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# Optimal Escapement Levels on Renewable Resource Management under Process Uncertainty: Some Implications of Convex Unit Harvest Cost*

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## Abstract

The terminology of renewable resource management becomes to span not only prototype harvesting problems but also various new types such as invasive species one. In all of these problems, process uncertainty of stock growth associated with environmental variability is one of the critical factors that significantly affects the management efficiency. While it may seem that a series of past researches fully examine optimal policy under process uncertainty, the case of convex unit harvest costs has not been fully characterized yet. Focusing on such a case, this paper addresses how the degree of process uncertainty affects optimal escapement level. The result suggests that optimal escapement level does not monotonically vary with process uncertainty. In many plausible cases, it should be adapted in a U-shaped manner, which is in contrast with the conventional wisdom.

**Key Words**: renewable resource management, constant escapement rule, process uncertainty, convex unit harvest cost

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* We are responsible for any remaining errors.

# 1 Introduction

Optimal harvesting policies on renewable resource management have been studied in various settings. One oft-taken approach is to consider uncertainty. While uncertainty can be categorized into several types, the focus of many past works is on the impact of process uncertainty associated with environmental variability. Such an issue would become more salient in the future since the presence of process uncertainty is unavoidable in most renewable resource management, and it is also reported that environmental fluctuation increases more due to various factors, such as climate change (see, e.g., Intergovernmental Panel on Climate Change (2001) and Karl et al. (1995)).

The pioneering work focusing on process uncertainty is Reed (1979) that provides the conditions for an interior constant escapement rule to be optimal and characterizes its escapement levels. An interior constant escapement rule is simply expressed as

$$h_t^* = \max(X_t - S^*, 0) \tag{1}$$

where $X_t$ is the current stock, $S^*$ is the optimal target escapement and $h_t$ is optimal harvest for $t = 0, 1, \ldots$.

However, the set-up in his model is somewhat tailored for a fishery problem, and also the analysis still remains unresolved for the case of convex unit harvest costs. The reason for this may be due to the argument that the convex unit harvest cost had been considered rare in fishery problems, although Clark (1990) gives some justification for such a case to hold. Another reason may be that analytical derivations are generally considered quite difficult or impossible in the case of convex unit harvest costs.

The terminology of renewable resource management has recently become to span not only prototype harvesting problems but also various new types such as invasive species management. A number of papers suggest that strictly convex unit harvest costs are plausible in such new types of problems. For instance, Bomford and O'Brien (1995) and several others argue that killing the first 99% of a target population can cost less than eliminating the last

1%. More precisely, the operational cost of removing one unit of invasive species may be escalated as the existing population decreases, and it is particularly evident in the last 10% population. This anecdote implies that the convexity could hold for unit harvest costs.

Given this state of affairs, the purpose of this paper is to extend the framework of the model to general renewable resource management.[1] We then examine the possibility that an interior constant escapement rule is optimal for more general problems when the unit harvest cost function is strictly convex. We finally demonstrate how the optimal escapement level could respond to the degree of process uncertainty.

The results suggest that the optimal escapement level could be non-monotonically changed with the degree of process uncertainty, which is in contrast with the conventional wisdom. In particular, it must be adapted in a U-shaped manner as the degree of process uncertainty rises.[2] To illustrate that this result holds for general renewable resource management, two distinct problems are introduced as examples: (i) fishery harvesting problem; and (ii) invasive species problem, in which a main common feature is that the unit harvest costs are strictly convex.

This paper seeks to cast some attention to the case of convex unit harvest costs under process uncertainty. The good point of such analysis on a unit harvest cost is that for the management agency, it is relatively easy to check its curvature from the field data such as the catch per unit of effort (CPUE), and thus it may be of some guidance for real practices. Together with the results obtained by Reed (1979), we believe that our results add to the understanding of optimal escapement levels with respect to the degree of process uncertainty.

This paper is organized as follows. In the next section, we introduce the model. The section is followed by presenting the analysis with some important results. In that section, numerical illustration is also given. Some conclusion is offered in the final section.

[1]The general renewable resource management in this paper includes any type of problems in which social benefit or damage is dependent on escapement or harvest of renewable resources in addition to harvest costs which are a function of both stock size and harvest.

[2]What we stand for by conventional wisdom here is that fact that the optimal escapement level under process uncertainty is larger than the one in the deterministic setting in most cases, and its escapement level must be adapted in a monotone increasing manner with respect to process uncertainty (see Reed (1979) and Clark (1990))

## 2 The Model

We set up an infinite-period stochastic model of renewable resource management with convex unit harvesting costs. Throughout this paper, of particular interest is "process uncertainty," which is also denoted by "growth uncertainty" or "environmental variability."[3] We introduce the condition under which a class of interior constant escapement rules is optimal and then demonstrate the possibility that the target escapement level should non-monotonically be changed with the degree of process uncertainty.

Process uncertainty in each period $t = 1, 2, \ldots, \infty$ is modeled by random variable $\{Z_t\}$ which is a sequence of independently and identically distributed random shocks with mean 1 and finite support $[1 - z_g, 1 + z_g]$, $z_g \in [0, 1]$, according to a common distribution function $\Phi$. With this specification of process uncertainty, the distribution is mean-preserving spread with respect to $z_g$ so that an increase in the degree of process uncertainty is equivalent to a rise in $z_g$. The stock of renewable resource evolves according to the following state equation:

$$X_{t+1} = Z_t f(x_t - h_t) = Z_t f(s_t), \tag{2}$$

where $f$ is the expected reproduction function that generates the next period stock $X_{t+1}$, depending on current period stock $x_t$ and harvest $h_t$, that is, the escapement level, $s_t = x_t - h_t$ at period $t$. We assume that $f$ is twice differentiable and strictly concave with $f(0) = 0$ and $f'(0) \in (1, \infty)$, and there exists an undisturbed stock level, $\tau > 0$ with $\tau = f(\tau)$ such that $f(s) > s$ if $s \in (0, \tau)$.

The social welfare in each period consists of social benefit that emanates from harvesting renewable resource stock and the cost associated with its operations. The social benefit in period $t$ is given by $B(s_t)$, where $B$ is concave in $s_t$. The harvesting cost in period $t$ is given by $C(h_t, x_t)$ and $C$ is increasing in $h_t$ with $C(h_t, x_t) \geq 0$ for any $h_t$ and $x_t$. The harvesting cost function in each period depends not only on the harvested stock but also on the existing

[3] It could be understood that process uncertainty is distinct from measurement and implementation uncertainty, as in the same way Sethi et al. (2005) interpret.

invasive species in that period. As in the most of previous researches, the harvesting cost in period $t$ is assumed to be:

$$C(h_t, x_t) = \int_{x_t - h_t}^{x_t} c(z)dz,$$

where $c(x_t)$ is the unit cost that depends on the current stock $x_t$ (see, e.g., Reed (1979), Costello et al. (2001), Moxnes (2003), and many others).

This study focuses on the case where the unit harvest cost is strictly decreasing and strictly convex in the stock level, that is, $c'(\cdot) < 0$ and $c''(\cdot) > 0$.[4] From the above, the social welfare in period $t$ is given by:

$$u(x_t, h_t) = B(x_t - h_t) - C(h_t, x_t).$$

The objective of a society is to maximize the expected present value of social welfare by choosing a sequence of harvest in each period, $\{h_t\}_{t=0}^{\infty}$:

$$\max_{h_t \in [0, x_t]} \mathbb{E}\left\{\sum_{t=0}^{\infty} \rho^t u(X_t, h_t)\right\}$$

subject to $X_{t+1} = Z_t f(s_t)$ and $s_t = x_t - h_t$, where $\rho \in (0,1)$ is the discount factor, and $\mathbb{E}$ is the expectation operator. The Bellman equation for this problem is posed as:

$$v(x_t) = \max_{h_t \in [0, x_t]} \left\{u(x_t, h_t) + \rho \mathbb{E}(v(f(x_t - h_t)))\right\},$$

where $v(\cdot)$ is the value function given the current stock size.

# 3 Analysis

This section seeks to explore how the increase in the degree of process uncertainty affects the optimal escapement level in the case of convex unit harvest costs. For this purpose,

[4]The rationale of these properties of the unit cost function is given by a series of past literature in the renewable resource management (see, e.g., Clark (1990)).

following Kotani et al. (2007), we transform the social welfare at period $t$ into $u(x_t, h_t) = B(x_t - h_t) - [Q(x_t - h_t) - Q(x_t)]$, where $Q(x) \equiv \int_x^m c(z)dz \in [0, \infty]$ with $m = \max\{(1 + z_g)f(s) | (1 + z_g)f(s) \geq s, s \geq 0\}$. The term of $Q(x)$ represents the operational cost of harvesting renewable resources from the possible maximum stock level $m$ to some stock level $x$. Applying the fact that $X_{t+1} = Z_t f(s_t)$, we rewrite the objective function as:

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \rho^t u(X_t, X_t - s_t)\right\} = Q(x_0) + \mathbb{E}\left\{\sum_{t=0}^{\infty} \rho^t \Gamma(s_t, Z_t)\right\}, \tag{3}$$

where $\Gamma(s_t, Z_t) = B(s_t) - Q(s_t) + \rho Q(Z_t f(s_t))$. We denote the expected discounted growth in the immediate value by:

$$g(s) = \int_a^b \Gamma(s, z)d\Phi(z) = B(s) - Q(s) + \rho \int_{1-z_g}^{1+z_g} Q(zf(s))d\Phi(z).$$

In the deterministic case of $z_g = 0$, the discounted growth in the immediate value corresponding to the above equation simplifies to:

$$l(s) = B(s) - Q(s) + \rho Q(f(s)).$$

Our focus is on how process uncertainty affects the optimal policy in a case where an interior constant escapement rule is optimal. For this purpose, it is assumed that $g(s)$ and $l(s)$ are strictly concave and unimodal in $s$ over the possible range $[\ell, m]$ with $\ell < m$, for given $z_g$.[5] With this assumption, optimal policy is an interior constant escapement rule, i.e.,

$$h_t^* = \max(X_t - S^*, 0), \tag{4}$$

where $S^* \in (\ell, m)$ is an optimal target escapement. For obtaining more concrete conditions, further assumptions on the functional form of $B(s)$ must be imposed, which we will discuss

[5]The lower bound of $\ell$ depends on the type of problems we analyze. For example, in a fishery problem $\ell$ is typically defined as the zero profit stock level (see Reed (1979)), while $\ell$ could be simply set as $\ell = 0$ in the other class of problems such as pest controls.

more later on.

For the characterization of optimal escapement levels, we first let $\bar{s}$ denote the escapement level attaining the maximum of $l(s)$ in our deterministic model. In this setting, the interior constant escapement rule is optimal, and the optimal target escapement level is equal to $\bar{s}$. However, the optimal policy in the stochastic setting is rather complex. Let $\sigma \in (\ell, m)$ denote the escapement level attaining the maximum of $g(s)$. We call it a "short-sighted optimal escapement level." Reed (1979) finds that if $\sigma$ is self-sustaining, i.e., $zF(\sigma) \geq \sigma$ for all $z$ such that $\Phi(z) > 0$, then the interior constant escapement rule with target level $S^* = \sigma$ is optimal. If $\sigma$ is not self-sustaining, then the interior constant escapement rule is still optimal, but the target escapement level $S^*$ is larger than $\sigma$, i.e., $S^* > \sigma$. The short-sighted optimal escapement level $\sigma$ could be just considered a lower bound of the optimal target escapement level $S^*$ in the stochastic model.

To compare the short-sighted optimal escapement level $\sigma$ in the stochastic case with the optimal escapement level $\bar{s}$ in the deterministic case, we take the differentiation of $g(s) - h(s)$:

$$g'(s) - l'(s) = \rho f'(s) \left[ c(f(s)) - \int_{1-z_g}^{1+z_g} zc(zf(s)) d\Phi(z) \right]. \tag{5}$$

Applying Jensen's inequality, our model specification implies the following three cases depending on the property of $xc(x)$:

**Case** 1 $\sigma < \bar{s}$ if $xc(x)$ is strictly convex;

**Case** 2 $\sigma = \bar{s}$ if $xc(x)$ is linear;

**Case** 3 $\sigma > \bar{s}$ if $xc(x)$ is strictly concave.

Notice that case 1 is possible only when the unit cost $c(x)$ is strictly convex. As mentioned in Reed (1979), case 1 is exactly a class of unresolved problems in that the optimal policy is not fully explored when process uncertainty increases. Thus, to fill the gap, the following subsection focuses on examining such a case and demonstrates that the optimal escapement level would be in a non-monotonic U-shaped manner.

### 3.1 Strictly Convex Unit Harvest Costs

In order for case 1 to hold, the unit cost function must satisfy the condition that $xc(x)$ is strictly convex, i.e., $2c'(x)+xc''(x) > 0$ for all $x \in [\ell, m]$. The assumption that $c(x)$ is strictly decreasing and strictly convex is not sufficient to guarantee that case 1 holds. However, the specification that have been extensively used in renewable resource management implies that case 1 could be plausible. For example, consider the following unit cost function:

$$c(x) = \frac{k}{bx^{\theta}}, \tag{6}$$

where $k$ is the cost per unit of effort, and $b$ is a parameter to be adjusted for stock units. The parameter $\theta$ could be interpreted as the sensitivity of catchability (see Kotani et al. (2006) and Kotani et al. (2007)).[6] This specification is employed in a series of literature including Reed (1979), Clark (1990), Moxnes (2003), and others. Case 1 can apply if the sensitivity of catchability is larger than unity ($\theta > 1$) so that $xc(x)$ is strictly convex. To focus on case 1, we assume $\theta > 1$ in the rest of the paper.

One question now arises: how likely is the sensitivity of catchability larger than unity? The justification for this is first provided by Clark (1990) for fishery problems. As of other renewable resource management, the invasive species problem is a noticeable example as noted in the introduction, in which the sensitivity of catchability, $\theta$, could be very large.

### 3.2 Illustration

This subsection illustrates how process uncertainty affects the optimal escapement level in the two distinct problems, fishery management and invasive species management, where the unit cost function takes the form of equation (6) with $\theta > 1$. By inspection of equation

[6]The stock-dependent catchability is specified as $q(x) = bx^{\theta-1}$. The catchability represents the extent to which the stock size is susceptible to one unit of effort for harvesting. Catch per unit of effort, or the stock size that can be harvested by one unit of effort, is given by $p(x) = xq(x)$. Assuming that the cost per unit of effort is constant at $k$, we write the unit harvest cost as $c(x) = k/p(x)$ (see, e.g., Clark (1990)). If the catchability is strictly increasing in $x$, i.e., $\theta > 1$, then both catch per unit effort and the unit harvest cost are strictly convex.

(5), the short-sighted optimal escapement $\sigma \equiv \sigma(z_g)$ is monotone decreasing in the degree of process uncertainty, $z_g$, due to the property of second order stochastic dominance. Since $\sigma$ is the optimal escapement level as long as it is self-sustaining, an increase in $z_g$ causes the optimal escapement level to decline.

It should also be noted that as $z_g$ increases, the region of the escapement levels that meet the self-sustaining property shrinks. If $z_g$ becomes large enough, the short-sighted optimal escapement $\sigma$ is not the optimal escapement level any longer and just indicates its lower bound. Combining these two facts, we can derive the following fact: There exists a critical level of the degree of process uncertainty, $z_g^* > 0$, such that if the degree of process uncertainty is relatively small with $z_g < z_g^*$, then $\sigma$ is self-sustaining, and hence it is the optimal escapement level. Otherwise, $\sigma$ is not self-sustaining, and it is nothing but a lower bound for optimal escapement levels.

Unfortunately, when $z_g$ is large enough that $\sigma$ is not self-sustaining, analytical derivation for identifying the optimal escapement level $S^*$ is impossible or unresolved as mentioned in Reed (1979). At this point, what we have in advance as information about $S^*$ is nothing but a lower bound of $\sigma$ that is monotone decreasing in $z_g$. From now on, we attempt to illustrate that in contrast to the case where $\sigma$ is self-sustaining, the optimal escapement level is increasing in $z_g$ when $z_g$ is large enough such that $\sigma$ is not self-sustaining, i.e., $z_g > z_g^*$. Thus, the optimal escapement level non-monotonically responds to an increase in the degree of process uncertainty, i.e., $S^*$ is decreasing in $z_g$ when $z_g$ is relatively small, but $S^*$ is increasing in $z_g$ when $z_g$ is relatively large. This feature is first noted in the discussion on the invasive species management in Kotani et al. (2007).

For the purpose of showing a non-monotonic, U-shaped property of optimal escapement levels for general renewable resource management, we consider two different types of problems: (i) fishery management; and (ii) invasive species management, by numerically solving for the optimal policy in a class of analytically intractable problems. In terms of fishery problems, Reed (1979) argues that an interior constant escapement rule can be optimal under process uncertainty irrespective of whether the sensitivity of catchability is larger or smaller than unity. He also claims that its escapement level under process uncertainty is larger than the deterministic one in most plausible cases of $\theta < 1$. Contrary to fishery problems, Kotani et al. (2007) show that in the invasive species management, an interior constant escapement rule can be optimal only when the sensitivity of catchability, $\theta$, is sufficiently large. Furthermore, if $\theta$ is larger than unity, then it is highly likely that an interior constant escapement rule is optimal.

In illustration of using these two problems, we impose some common functions and parameters for simplicity: $\rho = 0.95$ for the discount factor; the Bevertion-Holt reproduction function $f(s) = As/(1 + Bs)$ with $A = 10$ and $B = 1$; and the unit cost function $c(x) = k/(bx^{\theta})$ with $b = 1$ and $\theta = 1.1$.[7]

**Fishery Management**

Reed (1979) derives a set of conditions that must be satisfied for an interior constant escapement rule to be optimal. The parameters and functional forms we employ in this illustration are aligned to satisfy the conditions. We take $k = 1$ as the cost per unit of effort and set $B(s_t) = ph_t = p(x_t - s_t)$ as the social benefit or the profit in each period, where $h_t$ and $p = 1$ are respectively harvest and the price per harvest in that period.

In the deterministic setting of $z_g = 0$, the constant escapement rule with target $S^* \approx 2.5985$ is optimal. Figure 1 presents that the short-sighted optimal escapement level, $\sigma$ is monotone decreasing in the degree of process uncertainty, $z_g$, as in our analytical result. More importantly, Figure 2 exhibits that the optimal escapement level, $S^*$, is non-monotonic and inverse unimodal in $z_g$, and the critical level of the degree of process uncertainty is approximately $z_g^* \approx 0.61$. For $z_g > z_g^* \approx 0.61$, $S^*$ is increasing in $z_g$, while $S^*$ is decreasing in $z_g$ and is identical to $\sigma(z_g)$ for $z_g < z_g^*$. Overall, the optimal escapement level $S^*$ changes in a U-shaped manner as $z_g$ changes.

[7]We conduct sensitivity analysis by supposing other functional forms and parameter sets in current rewards and stock reproduction such as Riker or logistic. All of the results are qualitatively the same as the one presented in what follows.

**Invasive Species Management**

In terms of invasive species management, Kotani et al. (2007) derives a set of the conditions that must be satisfied for an interior constant escapement rule to be optimal. Similar to the previous arguments, the parameters and the functional form are taken to satisfy the conditions. We take $k = 50$ as the cost per unit of effort and set $B(s_t) = -a_1 s_t - a_2 s_t^2/2$ as the social damage that emanates from invasive species with $a_1 = 1$ and $a_2 = 2$.

In the deterministic setting of $z_g = 0$, the constant escapement rule with target $S^* \approx 3.9885$ is optimal. Figure 3 presents that $\sigma$ is decreasing in $z_g$, and Figure 4 shows that $S^*$ is non-monotonic and inverse unimodal in $z_g$. Notice that the qualitative features with respect to $\sigma$ and $S^*$ are the same as in the previous example of fishery management. The critical common feature in the two examples is a strictly convex unit harvest cost $c(x)$ associated with $\theta > 1$.

# 4 Discussion and Conclusion

The intuitions for the non-monotone property in optimal escapement levels could be explained as follows. When the degree of process uncertainty, $z_g$, is sufficiently small, there is not much worry or risk about the decline in the next period stock so that harvesting can be more focused upon improving the current reward. Since the unit harvest cost is convex, such small uncertainty works in the positive direction for the reward obtained from harvesting in the current period. That is, the optimal escapement level is decreasing in the degree of process uncertainty as long as the degree is sufficiently small.

On the other hand, once the degree of process uncertainty becomes larger over a certain level, the harvesting activities must be restrained. Since an increase in $z_g$ implies mean-preserving spread of process uncertainty, the support that can be taken in the next period will expand. In other words, the next period stock is more likely to decline so that utilizing much renewable resource in the current period yields high risk of the future value. This negative effect of large uncertainty in the future value could dominate the positive effect of uncertainty in the current reward. That is, the optimal escapement level is increasing in the degree of process uncertainty when the degree is sufficiently large.

Even though we do not delve into the concrete conditions for an interior constant escapement rule to be optimal, they are shown in Reed (1979) for fishery as well as in Kotani et al. (2007) for invasive species problems. Interestingly enough, the adaptation of the optimal escapement levels under process uncertainty are qualitatively the same in both profit maximization of fishery and damage minimization of invasive species problems. The remaining topic may be the problems of conflicts between wild animals and humans, such as the one analyzed by Rondeau and Conrad (2003), in which the conditions for an interior constant escapement rule to be optimal are not derived in the discrete time setting yet.

We are hopeful that the result would help on the management-decision under uncertainty and raise some caution in the case of convex unit harvest cost, which could be plausible especially when the invasive species or some nuisances must be culled to control social welfare.

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Figure 1: $\sigma$ as a function of $z_g$ in fishery problems

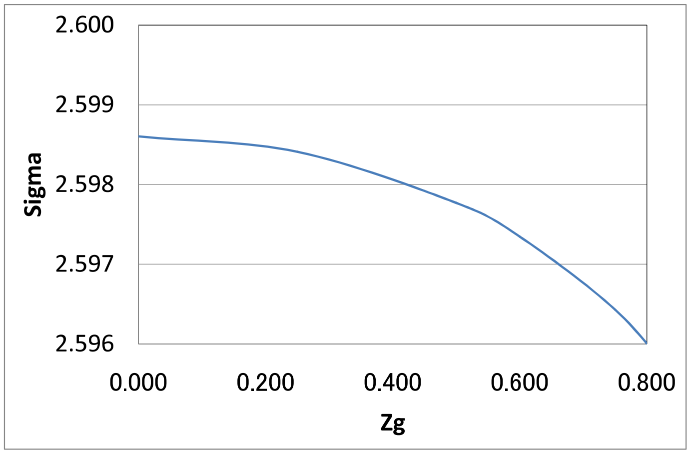


Figure 2: $S^*$ as a function of $z_g$ in fishery problems

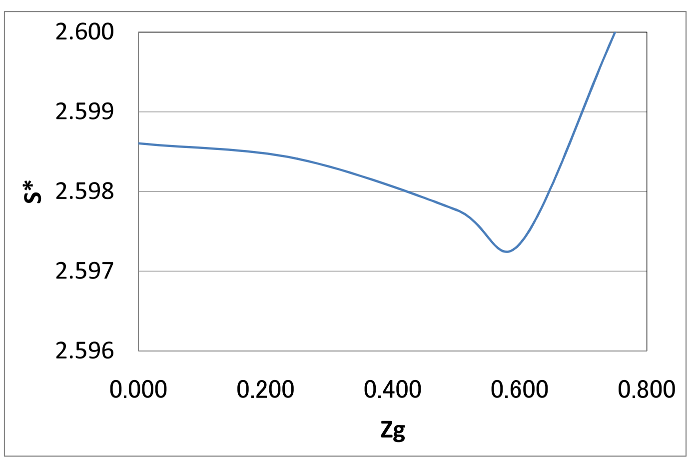

Figure 3: $\sigma$ as a function of $z_g$ in invasive species problems

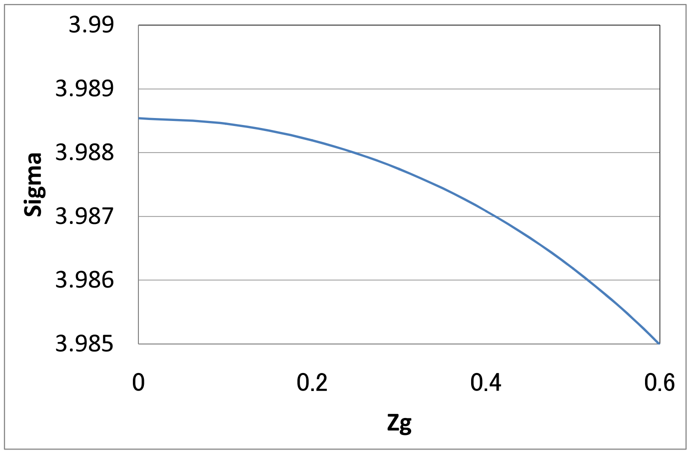


Figure 4: $S^*$ as a function of $z_g$ in invasive species problems