirm that when the degree of growth uncertainty is large enough, say $z_g \geq 1.0$, the optimal policy finally come to be giving-up without any control for any stock level.

The intuition behind this result is similar to that in the non-eradication case. When the degree of growth uncertainty is relatively large, the stock level is highly likely to be in a non-self-sustaining region, i.e., the invasive species stock is naturally likely to decline even without any control. This implies that a larger degree of growth uncertainty causes the current removal to be less attractive for the society officials. Thus, waiting is optimal until the stock level happens to become relatively small. We believe that this simple result is the first to show such a qualitative feature of the shift of the Skiba point on the invasive species management due to growth uncertainty.

4 Measurement Error and Growth Uncertainty

This section incorporates measurement error into the previous setup with growth uncertainty. We adopt the same specifications of measurement error as in the stochastic model of Sethi, Costello, Fisher, Hanemann, and Karp (2005), i.e., measurement error is Markovian; a signal of true stock is in a multiplicative fashion; and Z_t^m is uniformly distributed over the interval $[1-z_m, 1+z_m]$, where the parameter $z_m \in [0, 1)$ represents the degree of measurement error. As in the previous section, we attempt to analyze the two cases: (i) the non-eradication case and (ii) the eradication case, in order. In each case, two different values of the degree of measurement error, $z_m \in \{0.25, 0.50\}$, are examined, and the results are compared to the cases in the absence of measurement error.

For the clarity of our results, two sets of figures are presented in each of the two cases. Figures 3 and 4 are the non-eradication cases corresponding to $\theta = 1.1$, and Figures 5 and 6 are the eradication cases corresponding to $\theta = 0.5$. Figures 3 and 5 shows the base case where the parameter set is the same as that in the previous section, while Figures 4 and 6 shows one of the results adopted from sensitivity analysis, which is denoted by 'sensitivity analysis' in the caption of figures. The set-up of sensitivity analysis differs from the base case in the parameter value of a_2, which represents the degree of convexity in the social damage function. Instead of $a_2 = 2$ in the base case (Figures 3 and 5), we use the value of $a_2 = 1$ in the case of sensitivity analysis (Figures 4 and 6) so that the degree of convexity is weakened compared to the base case. We can demonstrate our qualitative results more clearly by showing the sensitivity analysis cases in tandem with the base cases. Some of our results are common to the previous work in stochastic fishery models, but we obtain new findings which emanate from the interplays between measurement error and growth uncertainty.

4.1 Non-Eradication Case

The previous section has shown that in non-eradication cases, growth uncertainty does not change the qualitative feature of the optimal policy from constant escapement rules, although its target escapement level is affected. This subsection analyzes how measurement error influences the optimal policy in non-eradication cases. Figures 3 and 4 represent the optimal policies in the base case and sensitivity analysis case, respectively. Each of them consists of six sub-figures.

At the left side in Figures 3 and 4, three sub-figures describe optimal policies in a situation where the degree of measurement error is fixed at $z_m \in \{0, 0.25, 0.5\}$, respectively. Each sub-figure shows three optimal policies, each of which corresponds to some level of the degree of growth uncertainty $z_g \in \{0.25, 0.5, 0.75\}$. On the other hand, at the right side, three sub-figures describe optimal policies in a situation where the degree of growth uncertainty is fixed at $z_g \in \{0.25, 0.5, 0.75\}$. Each sub-figure shows three optimal policies, each of which corresponds to some level of the degree of measurement error $z_m \in \{0, 0.25, 0.5\}$. Notice that the target removal is taken as the vertical axis, and the measured stock as the horizontal axis. In this way, it is easy to see how optimal removal actions should be adapted to uncertainty. An interior constant escapement rule is described by the graph in which the target removal is zero if the measured stock is below a certain level, otherwise it linearly increases in the measured stock with its slope of unity. Figure 1 and sub-figure of $z_m = 0$ in Figure 3 represent the identical constant escapement rules, while they use different variables as the vertical axis.

Our findings in the non-eradication cases are as follows. First, the introduction of measurement error could change the qualitative feature of the optimal policy from a constant escapement rule to a non-constant escapement rule (see sub-figures in Figures 3 and 4). If the measured stock is below a certain level, no control is optimal as in the case of the absence of measurement error. However, if the measured stock is above the certain level, the optimal target removal increases non-linearly (with some concavity) in the measured stock, which is in contrast to the case of the absence of measurement error, where the optimal target removal increases linearly in the measured stock. Notice that the deviation of the optimal policy from constant escapement rules becomes more distinguished as the degree of measurement error gets larger (see sub-figures related to the cases of $z_m = 0.25$ and $z_m = 0.5$ in Figures 3 and 4). These results could be consistent with those in stochastic fishery models of Clark and Kirkwood (1986) and Sethi, Costello, Fisher, Hanemann, and Karp (2005), where the presence of measurement error could fundamentally alter the optimal policy.

Second, more interestingly, sub-figures at the left in Figures 3 and 4 illustrate that the relation of the intensity of target removal operations associated with growth uncertainty could get reversed in the presence of measurement error. Sub-figures of $z_m = 0$ in both Figures 3 and 4 show that the optimal removal declines as the degree of growth uncertainty rises, while in contrast, sub-figures of $z_m = 0.5$ in both Figures 3 and 4 show that the optimal target removal increases as the degree of growth uncertainty rises.

One possible explanation for this may emanate from the fact that measurement error gives rise to a new important role of growth uncertainty in our dynamic problem, i.e., the degree of growth uncertainty now has an impact on the expectation of current rewards as well. Without measurement error, growth uncertainty only affects the expectation of the continuation value in the dynamic programming equation (value in the next period). However, if measurement error is present, growth uncertainty contributes to the uncertainty associated with not only future but also "current" values. This is the critical difference between the cases with and without measurement error. In the presence of measurement error, a rise in the degree of growth uncertainty intensifies the uncertainty on current rewards and thus reduces the expected current reward associated with the stock of invasive species compared to the case without measurement error.[12] An important implication is that when growth uncertainty is intensified in the presence of large measurement error, the importance of controlling invasive species and current social damage rises so that the target removal becomes more aggressive.

Third, from sub-figures at the right in Figures 3 and 4, we confirm that holding the degree of growth uncertainty fixed, intensified measurement error could change the sensitivity of the optimal target removal in response to the measured stock.[13] Generally, an increase in the measured stock makes the optimal target removal less aggressive compared to the case without measurement error. To understand this, we notice that given a positive value of z_m, informational quality of measurement in hand gets worsen as the measured stock increases, due to the multiplicative stochastic term of Z_t^m. We would also say that z_m can be reinterpreted as a measure of how fast informational quality of measurement gets worse with an increase in the measured stock. In the presence of measurement error z_m, any control is likely to be less effective as the measured stock becomes larger, since costly removal operation must be implemented based on more imprecise information.

Fourth, the role of measurement error on informational quality also affects the critical measured stock level at which the target removal operation is triggered. Sub-figures at the right in Figure 3 suggest that the critical level in the case of $z_m = 0.25$ is almost identical to that in the case without measurement error $z_m = 0$. However, once the degree of measurement error becomes sufficiently large, i.e., $z_m = 0.5$, the critical level becomes smaller than that in the case without measurement error. That is, around the measured stock level just above the critical value, an increase in the degree of measurement error causes the optimal target removal to rise. This result is quite similar to the one obtained in stochastic fishery models of Sethi, Costello, Fisher, Hanemann, and Karp (2005). Noticing

[12] This holds when the strict convexity of social damage is large enough. Jensen's inequality can be applied for the proof.

[13] Sub-figures at the left in Figure 3 show the comparison of optimal policies holding the degree of measurement error, while sub-figures at the right show the comparison of optimal policies holding the degree of growth uncertainty. In particular, sub-figures at the right can starkly clarify how measurement error causes the optimal policy to deviate from constant escapement rules.

that z_m represents the speed at which informational quality of measurement gets degraded, we might conclude that as z_m becomes larger, the society officials have a stronger incentive to control the stock before the measured stock becomes larger and informational quality becomes much lower. Thus, an increase in the degree of measurement error z_m could cause the trigger level of the measured stock initiating removal operation to be small compared to the case without measurement error.

However, we also confirm that this result is not quite robust on invasive species management. This may be due to the fact that optimal policies are also dependent on other factors, such as the degree of the convexity in the social damage function, which are unique on invasive species. If we just change the parameter $a_2 = 2$ into $a_2 = 1$, then the introduction of measurement error might cause the critical level of the measured stock in fact to get larger (see sub-figure related to $z_g = 0.25$ in Figure 4). In this case, the effect of the speed at which informational quality gets worse dominates the incentive of controlling the stock earlier than in no measurement error case which comes from convexity of social damage functions.

4.2 Eradication Case

This subsection examines how measurement error with growth uncertainty affects the optimal policy in the eradication case where immediate eradication action can be optimal for some levels of the measured stock. Figures 5 and 6 respectively present the base case and sensitivity analysis case, both of which correspond to $\theta = 0.5$. The difference between them is only the value of a_2, which represents the degree of convexity in the social damage function, as in the previous subsection. In the absence of both growth uncertainty and measurement error, the optimal policy in the base case is immediate eradication for any stock level, while in the sensitivity analysis case it is immediate eradication or giving-up without any control depending on whether the stock level is smaller or larger than a Skiba point (which is around 4.0 in this sensitivity analysis case).

All graphs in each sub-figure of Figures 5 and 6 show that as in the case of the absence of measurement error, there exists a critical level of the measured stock, which is a Skiba point, such that immediate eradication is optimal if the measured stock is smaller than the critical level, otherwise giving-up without control is optimal.[14]

In the presence of measurement error in addition to growth uncertainty, the optimal policy may be more elusive in the sense that a clear pattern of the impacts of the uncertainties on the optimal policies cannot be observed. Sub-figures in Figures 5 and 6 show that the Skiba points are affected by the degrees of measurement error as well as growth uncertainty, but they do not display a systematic shift of the Skiba point at all. One thing we may be able to say is that as the degree of measurement error gets larger, the Skiba point becomes less sensitive to a change in the degree of growth uncertainty (see sub-figures at the left in Figures 5 and 6).

In general, however, such complexity arises from various other eradication cases in which some parameters and functional forms are changed as alternative specifications. Therefore, we would say that the issue on how multiple uncertainties affect the optimal policy remains as an open question and would be a very interesting topic to be addressed in the future.

4.3 Value of Optimal Program

The previous subsections have focused on the qualitative change of the optimal policy in response to a change in the degrees of growth uncertainty and/or measurement error. This subsection examines the impact of multiple uncertainties on social welfare through numerical analysis on the value function. We show the results in non-eradication and eradication cases in order.

Figures 7 and 8 respectively correspond to the base case and sensitivity analysis case, as

[14]Some sub-figures in Figure 5 describe a situation where the optimal target removal is constant at some positive level over some region of the measured stock. This constant target removal arises from the fact that given some degree of measurement error, there exists a possible upper level of true stock. The society officials just set the target removal at the upper level when the current measured stock is relatively large so that the next period's stock can be reached to the upper level with some positive probability, and when their optimal policy is immediate eradication. Thus, even though some positive level of constant target removal is observed over some region of the measured stock in sub-figures, it is aimed for eradication.

in the previous subsections. They show some systematic features of the impact of growth uncertainty and measurement error on the value function in the non-eradication case. First, holding the degree of measurement error fixed, a rise in the degree of growth uncertainty causes the graph of the value functions to shift up (see sub-figures at the left in Figures 7 and 8). This result might be surprising since growth uncertainty representing environmental variability could improve social welfare if the society officials behave optimally. This may be partly because a large degree of growth uncertainty enlarges the non-self-sustaining region of the stock so that the future stock of invasive species is likely to be decreased even without any control.

Second, holding the degree of growth uncertainty unchanged, a rise in the degree of measurement error causes the graph of the value functions to shift down (see sub-figures at the right in Figures 7 and 8). In other words, a rise in the degree of measurement error would deteriorate social welfare, which is in sharp contrast to the impact of a change in degree of growth uncertainty. This result is quite consistent with the one in the stochastic fishery models of Clark and Kirkwood (1986). In other words, more accurate information about the current status of invasive stock in general benefit a society in the non-eradication case.

What kind of policy implications can we suggest out of these results concerning the value of optimal programs in the non-eradication case? First, given the same degree of social damage from invasive species, governments that need to control various kinds of invasive species should prioritize the species that highly fluctuates owning to environmental variability. In this way, governments could pursue more efficient allocation. Second, concerning measurement error, when governments decide not to aim at eradication, improving the quality of measurement, perhaps through technological improvement or more efforts on stock survey, could be beneficial for a society.

We next turn to the eradication case under growth uncertainty and measurement error. Figure 9 corresponds to the base case and exhibits complex patterns of shifts in the value functions, as in the discussion of the optimal policies in the previous subsection. One thing to be noticed is that in the absence of measurement error, the graph of the value functions shifts up as the degree of growth uncertainty rises (see sub-figure related to $z_m = 0$ in Figure 9). However, once measurement error is introduced in the model specification, this feature collapses. The value functions cross each other with some complexity, and thus we could not draw clear conclusion on the impact of growth uncertainty and measurement error on social welfare. The results concerning the value function in the eradication case suggests that improving the informational quality related to measurement error does not necessarily benefit management programs. This result is also opposite to the one in non-eradication cases.

5 Conclusion

This paper has examined how growth uncertainty and measurement error affect the optimal policy on invasive species management. Although there might be other factors that complicates the management decisions, such multiple uncertainties are among the main factors that government officials should take into account in advance. We have shown that such uncertainties could significantly alter not only qualitative features of the optimal policy, but also the value of management practices. While some of our results are common to the findings of stochastic fishery models in the past literature, we found a series of novel results, focusing on the two cases: the non-eradication case with a relatively large sensitivity of catchability in which the constant escapement rule would be optimal in the deterministic setting; and the eradication case with a relatively small sensitivity of catchability in which immediate eradication would be optimal in the deterministic setting. We believe that some of our results would be valuable for real management practices.

We demonstrate that in the non-eradication case, growth uncertainty does not cause the qualitative feature of the optimal policy to change from the constant escapement rule, but it alters the target escapement level in the optimal policy, as in Reed (1979). One of the important and new findings is that the target escapement level may not monotonically vary with the degree of growth uncertainty. In fact, the impact could be U-shaped in the sense that the target escapement level is decreasing and then is increasing in the degree of growth uncertainty. On the other hand, in the eradication case, growth uncertainty could yield a Skiba point in the optimal policy, which suggests that waiting with more patience is required before eradication actions are taken in optimal policies, as the degree of growth uncertainty increases. In addition, the value of optimal programs in both cases is increasing in the degree of growth uncertainty.

Concerning multiple uncertainties, we identify that the introduction of measurement error, in addition to growth uncertainty, significantly affects the qualitative features of the optimal policy in both the non-eradication and the eradication cases. In the non-eradication case, measurement error could cause the optimal policy to deviate from the constant escapement rules, and also the impact of measurement error is significantly affected by the degrees of growth uncertainty. In particular, given a small degree of measurement error, less removals are required as the degree of growth uncertainty rises. In contrast, given a large degree of measurement error, more removals are required as the degree of growth uncertainty rises. These imply that the adaptation to the growth uncertainty in the optimal policy could get reversed in the presence of measurement error. On the other hand, in the eradication case, the presence of measurement error also affects the optimal policy in that a Skiba point moves in complex manners. However, we could not find some systematic patterns, and this issue leaves us an open question.

This study has also examined the values of the optimal program in the presence of both growth and measurement uncertainty. In the non-eradication case, the value of optimal programs is generally monotone increasing in the degree of growth uncertainty, while it is monotone decreasing in the degree of measurement error. These findings suggest that given the same degree of social damage from invasive species, governments that need to control various kinds of invasive species should prioritize the species that highly fluctuates owning to environmental variability. Moreover, improving the quality of measurement generally benefits a society. On the other hand, in the eradication case, the value of the optimal program could be monotone increasing in the degree of growth uncertainty in the absence of measurement error. However, once measurement error is present, the values exhibit complex curvature in the sense that they could cross each other. Thus, we could not draw a clear conclusion on this. In other words, if the optimal policy involves some eradication actions, there is no clear agreement on whether uncertainty is better or not for a society.

Although we have confirmed through sensitivity analysis that the above results could be obtained from other different functional forms and parameter sets that satisfy the basic assumptions we imposed in this paper, some attention must be paid to the specification of uncertainty. We assume that growth uncertainty and measurement error are a Markovian, and the probability distributions are fully known with parameters. In reality, these suppositions can be questioned. What we can do instead in future researches may be to apply the Bayesian learning model for unknown parameters or the Kalman filter for alternative assumptions of state space modeling. However, it is our belief that this paper could be considered a starting point of researches on invasive species management in the presence of uncertainty, and we hope that some potential extensions as listed above would be made in the near future.

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Figure 1: Base case: Optimal policy in non-eradication case under growth uncertainty

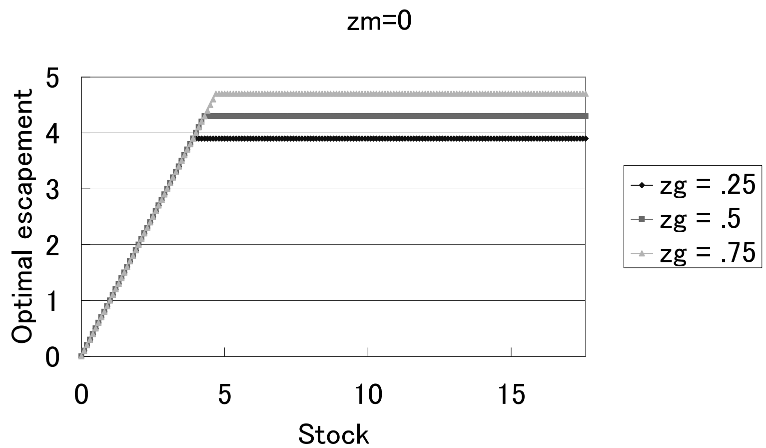

Figure 2: Base case: Optimal policy in eradication case under growth uncertainty

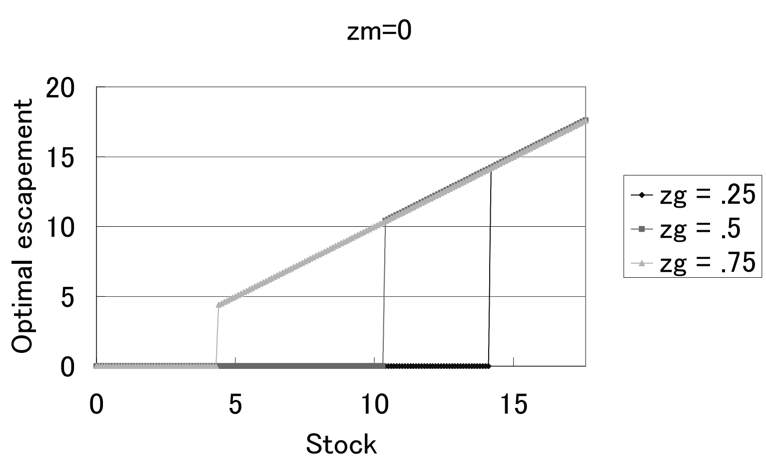

Figure 3: Base case: Optimal policy in non-eradication case under growth uncertainty and measurement error [Fix z_m (Left); Fix z_g (Right)]

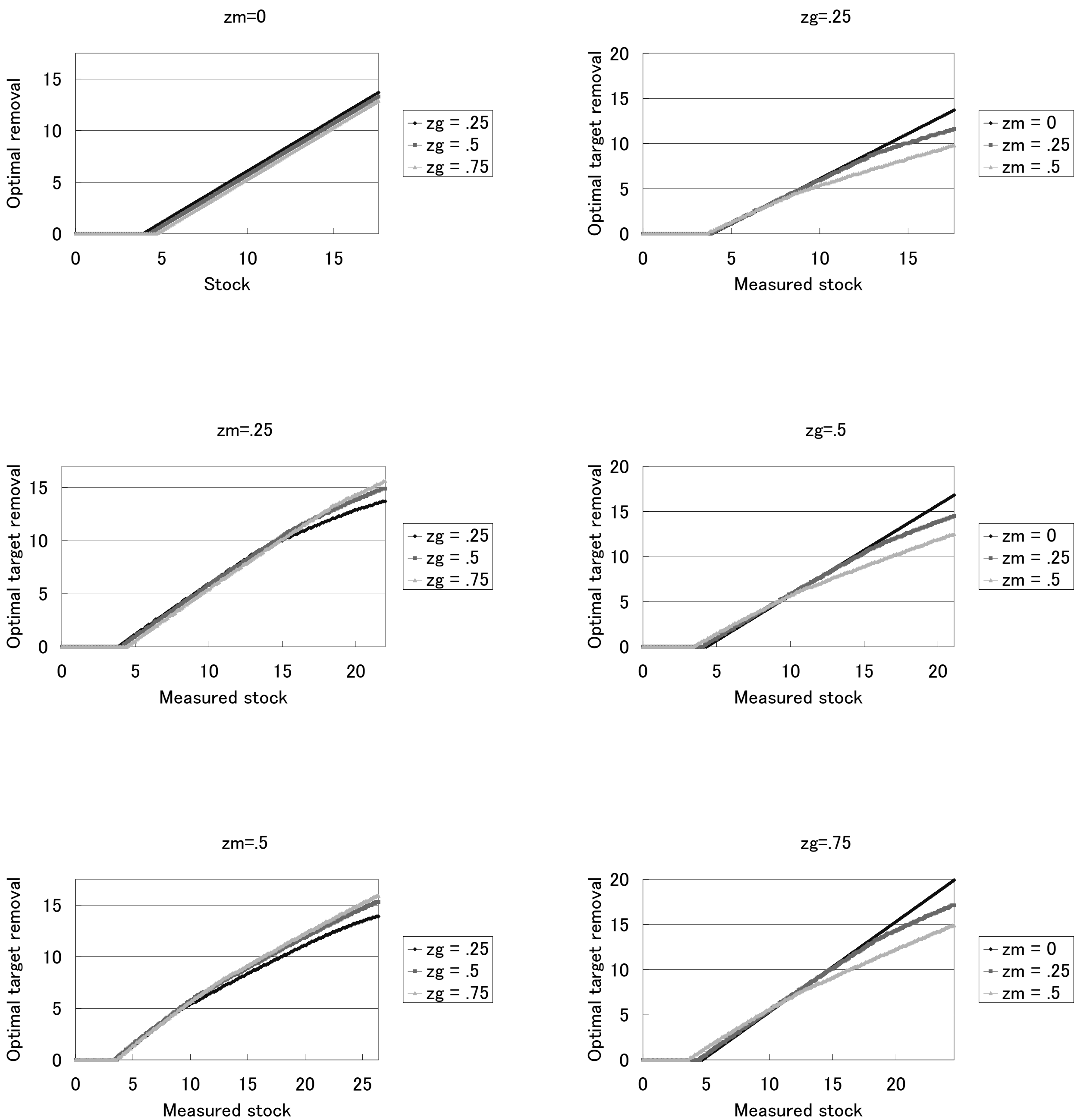

Figure 4: Sensitivity analysis: Optimal policy in non-eradication case under growth uncertainty and measurement error [Fix z_m (Left); Fix z_g (Right)]

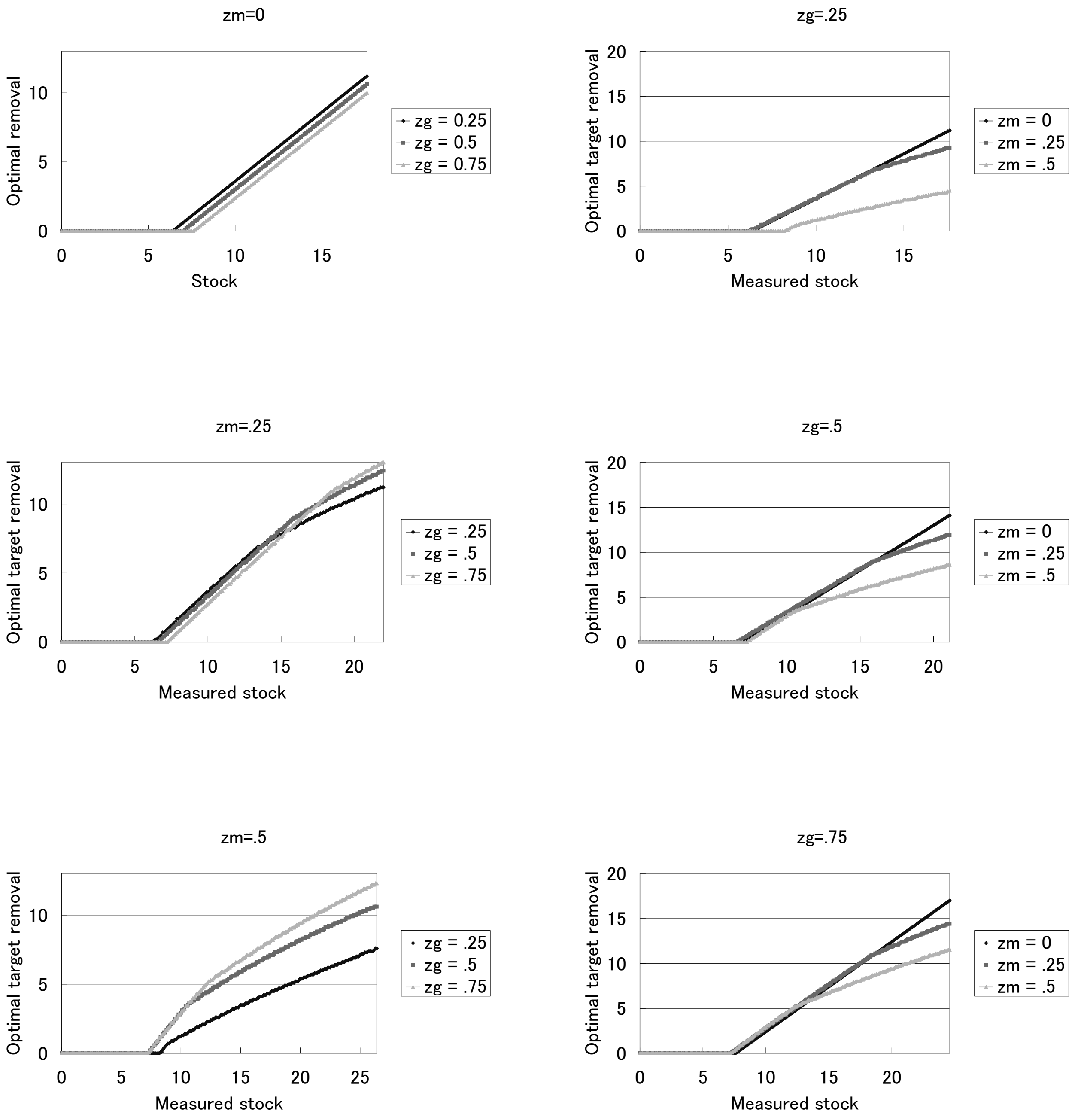

Figure 5: Base case: Optimal policy in eradication case [Fix z_m (Left); Fix z_g (Right)]

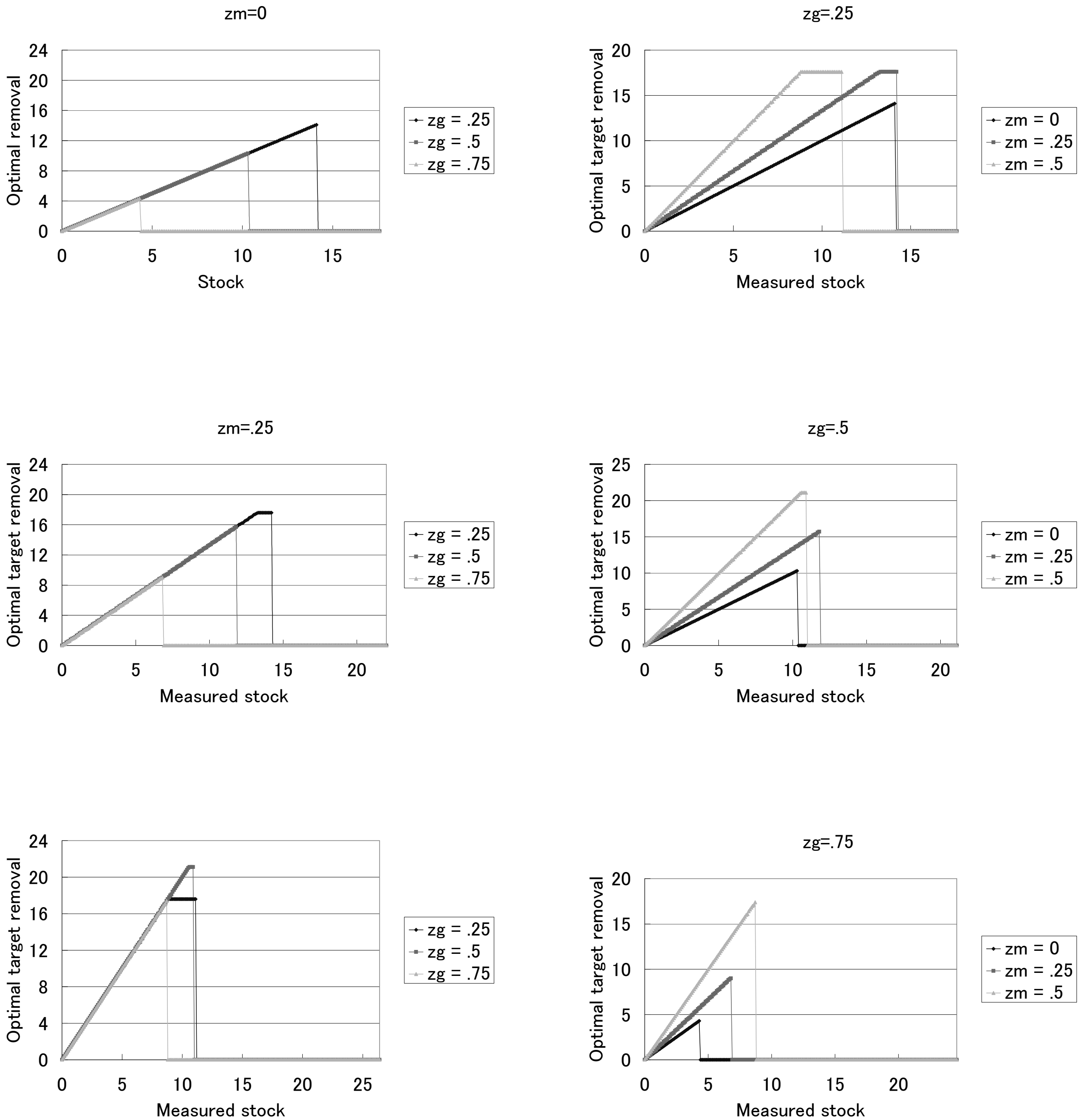

Figure 6: Sensitivity analysis: Optimal policy in eradication case under growth uncertainty and measurement error [Fix z_m (Left); Fix z_g (Right)]

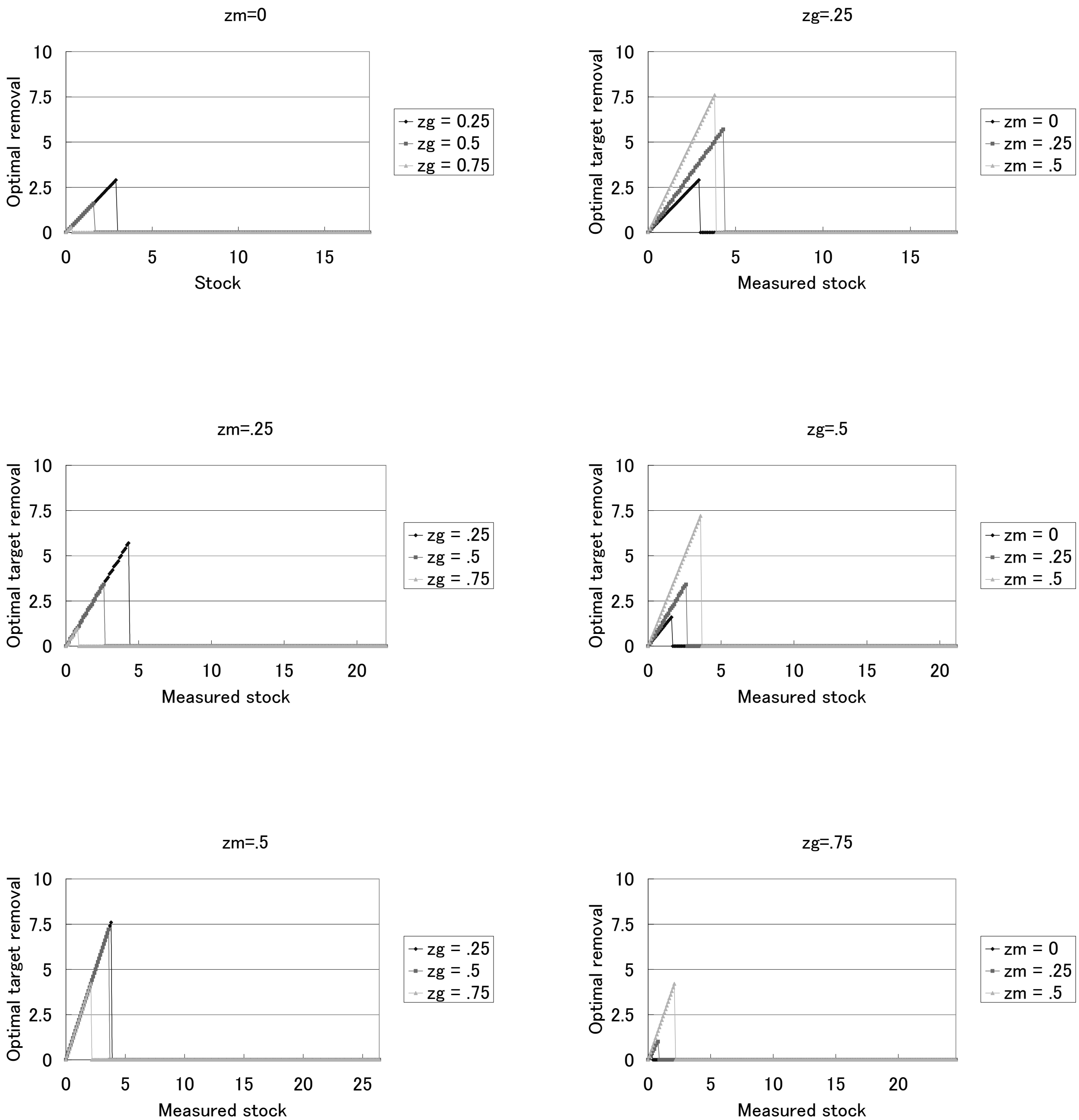

Figure 7: Base case: Value function in non-eradication case [Fix z_m (Left); Fix z_g (Right)]

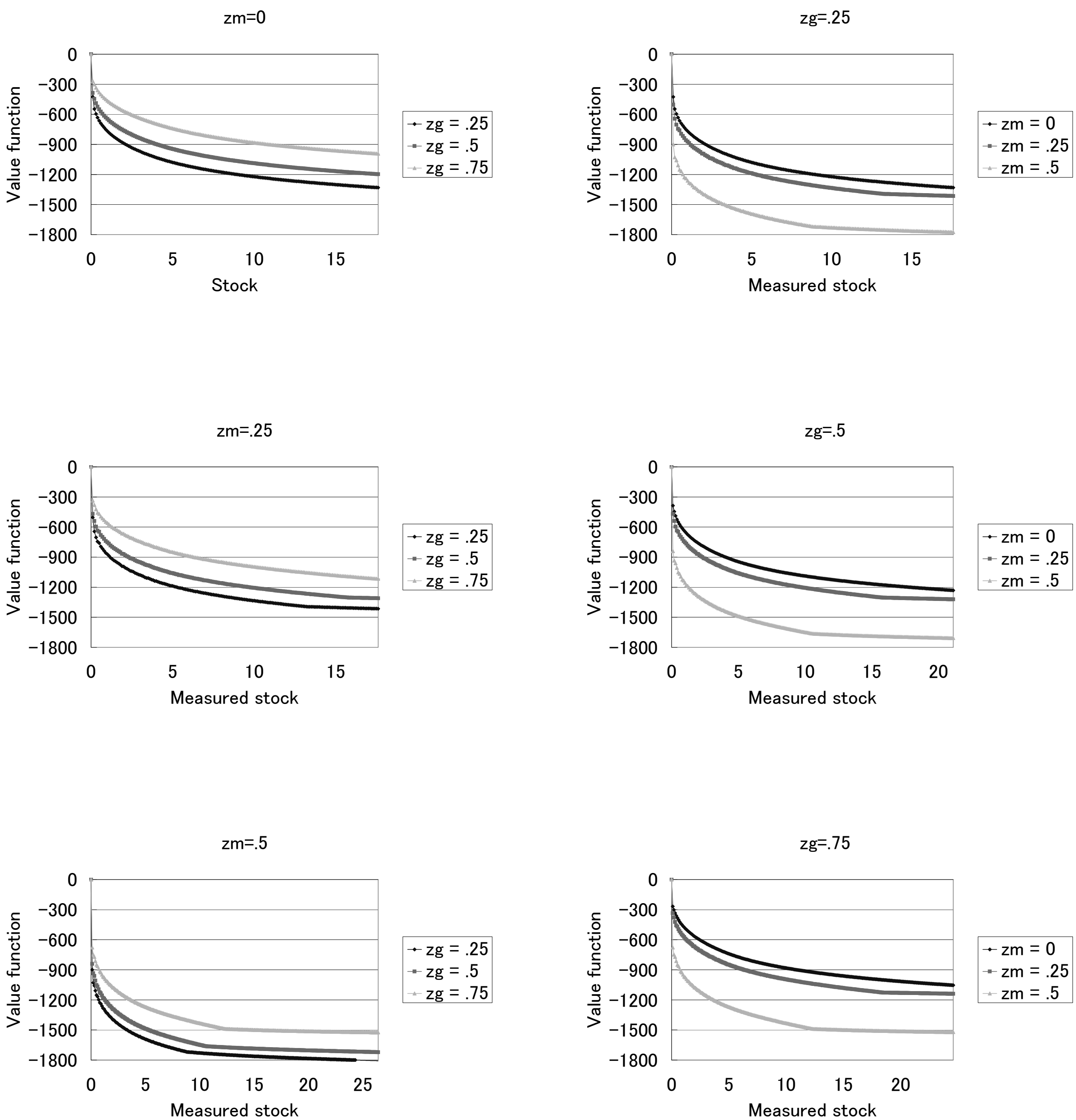

Figure 8: Sensitivity analysis: Value function in non-eradication case [Fix z_m (Left); Fix z_g (Right)]

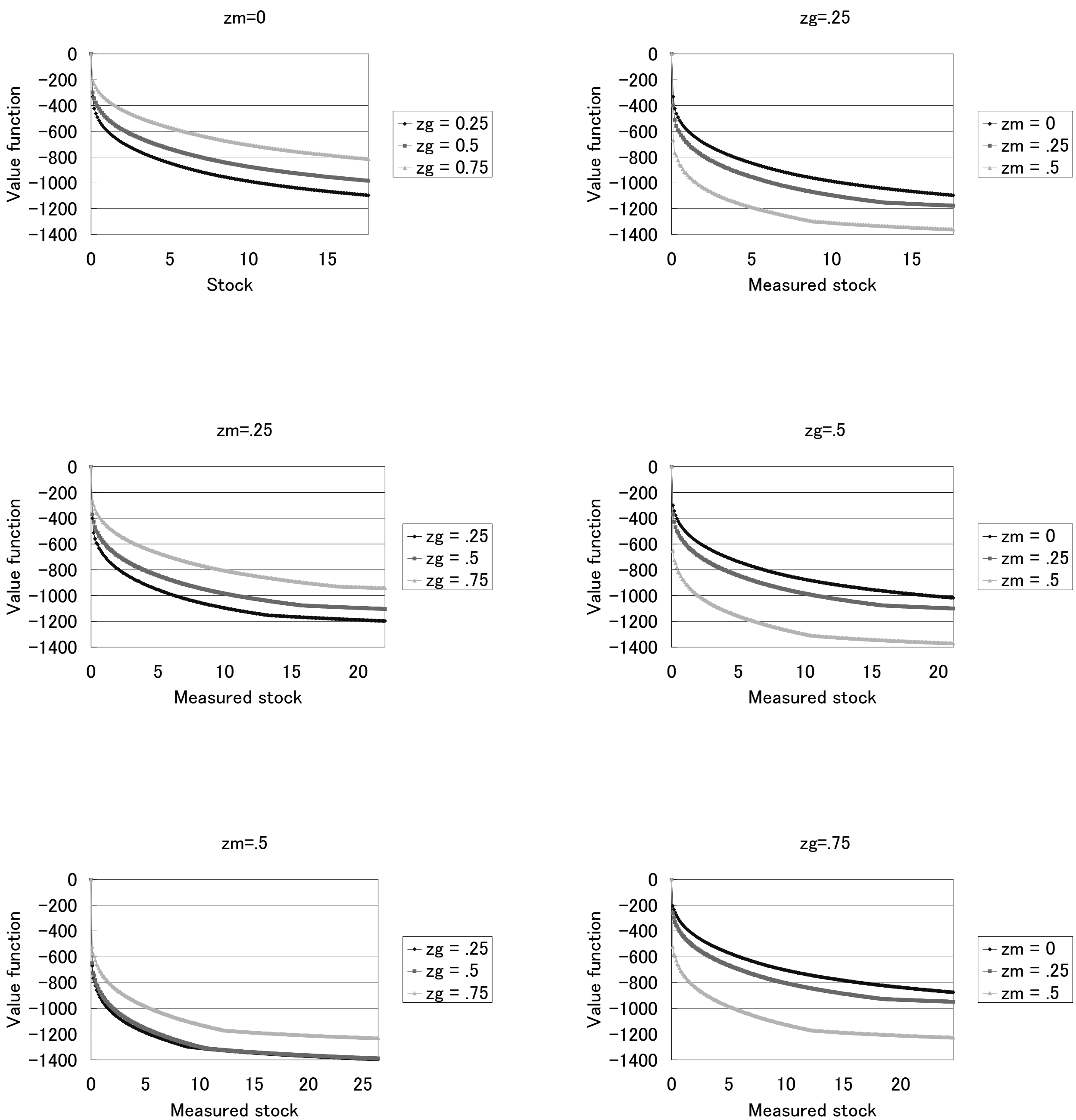

Figure 9: Base case: Value function in eradication case [Fix z_m (Left); Fix z_g (Right)]