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Abstract

This paper re-visits effects of fiscal expansion on employment and unemployment by focusing on both hiring and firing margins. We develop a dynamic stochastic general equilibrium model with labor search frictions in which job separation is endogenously determined. We study effects of fiscal stimuli in the form of government spending and hiring subsidies. The prediction of our model is in contrast with earlier studies that assume exogenous job separation. First, our model generates a larger size of the impact of a government spending shock on labor market variables than the model without endogenous separation. Second, while an increase in hiring subsidies increases employment and reduces unemployment in the model without endogenous separation, it reduces employment and increases unemployment in our model.

JEL Classification: E24; E62; J64

Keywords: Fiscal Policy; Unemployment; Labor market; Search and matching; Endogenous separation

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1 Introduction

Recently, a number of studies examine effects of fiscal expansion on unemployment in dynamic stochastic general equilibrium (DSGE) models (Yuan and Li, 2000; Monacelli et al., 2010; Campolmi et al., 2011; Brükner and Pappa, 2012). They incorporate labor market search and matching into an otherwise standard DSGE model and study effects of fiscal stimuli in the form of government spending and hiring subsidy on the economy. One of common features of these studies is exogenous job separation. They assume that while a worker' transition rate from unemployment to employment is endogenously determined through a matching market, employed workers lose their jobs due to exogenous separation shocks and thus a transition rate from unemployment to employment is exogenous. However, recent empirical studies demonstrate that unemployment dynamics is determined by both inflow and outflow rates of unemployment, and find large variations in both inflow and outflow rates over business cycles (Elsby et al., 2009; Fujita and Ramey, 2009).[1] This suggests that in order to study effects of fiscal policies on unemployment, it is necessary to use a model in which worker's transitions between employment and unemployment are endogenously determined.

This paper re-visits the effects of fiscal policies on unemployment and employment by focusing on both hiring and firing margins. We develop a DSGE model with search frictions in which workers' transition between employment and unemployment are endogenously determined. In the model, jobs differ by idiosyncratic operating cost. When the cost is too high, production is not profitable. A firm-worker pair then chooses to destroy the job. Thus, in our model job separation is endogenously determined, while in models in existing studies, the separation rate is exogenous and assumed to be constant over time. By using the extended model, we study the effects of fiscal stimuli in form of traditional government spending and hiring subsidies on the labor market.[2]

The prediction of our model is in contrast with earlier studies that assume exogenous job separation. While both models with and without endogenous separation generate similar pattern of responses of unemployment and employment to a positive government spending shock, the model with endogenous separation generates a larger impact of the shock on these labor market variables than the model without endogenous separation. This difference can be explained

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[1]Recently, several studies investigate the contribution of inflow and outflows rates to the unemployment variability over the business cycle. For the United States, Hall (2005) and Shimer (2012) claim that the outflow rate dominates and the inflow rate is acyclical. In contrast, Elsby et al. (2009) and Fujita and Ramey (2009) find a greater role for inflow rates that account for around half of cyclical changes in unemployment. For European countries, Petrongolo and Pissarides, (2008) and Elsby et al. (2009) find approximately a 50:50 inflow/outflow split to unemployment variation.

[2]Note that Campolmi et al. (2011) is a complementary contribution which studies the effect of fiscal stimuli in the form of hiring subsidies on the labor market in a New Keynesian model with matching frictions and exogenous job separation.

as follows. In the model with endogenous separation, a positive government spending shock substantially reduces job separation, leading to more employment and less unemployment.

The most striking finding is that the effect of increases in hiring subsidies on unemployment and employment differs between models with and without endogenous separation. While a positive hiring subsidy shock increases employment and reduces unemployment in the model without endogenous separation, it reduces employment and increases unemployment in our model. Furthermore, in our model, a traditional government spending policy delivers larger multipliers compared to a hiring subsidy policy. This result is in contrast with those of earlier studies that employ exogenous separation.[3]

These differences are results of incorporating endogenous job separation into the model. The incorporation of endogenous separation gives rise to a new channel through which fiscal policies affect employment and unemployment: an increased separation rate. When a hiring subsidy increases, the cost of positing vacancies falls. A decrease in vacancy cost has two counteracting effects on employment. On one hand, it increases firms' incentive to post vacancies, leading to higher employment and lower unemployment. On the other hand, it induces more separation by reducing the opportunity cost of continuing existing matches. Under the plausible parameter values, the latter effect dominates the former one. As a result, a positive hiring subsidy shock reduces employment and increase unemployment.

Our work is related to a number of recent papers that study effects of fiscal policies on the labor market. Yuan and Li (2000) and Monacelli et al. (2010) study effect of government spending shock on the U.S. labor market by incorporating search frictions into a standard RBC model. Brükner and Pappa (2012) develop a New Keynesian model with search frictions and workers' participation choices, and examine the effects of fiscal stimuli in the form of government spending. None of these papers considers hiring subsidy and endogenous job separation. Campolmi et al. (2011) explore the effectiveness of fiscal stimuli in the form of government spending and hiring subsidy in a New Keynesian model with search frictions and endogenous participation. While they assume that job separation is exogenous, our paper considers endogenous job separation.

The remainder of the paper is organized as follows. Section 2 describes the theoretical model. We develop a DSGE model with labor market frictions and endogenous job separation. In Section 3, we calibrate the model parameters and present the quantitative results of effects of fiscal expansion on the economy. Section 4 discusses the sensitivity of the numerical results to our choice of parameters. Section 5 concludes.

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[3]See, for example, Campolmi et al. (2011).

2 The model

We consider a stochastic dynamic general equilibrium model with search frictions in the labor market.[4] In our model, job separation is endogenously determined. Jobs differ by idiosyncratic cost of a non-productive intermediate input (an operating cost). When the cost is too high, production is not profitable. Thus, a firm-worker pair chooses a reservation cost and destroys the job whose cost rises above it.[5] We consider fiscal stimuli in the form of government spending and hiring subsidy.

An economy consists of households, firms, and the government. Each household consists of a continuum of infinitely-lived workers normalized to one. They search for jobs when unemployed, while they supply labor services and earn wages when employed. Firms hire workers in a frictional labor market and produce outputs by using capital and labor. Firms sell their products to households in a competitive market. Employment is the outcome of workers' and firms' search behavior, while wages and labor supply are outcome of a bargaining process. Time is discrete.

Household's problem   A representative household consists of a continuum of individuals of mass one. A member of the household is either employed or unemployed. In the period t, a fraction n_t of the household's members are employed and, a fraction of u_t are employed. The household pools incomes and shares total income and risk among all family members as in Merz (1995).

The household's expected life time utility is given by

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_0 n_t \frac{h_t^{1+\mu}}{1+\mu} \right] \right\}, \tag{1}$$

where $\beta \in (0,1)$ is the household's subjective discount factor, C_t is aggregate consumption of the household, and h_t is the individual hours worked and $\Phi_0 > 0$ measures the disutility of working, μ is the inverse of the Frisch elasticity of labor supply, and $1/\sigma$ is the intertemporal elasticity of substitution.

Employed household members earn wage and unemployed household members receive unemployment benefits z. The household receives profits from firms Π and pays lump sum taxes

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[4]Andolfatto (1996) and Merz (1995) consider a stochastic real business cycle model with search frictions and exogenous job separation. Den Hann et al. (2000) develop a dynamic general equilibrium model with search frictions and endogenous job separation and study propagation of aggregate shocks. While they focus on cyclical behavior of labor market variables, this paper studies effects of fiscal stimuli on the labor market.

[5]In the most widely used endogenous separation model developed by Mortensen and Pissarides (1994), jobs differ by match-specific idiosyncratic productivity and job separation takes place based on the level of the idiosyncratic productivity. In contrast, in our model, jobs are characterized by match-specific operating costs and job separation decisions are based on the level of operating costs.

to the government τ. The household may either consume C or accumulate capital K through investment I according to $K_{t+1} = (1 - \delta) K_t + I_t$, where δ is the depreciation rate.

The budget constraint of the representative household is

$$C_t + K_{t+1} + \tau_t = \bar{W}_t + (1 - n_t) z + (1 - \delta) K_t + r_t K_t + \Pi_t,$$

where r_t denotes the real rental rate of capital and $\bar{W}$ is the total wages paid to the workers of the household.

The household chooses C_t and K_{t+1} to maximize (1) subject to the budget constraint. Let λ_t be the Lagrange multiplier on the budget constraint. Then, the household's problem yields following first-order conditions:

$$C_t^{-\sigma} = \lambda_t,$$

and

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} (1 - \delta + r_{t+1}).$$

These first-order conditions yield the following Euler equation:

$$C_t^{-\sigma} = \beta \mathbb{E}_t C_{t+1}^{-\sigma} (1 - \delta + r_{t+1}).$$

Labor market The labor market is subject to frictions and firms and workers cannot meet instantaneously but must go through a time-consuming search process. The labor market is modeled in the style of a search and matching model developed by Mortensen and Pissarides (1994). The number of successful job matches is determined by the matching function $m = m(u_t, v_t)$. The matching function is continuous, twice differentiable, increasing in its arguments, and exhibits constant returns to scale. The probability of a firm with a vacancy is matched with a worker is given by $m(u_t/v_t, 1) \equiv q(\theta_t)$, where $\theta_t \equiv v_t/u_t$ is labor market tightness. Then, the probability that an unemployed worker is matched with a firm with a vacant job is $m(1, v_t/u_t) = \theta_t q(\theta_t) = p(\theta_t)$. Note that both firms and workers take q_t and p_t as given.

Firm's problem Production takes place when one firm is matched with one worker. When a firm hires a worker, the firm produces output according to a constant returns to scale production function $y_t = A_t f(k_t, h_t)$, where A_t is an aggregate productivity common to all firms, k_t is capital per worker, and h_t is hours worked per worker.

We assume that in order to produce output, a firm-worker pair needs to pay an operating cost x_t besides labor and capital costs. The operating cost is idiosyncratic to each match. The match-specific operating cost x_t is assumed to be independent and identically distributed across firms and time, with a cumulative distribution function $G : [\underline{x}, \bar{x}] \to [0, 1]$. Every period an existing match draws a new idiosyncratic cost and decides whether producing output at the new level of cost or terminating the employment relationship. Each match chooses a reservation

5

value $\tilde{x}$; if the match-specific cost falls below $\tilde{x}$, they continue producing output. The reservation value is chosen so as to maximize the match's present value.

Match separation occurs as the results of one of two distinct events. First, a job can be terminated by an exogenous shock that occurs with probability s. Second, an idiosyncratic shock arrives and a job becomes no longer profitable, the firm chooses to close down the job. When job separation takes place, the firm can either reopen a job as a new vacancy or withdraw from the labor market, while the worker becomes unemployed.[6]

Timing of the model   The timing of the model is as follows. At the beginning of each period, every firm with a filled job draws an idiosyncratic cost and determines the reservation value. After endogenous separation takes place, the levels of employment and unemployment are determined. At the point, matched firms start production and unemployed workers search for jobs. At the end of the period, wages are paid and the firm's profits are distributed to the households, and households make consumption decision. Finally, some matches are destroyed due to the exogenous separation shock.

Value functions   The problems of firms and workers are characterized by the Bellman equations. Let $\mathcal{J}_t(x_t)$ denote the asset value for a firm with a filled job with an idiosyncratic operating cost x. The value of a filled job with an idiosyncratic operating cost x satisfies

$$
\begin{aligned}
\mathcal{J}_t(x_t) \;=\; & \max_{k_t} \big\{ A_t f(k_t, h_t) - x_t - w_t(x_t)h_t - r_t k_t \\
& + \mathbb{E}_t \beta_t \left[(1-s) \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{J}_{t+1}(x_{t+1}) dG(x_{t+1}) + \left[1 - (1-s)\, G(\tilde{x}_{t+1}) \right] \mathcal{V}_{t+1} \right] \big\}, \quad (2)
\end{aligned}
$$

where $\beta_t = \beta \lambda_{t+1}/\lambda_t$ is the stochastic discount factor, $w_t(x_t)$ is wage paid to the employee, and $\mathcal{V}$ is the value of a firm with a vacant job. The value $\mathcal{J}_t(x_t)$ is determined by several factors. During the current period, a firm with a filled job produces $A_t f(k_t, h_t)$, and pays wages $w_t(x_t)h_t$, the rental cost of capital $r_t k_t$, and the operating cost x_t. In the following period, if the match is not destroyed by an exogenous shock, and if the idiosyncratic cost is below the reservation value $\tilde{x}_{t+1}$, the match continues and obtains $\mathcal{J}_{t+1}(x_{t+1})$, otherwise the match is destroyed and the firm gets the value of posting a vacancy $\mathcal{V}_{t+1}$.

The first-order condition for the capital is

$$
A_t f_k(k_t, h_t) = r_t.
$$

This implies that the optimal capital is chosen to equate the marginal product of capital to the rental rate.

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[6]Note that in equilibrium firms are indifferent between these two options due to free entry.

The value of a firm with a vacant job is

$$\mathcal{V}_t = -(1 - \tau_t^v)\gamma + \mathbb{E}_t \beta_t \left[q_t \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{J}_{t+1}(x_{t+1}) dG(x_{t+1}) + [1 - q_t G(\tilde{x}_{t+1})] \mathcal{V}_{t+1} \right], \qquad (3)$$

where γ is a flow cost of posting a vacancy and τ_t^v is a subsidy to the cost of posting the vacancy.[7]

In equilibrium, all profit opportunities from new jobs are exploited, so that the following free entry condition holds:

$$\mathcal{V}_t = 0.$$

We now turn to the worker's side. The value of an employed worker in a job with idiosyncratic cost x, $\mathcal{W}_t(x_t)$, is characterized by the following Bellman equation:

$$\begin{aligned} \mathcal{W}_t(x_t) &= w_t(x_t)h_t - \frac{\Phi(h_t)}{\lambda_t} \\ &+ \mathbb{E}_t \beta_t \left[(1 - s) \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{W}_{t+1}(x_{t+1}) dG(x_{t+1}) + [1 - (1 - s) G(\tilde{x}_{t+1})] \mathcal{U}_{t+1} \right], \end{aligned} \qquad (4)$$

where $\mathcal{U}$ is the value of an unemployed worker and $\Phi(h_t)/\lambda_t = \Phi_0 h_t^{1+\mu}/\lambda_t(1+\mu)$ is the disutility from supplying labor in terms of consumption. The value of an employed worker is composed of the wage, the disutility from supplying labor, and the continuation value, which is the value of being employed if the match is not destroyed, or the value of being unemployed if it is destroyed.

The value of an unemployed worker is

$$\mathcal{U}_t = z + \mathbb{E}_t \beta_t \left[p_t \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{W}_{t+1}(x_{t+1}) dG(x_{t+1}) + [1 - p_t G(\tilde{x}_{t+1})] \mathcal{U}_{t+1} \right]. \qquad (5)$$

An unemployed worker receives the unemployment benefit z, and matched with a firm with a vacant job with probability p_t. If the idiosyncratic cost for the match is below the reservation value $\tilde{x}_{t+1}$, the worker will be employed in the following period and obtain the value of being employed; otherwise, she remains unemployed and obtains the value of being unemployed.

Wage determination and hours choice   Wages and hours worked are determined as the outcome of a bilateral bargaining process between workers and firms. In each period, firms and workers negotiate through Nash bargains. Thus, wage and hours worked are chosen to maximizes the Nash product

$$\max_{w_t(x_t), h_t} (\mathcal{W}_t(x_t) - \mathcal{U}_t)^\eta (\mathcal{J}_t(x_t) - \mathcal{V}_t)^{1-\eta},$$

---

[7]As Campolmi et al. (2011) suggested, fiscal stimuli have taken various forms in practice. In the aftermath of the 2007-2008 crisis, expansionary fiscal package implemented in various countries were largely devoted to facilitate job creation. The American Jobs Act passed by the Obama administration is an example. Following Campolmi et al. (2011), we incorporate subsidies to the cost of positing vacancies in order to study effects of fiscal policies targeted particularly at the labor market.

where $\eta \in (0,1)$ is a worker's bargaining power.

Taking first-order conditions with respect to $w(x)$ and h, we have the wage equation

$$w_t(x_t)h_t = \eta \left[A_t f(k_t, h_t) - x_t - r_t k_t + (1 - \tau_t^v)\gamma\theta_t \right] + (1 - \eta)\left(\frac{\Phi(h_t)}{\lambda_t} + z \right),$$

and the hours supply equation

$$A_t f_h(k_t, h_t) = \frac{\Phi'(h_t)}{\lambda_t}.$$

The wage equation is similar to the one in the search and matching literature.[8] The wage is a weighted average of the marginal revenue product and the cost of replacing the worker, and of the outside option of the worker, which consists of unemployment benefits and the marginal disutility of labor.

The hours supply equation states that hours worked is determined by equalizing the marginal product of hours and the worker's marginal rate of substitution between leisure and consumption.

Job creation and separation By using the free entry condition, (3) can be rewritten as

$$(1 - \tau_t^v)\gamma = \mathbb{E}_t \beta_t q_t \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{J}_{t+1}(x_{t+1}) dG(x_{t+1}).$$

By using this, the value function of can be rewritten as

$$\mathcal{J}_t(x_t) = A_t f(k_t, h_t) - x_t - w_t(x_t)h_t - r_t k_t + (1 - s)(1 - \tau_t^v)\frac{\gamma}{q_t}.$$

We then have the following job creation condition

$$
\begin{aligned}
(1 - \tau_t^v)\frac{\gamma}{q_t} = \; & \mathbb{E}_t \beta_t \int_{\underline{x}}^{\tilde{x}_{t+1}} \left[A_{t+1}f(k_{t+1}, h_{t+1}) - x_{t+1} - w_{t+1}(x_{t+1})h_{t+1} - r_{t+1}k_{t+1} \right. \\
& \left. + (1 - s)(1 - \tau_{t+1}^v)\gamma/q_{t+1} \right] dG(x_{t+1}).
\end{aligned}
\tag{6}
$$

The job creation condition states that expected cost of positing a vacancy, the left-hand side of (6), is equal to the firm's share of the expected new surplus from a new job match, the right-hand side of (6).

A match is destroyed when the idiosyncratic cost is so high that it makes the match surplus to zero. Let $\mathcal{S}(x)$ be the joint gross return from a match with idiosyncratic cost x. Then, the match surplus function is given by

$$\mathcal{S}_t(x_t) = \mathcal{J}_t(x_t) + \mathcal{W}_t(x_t) - \mathcal{U}_t - \mathcal{V}_t. \tag{7}$$

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[8]See, for example, Mortensen and Pissarides (1994) and Pissarides (2000).

Using equations (2), (4), (5), and (7) with the free entry condition, we obtain

$$
\begin{aligned}
\mathcal{S}_t(x_t) &= A_t f(k_t, h_t) - x_t - r_t k_t - \frac{\Phi(h_t)}{\lambda_t} - z \\
&\quad + \mathbb{E}_t \beta_t \left(1 - s + p_t \eta\right) \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{S}_{t+1}(x_{t+1}) dG(x_{t+1})
\end{aligned}
\tag{8}
$$

Since the surplus function $\mathcal{S}_t(x_t)$ is strictly decreasing in x_t, the firm and the worker choose a reservation policy, i.e., they will continue their match if $\mathcal{S}_t(x_t) \geq 0$ but stop if $\mathcal{S}_t(x_t) < 0$. Thus, separation takes place at $x_t \leq \tilde{x}_t$, where $\tilde{x}_t$ is defined by $\mathcal{S}_t(\tilde{x}_t) = 0$. Note that the reservation productivity at the time the match is formed is the same as the one at match dissolution.

Evaluating (8) at $x_t = \tilde{x}_t$, we obtain the expression for the reservation threshold:

$$
A_t f(k_t, h_t) - \tilde{x}_t - r_t k_t + \mathbb{E}_t \beta_t \left(1 - s + p_t \eta\right) \int_{\underline{x}}^{\tilde{x}_{t+1}} \mathcal{S}_{t+1}(x_{t+1}) dG(x_{t+1}) = z + \frac{\Phi(h_t)}{\lambda_t}
\tag{9}
$$

We refer to this as the job destruction condition. The left-hand side of (9) is the marginal value of job continuation under the reservation value $\tilde{x}$. The first three terms represent the current productivity gain, and the fourth term is the option value of retaining an existing job. On the right-hand side of (9) is the marginal value of destruction (or the marginal opportunity cost of continuation) of a job. The job destruction condition says that the optimal reservation value $\tilde{x}$ should be set so as to equalize marginal benefit of continuation and destruction of the job.

Government policy and resource constraint   The government has to finance government spending G_t, unemployment benefits $u_t z$, and the subsidy to the cost of posting the vacancy $\tau_t^v \gamma v_t$ by imposing the lump-sum tax τ_t to households. The government budget constraint is thus given by

$$
\tau_t = G_t + u_t z + \tau_t^v \gamma v_t.
$$

The government spending, G_t, follow the exogenous stochastic processes:

$$
\log(G_t) = (1 - \rho_G) \log(G^*) + \rho_G \log(G_{t-1}) + \varepsilon_{G,t}
$$

where $\varepsilon_{G,t}$ i.i.d. and G^* denotes the steady-state share of government spending.

Similarly, a hiring subsidy takes the following form:

$$
\log(\tau_t^v) = (1 - \rho_{\tau^v}) \log(\tau^{v*}) + \rho_{\tau^v} \log(\tau_{t-1}^v) + \varepsilon_{\tau^v,t}
$$

where $\varepsilon_{\tau^v,t}$ i.i.d. and τ^{v*} denotes the steady-state share of government spending.

Aggregate output and capital are obtained by

$$
Y_t = n_t y_t \text{ and } K_t = n_t k_t,
$$

respectively.

Total profit of firms in the economy is

$$\Pi_t = [A_t f(k_t, h_t) - \bar{x}_t - \bar{w}_t h_t - r_t k_t] n_t - (1 - \tau_t^v) \gamma v_t,$$

where

$$\bar{w}_t = \frac{1}{G(\tilde{x}_t)} \int_{\underline{x}}^{\tilde{x}_t} w_t(x_t) dG(x) \text{ and } \bar{x}_t = \frac{1}{G(\tilde{x}_t)} \int_{\underline{x}}^{\tilde{x}_t} x dG(x).$$

By combining the household and government budget constraint, we can obtain the aggregate resource constraint. Aggregate production must equal private and public demand:

$$C_t + K_{t+1} - (1 - \delta) K_t + G_t + \gamma v_t + \bar{x}_t n_t = Y_t.$$

Labor market dynamics The evolution of employed workers, defined as $n_t = 1 - u_t$, is given by

$$n_t = m_{t-1} G(\tilde{x}_t) + (1 - s) G(\tilde{x}_t) n_{t-1}.$$

Note that the job finding rate and the separation rate are given by $p_{t-1} G(\tilde{x}_t)$ and $1 - (1 - s) G(\tilde{x}_t)$, respectively.

3 Quantitative analysis

In this section, we simulate a quantitative version of the model. We first calibrate the model to match several dimensions of the U.S. data. We then solve the model by approximating the equilibrium conditions around a non-stochastic steady state and simulate it.

3.1 Basic calibration

In order to study effects of fiscal stimulus on labor market variables, we calibrate the model to match certain U.S. economy facts. We choose one quarter as the length of a model period. We set the discount factor $\beta = 0.99$ to match the annual real interest rate of approximately 4 percent. The relative risk aversion parameter σ is set to 2. We normalize the level of Φ_0 in the disutility of labor to unity without loss of generality. The elasticity of intertemporal substitution in the hours supply given by $1/\mu$, and the value of this elasticity has been the subject of some discussion. As our baseline, we choose a quadratic disutility, $\mu = 2$.

We assume that the matching function is Cobb-Douglas,

$$m(u_t, v_t) = m_0 u_t^{\xi} v_t^{1-\xi},$$

where m_0 is the matching constant and ξ is the matching elasticity with respect to unemployment. The elasticity parameter ξ is set to 0.5, as suggested by the estimates in Petrongolo and Pissarides (2001). We use the Hosios (1990) condition to pin down the worker's bargaining power, so $\eta = \xi$.

We target a mean value of the vacancy–unemployment ratio of 0.72, which is reported by Pissarides (2009).[9] Monthly transitions data from Shimer (2005) gives a mean value of 0.594 for the job finding rate and 0.036 for the job separation rate between 1960 and 2004. In order to pin down the scale parameter m_0, we combine the monthly job finding rate with the vacancy-unemployment ratio.

Silva and Toledo (2009) use evidence provided by Davis, Faberman, and Haltiwanger (2006) and Nagypál (2004) to determine the exogenous and endogenous components of the separation rate. They assume that endogenous job separation accounts for, on average, 35% of total separations. Since I target a total separation rate of 0.036, I set the quarterly exogenous separation rate at $s = 0.0702$.

Following Mortensen and Pissarides (1994), I assume that the idiosyncratic cost distribution G is uniform in the range $[0, \zeta]$, so that $G(x) = x/\zeta$. Following Pissarides (2007) and Elsby and Michaels (2008), the parameter ζ is chosen to match the monthly endogenous job separation rate.

The production function is specified by $y = k^\alpha h^{1-\alpha}$, and we set the capital share $\alpha = 1/3$. The standard annual capital depreciation rate of 10% corresponds to a value of δ to 0.025 per quarter. Following Shimer (2005), the vacancy cost γ is obtained from the steady-state solutions of the model.

We target the unemployment benefits z to be 40% of the average wage of employed workers in the economy.[10] Following Campolmi et al. (2011), we set the steady-state value for government spending to output ration $G/Y = 0.15$ and the steady-state level of the hiring subsidy $\tau^{v*} = 0.01$. The autocorrelations of government spending ρ_G and of the hiring subsidy ρ_{τ^v} are set to 0.9. The parameter values are summarized in Table 1.

Selected endogenous variables in the steady-state under the calibrated parameter values are reported in Table 2. Labor market tightness, the job-finding rate, the separation rate are equal to their target values.

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[9]The sample mean for the vacancy-unemployment ratio in 1960-2006 is derived by using JOLTS data since December 2000 and the Help-Wanted Index adjusted to the JOLTS units of measurement before then.

[10]This parameter has been the subject of some discussion. Shimer (2005) sets $z/\bar{w} = 0.4$, where $\bar{w}$ is the average worker's wage, in order to capture the unemployment benefits. Hagedorn and Manovskii (2008) argue that Shimer's choice of the value of the opportunity cost of employment is too low because it does not allow for the value of leisure, home production, or unemployment benefits. They calibrate the opportunity cost of employment and the worker's bargaining power to match the observed cyclical response of wages and average profit rate. Their results are $z/\bar{w} = 0.955$ and $\eta = 0.052$. Mortensen and Nagypál (2007) criticize Hagedorn and Manovskii (2008) for using these parameters because these parameters yield workers a gain of 2.8% inflow utility by going from unemployment to employment. Hall and Milgrom (2008) use the empirical literature on household consumption and labor supply and estimate the value of $z/\bar{w} = 0.71$.

Table 1: Parameter values

Parameter	Description	Value	Source/Target
β	Discount factor	0.99	Data
δ	Depreciation rate	0.025	Data
α	Parameter in production function	0.333	Data
m_0	Matching efficiency	2.189	Job-finding rate
ξ	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
s	Exogenous separation rate	0.07	65% of total separations
ζ	The upper support of G	0.564	35% of total separations
σ	Relative risk aversion parameter	2.0	See text
Φ	Disutility of labor	1.0	Normalization
μ	Frisch elasticity	2.0	See text
z	Unemployment benefits	0.795	Replacement rate 40%
η	Worker's bargaining power	0.5	$\eta = \xi$ (efficiency condition)
γ	Vacancy cost	0.335	$v - u$ ratio
τ^{v*}	Hiring subsidy rate	0.01	See text
ρ_G	Gov. spending autoregressive parameter	0.9	See text
ρ_{τ^v}	Hiring subsidy autoregressive parameter	0.9	See text

3.2 Effects of government spending shock

We now study dynamic responses of the economy to government spending shocks. The solid lines in Figure 1 display impulse responses of relevant labor market variables to a one standard-deviation shock to government spending.

An increase in government spending leads to a significant fall in the unemployment rate and an increase in employment. It also increases job-finding rate and reduces the separation rate. The pattern of responses of these labor market variables to the government spending shock is in line with what existing empirical studies found (See for example, Monacelli et al. 2010). However, the model cannot consistently capture the empirical pattern of responses of vacancies. While an increase in government spending increases vacancies in the data, it reduces vacancies in the model.

The prediction of our model is in contrast with that of a model without endogenous job separation. For comparison, we develop a version of our model in which separation takes place due to only exogenous shocks. We then simulate a quantitative version of the model using our calibration strategy. The results are also shown in Figure 1.

First, the pattern of responses of vacancies differs between models with and without endogenous job separation. The main reason why a positive government shock reduces vacancies in the model with endogenous separation is as follows. In the model with endogenous separa-
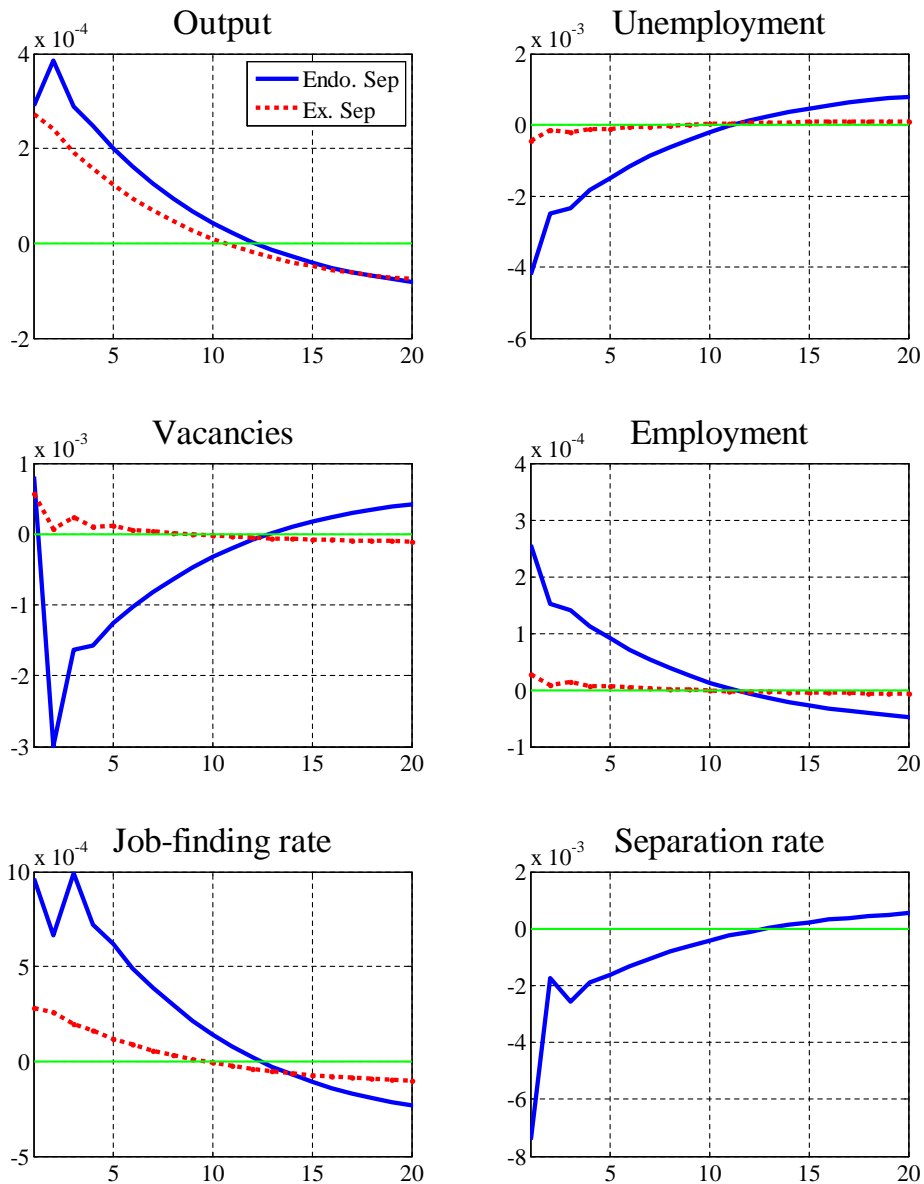
Figure 1: Dynamic responses of labor market variables to a positive government spending shock
Note: the solid line labeled "Endo. Sep" plots impulse response functions of our model. The dashed line labeled "Ex. Sep" plots impulse response functions in a model without endogenous job separation.

Table 2: Model solutions

Variables	Description	Solution
θ	Labor market tightness	0.720
$\tilde{x}$	Reservation cost	0.541
u	Unemployment rate	0.057
v	Vacancy	0.041
n	Employment rate	0.943
p	Job-finding rate	1.857
-	Separation rate	0.108
h	Hours worked	0.976
C	Aggregate consumption	1.468
G	Government spending	0.425
T	Lump-sum tax	0.471
Y	Aggregate output	2.836
I	Aggregate investment	0.673

tion, a positive government spending shock can substantially reduce the number of job seekers (unemployed workers) by lowering job separation, which in turn makes vacancy posting less attractive.

Second, the model with endogenous job separation generates a larger impact of the government spending shock on labor market variables than the model without endogenous job separation. To capture the size of the impact of the government spending shock on labor market variables, following Uhlig (2000), we compute net present value fiscal multiplier

$$\mathcal{M}^x_{t,t+j} = \frac{\sum_{i=1}^{j} \beta^{i-1} \left(x_{t+i} - x_{t+i-1} \right)}{\sum_{i=1}^{j} \beta^{i-1} \left(G_{t+i} - G_{t+i-1} \right)},$$

where x is a variable of interest. We compute short-run and medium-run multipliers. Short-run multipliers are defined as $\mathcal{M}^x_{0,1}$ with the system being in its steady-state at time 0 and fiscal shock being realized at time 1. We define medium-run multipliers as $\mathcal{M}^x_{0,10}$. In the model with endogenous job separation, the short-run and medium-run employment multipliers are 0.056 and 0.011, respectively. On the other hand, in the model without endogenous job separation, they are 0.007 and -0.0001, respectively.

We now turn to see the dynamic responses of macroeconomic variables to the government spending shock. The solid lines in Figure 2 shows the results. An increase in government spending increases output and hours worked per worker. On the impact, both output and hours worked per worker rise and in the following periods, they decrease and gradually return to their steady-state value. This is due to a negative wealth effect. Since higher government spending is financed by higher taxes, a household increases their labor supply and reduces consumption as

consumption and leisure are normal goods. An increase in government spending also crowds out investment. These results are consistent with the prediction of a standard neoclassical model (see, for example, Baxter and King, 1993).

For comparison, in Figure 2, we also present impulse response functions for these macroeconomic variables to a positive government spending shock in the model without endogenous separation. It shows that two models generate similar results both qualitatively and quantitatively.

3.2.1 The effect of hiring subsidy

We now examine the effect of an increase in the subsidy to the cost of posting a vacancy. Figure 3 shows impulse responses of labor market variables to a one standard-deviation shock to the hiring subsidy.

An increase in the hiring subsidy has two counteracting effects on employment. On one hand, it increases firms' incentive to post vacancies by reducing vacancy posting costs. This increases employment. On the other hand, an increase in the hiring subsidy induces more job separation and thus lowers employment by increasing the reservation cost. This is because the job creation costs that are incurred when firms destroy current jobs and re-enter the market, are low due to the hiring subsidy.

Figure 3 shows that a positive hiring subsidy shock increases vacancies and separation. On the impact, employment falls and unemployment increases due to high separation. In the following periods, they gradually return to their steady-state levels. The main reason why employment falls and unemployment increases is that under calibrated parameter values, the above-mentioned latter effect dominates the former one.

In order to assess the contribution of endogenous job separation to our results, we study effects of the hiring subsidy shock in the model without endogenous job separation. Results are also shown in Figure 3.

The most striking finding is that the effect of a positive hiring subsidy on employment and unemployment is different between models with and without endogenous job separation. The model without endogenous job separation predicts an increased employment and a decreased unemployment in response to the positive hiring subsidy shock, which is opposite to what the model with endogenous job separation predicts. Furthermore, the response of output to the shock is also different between two models. While a positive hiring subsidy shock leads to an increase in output in the model without endogenous separation, it leads to a fall in output in the model with endogenous separation. This is because the response of employment to the shock differs between these two models.

These differences are the results of incorporating endogenous job separation into the model. The incorporation of endogenous job search gives rise to a new channel through which a positive
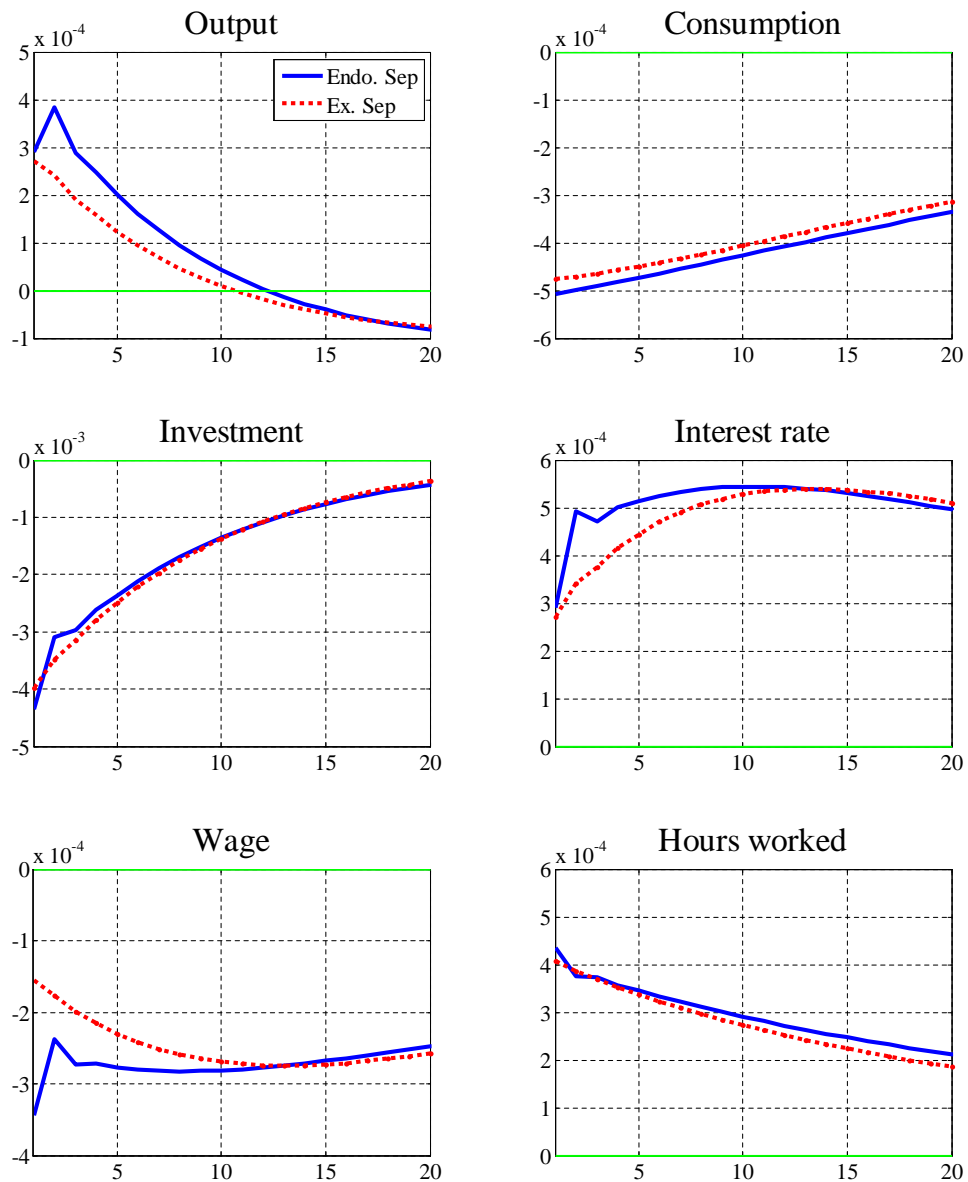
Figure 2: Dynamic responses of macroeconomics variables to a positive government spending shock

Note: the solid line labeled "Endo. Sep" plots impulse response functions of our model. The dashed line labeled "Ex. Sep" plots impulse response functions in a model without endogenous job separation.
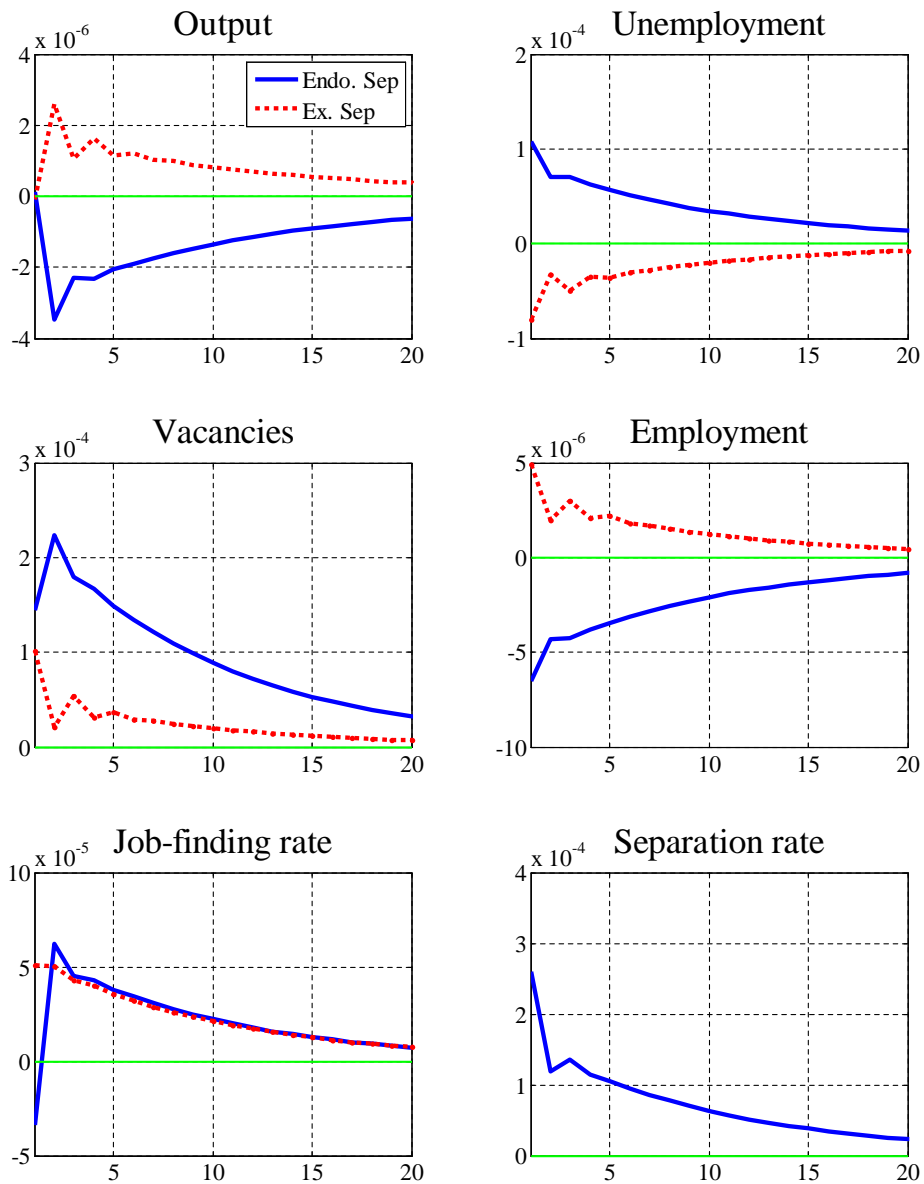
Figure 3: Dynamic responses of labor market variables to a positive hiring subsidy shock
Note: the solid line labeled "Endo. Sep" plots impulse response functions of our model. The dashed line labeled "Ex. Sep" plots impulse response functions in a model without endogenous job separation.

hiring subsidy shock reduces employment and increases unemployment: an increased separation rate. When the hiring subsidy increases, the cost of positing vacancies falls. A decrease in job creation cost also reduces the opportunity cost of continuation of an existing job. This induces more separation, leading to lower employment and higher unemployment.

Equally important, models with and without endogenous separation generate quantitatively different results. Specifically, although both models predict an increase in vacancies in response to positive hiring subsidy shocks, their sizes of the impact differ. To see this, as in the previous discussion, we compute short-run and medium-run vacancy multipliers in the two models.[11] In the model with endogenous separation, the short-run and medium-run vacancy multipliers are 0.059 and 0.093, respectively. On the other hand, in the model without endogenous separation, they are 0.042 and 0.021, respectively. Thus, the model with endogenous separation generates a larger impact of the hiring subsidy shock on vacancies than the model without endogenous separation. This is because, in the model with endogenous separation, an increase in unemployment due to the shock expands the number of job seekers and facilitates firms to find workers. This induces more vacancy postings.

3.3 Fiscal multipliers

In the literature, it is shown that hiring subsidies deliver larger multipliers than government spending. However, this result is obtained in models without endogenous separation. We now compute a variety of multipliers for both traditional increases in government spending and increases in hiring subsidies in our model, and compare our results with those in the model without endogenous separation.

Figure 4-(a) shows employment multipliers for both models with and without endogenous separation. When separation is not endogenously determined, the employment multiplier for hiring subsidies is larger than that for government spending. This implies that hiring subsidies are more effective than government spending if the fiscal authority is concerned about employment. This result does not hold anymore once we incorporate endogenous separation into the model. In the model with endogenous separation, an increase in hiring subsidies reduces employment, and thus the employment multiplier is negative. On the other hand, as in the model without endogenous separation, a positive government shock increases employment. Therefore, in general, the multiplier for government spending is larger than that for hiring subsidies. Furthermore, if we look at short-run multipliers, interestingly, the multiplier for government spending in the model with endogenous separation (0.056) is larger than that for hiring subsidies in the model without endogenous separation (0.042).

Figure 4-(b) reports unemployment multipliers for government spending and hiring subsi-

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[11]Multipliers for hiring subsidy are computed by $\mathcal{M}_{t,t+j}^x = \sum_{i=1}^{j} \beta^{i-1} \left(x_{t+i} - x_{t+i-1} \right) / \sum_{i=1}^{j} \beta^{i-1} \left(\tau_{t+i}^v - \tau_{t+i-1}^v \right)$, where x is variable of interest.
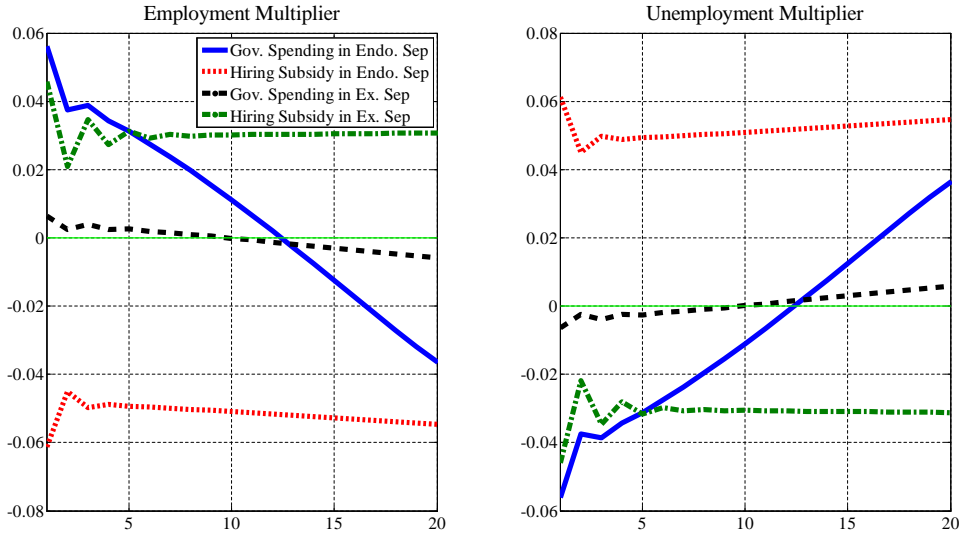
Figure 4: Employment and Unemployment Multipliers

dies in both models with and without endogenous separation. Similar to employment multipliers, in the model without endogenous separation, the unemployment multiplier for hiring subsidy is higher (in absolute value) than that for government spending. In contrast, in the model with endogenous separation, a change in government spending is more effective to reduce unemployment than a change in hiring subsidies.

4   Discussion

This section evaluates the robustness of our results in the previous section. We first discuss the sensitivity of the results to our choice of parameter values. We then examine how the degree of persistence of fiscal policy shocks affects the model's predictions.

4.1   Sensitivity analysis

We now study how our results vary with the worker's bargaining power η, the Frisch elasticity $1/\mu$, and the flow value of unemployment z. When we change these parameters, we also re-calibrate parameters m_0, γ, and ζ to maintain our calibration target values. Table 3 reports sensitivity checks for both short-run and medium-run employment multipliers.

We first discuss the sensitivity of the results to our choice of the worker's bargaining power η. Table 3 shows that a larger value of η yields a smaller effect of the government spending shock and a larger effect (in absolute value) of the hiring subsidy shock. However, the change

Table 3: Sensitivity analysis: employment multipliers

	Increase in government spending		Increase in hiring subsidies	
	Short-run	Medium-run	Short-run	Medium-run
Benchmark	0.056	0.011	-0.061	-0.051
Bargaining power				
$\eta = 0.3$	0.065	0.012	-0.055	-0.043
$\eta = 0.7$	0.052	0.011	-0.063	-0.055
Frisch elasticity				
$1/\mu = 0.2$	0.022	-0.026	-0.081	-0.062
$1/\mu = 1.0$	0.117	0.082	-0.029	-0.027
Unemployment benefit (z)				
Replacement rate=20%	0.041	0.012	-0.080	-0.060
Replacement rate=60%	0.092	0.026	-0.024	-0.025

in η does not substantially alter values of employment multipliers.

Next, we consider the impact of the elasticity of intertemporal substitution in the supply of hours $1/\mu$. The value of the Frisch elasticity has been the subject of some discussion. While most microeconomic studies estimate this elasticity to be small, between 0 and 0.5, students of the business cycle tend to work with higher elasticities, typically unity and above. We consider two different values of $\mu = 1$ and $\mu = 5$. Table 3 shows that the change of μ is crucial to the result. A larger value of μ yields a smaller effect of the government spending shock on employment and a larger (in absolute value) effect of the hiring subsidy shock. In our model, the marginal cost of production equals to the marginal rate of substitution between consumption and leisure. An increase in convexity in the disutility of labor increases the marginal costs of production, lowing marginal profits. This discourages firms from posting vacancies, and it becomes harder for an unemployed worker to find a job.

Finally, we discuss the sensitivity of the results to our choice of the flow value unemployment z. This parameter has been the subject of some discussion in the literature on cyclical properties of search and matching models. We consider two different values of z that are obtained by targeting the replacement rate of 20% and 60%.[12] A larger value of z yields a smaller effect of the government spending shock on employment and a larger (in absolute value) effect of the hiring subsidy shock. Our finding can be explained by the argument of Hagedorn and Manovskii (2008). Hagedorn and Manovskii (2008) argue that with a high value of z, firms make small profits and are more responsive to productivity shock. Thus, a higher value of z increases the volatility of labor market variables in a search and matching model in response to productivity shocks. Our results suggest that this argument is also applied when we analyze the effect of

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[12]When the replacement rate is 20% and 60%, the values of z are 0.425 and 1.126, respectively.

fiscal policies.

4.2 Role of persistence

We now examine how the degree of persistence of fiscal policies affects models' outcomes. As Faia et al. (forthcoming) argue, although the time series data of government spending is fairly persistent in normal times, the government might use fiscal stimulus in a discretionary and episodic fashion during extreme recessions. Furthermore, Mayer et al. (2010) demonstrate that effect of a government spending shock on the labor market depends on the degree of persistence of the shock in a model without endogenous separation. Thus, it is worth studying how the degree of persistence of fiscal policies affects the labor market in a model with endogenous separation.

Figure 5 displays dynamic responses of labor market variables to a government spending shock for three different values of ρ_g. Effects of the government spending shock on labor market variables depend on the degree of persistence. Figure 5 shows that only highly persistent government spending shock can generate a sustained increase in employment and decrease in unemployment. When the shock is not highly persistent, it rather reduces employment and increases unemployment.

We also examine how the degree of persistence of a hiring subsidy shock affects labor market outcomes. Figure 6 shows that, similar to the case of the government spending shock, the degree of persistence affects the model's results. When the shock is highly persistent, the hiring subsidy shock generates a sustained decrease in employment and increase in unemployment. In contrast, when the shock is short-lived, these effects are smaller.

5 Conclusion

Focusing on both hiring and firing margins, this paper studies the effect of fiscal expansion on the labor market. Recent empirical studies of the U.S. labor market suggest that in order to study the unemployment dynamics, it is important to take into account both hiring and firing margins. We develop a dynamic stochastic general equilibrium model with search frictions in which job separation is endogenously determined. We then consider fiscal stimuli in the form of government spending and hiring subsidies.

The prediction of our model is in contrast with earlier studies that use model with exogenous separation. First our model generates a larger size of the impact of a government spending shock on labor market variables than the model without endogenous job separation. Second, while an increase in hiring subsidies increases employment and reduces unemployment in models without endogenous job separation, it reduces employment and increases in our model.

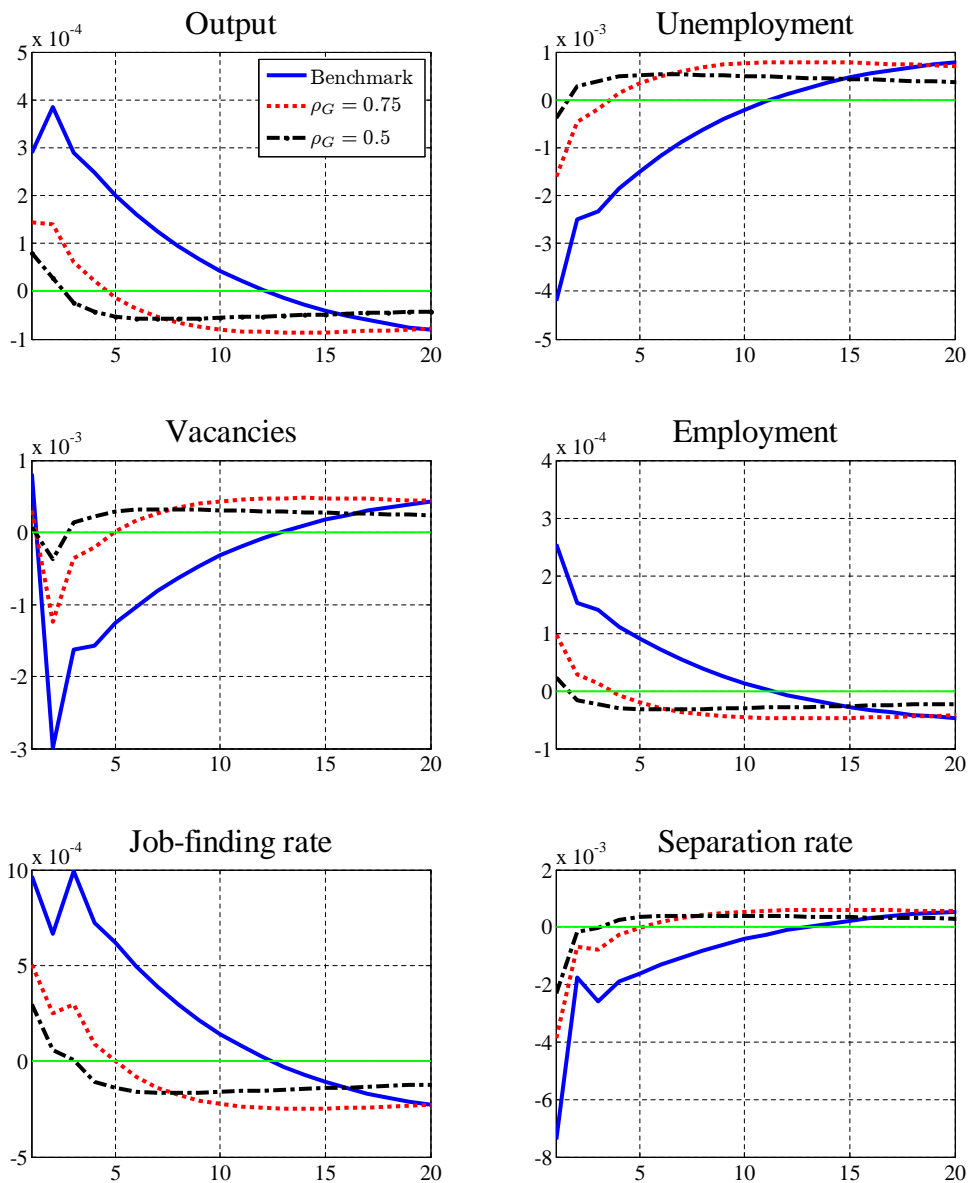A number of important issues remain for future research. One issue to be considered is

Figure 5: The role of the degree of persistence of government spending shocks
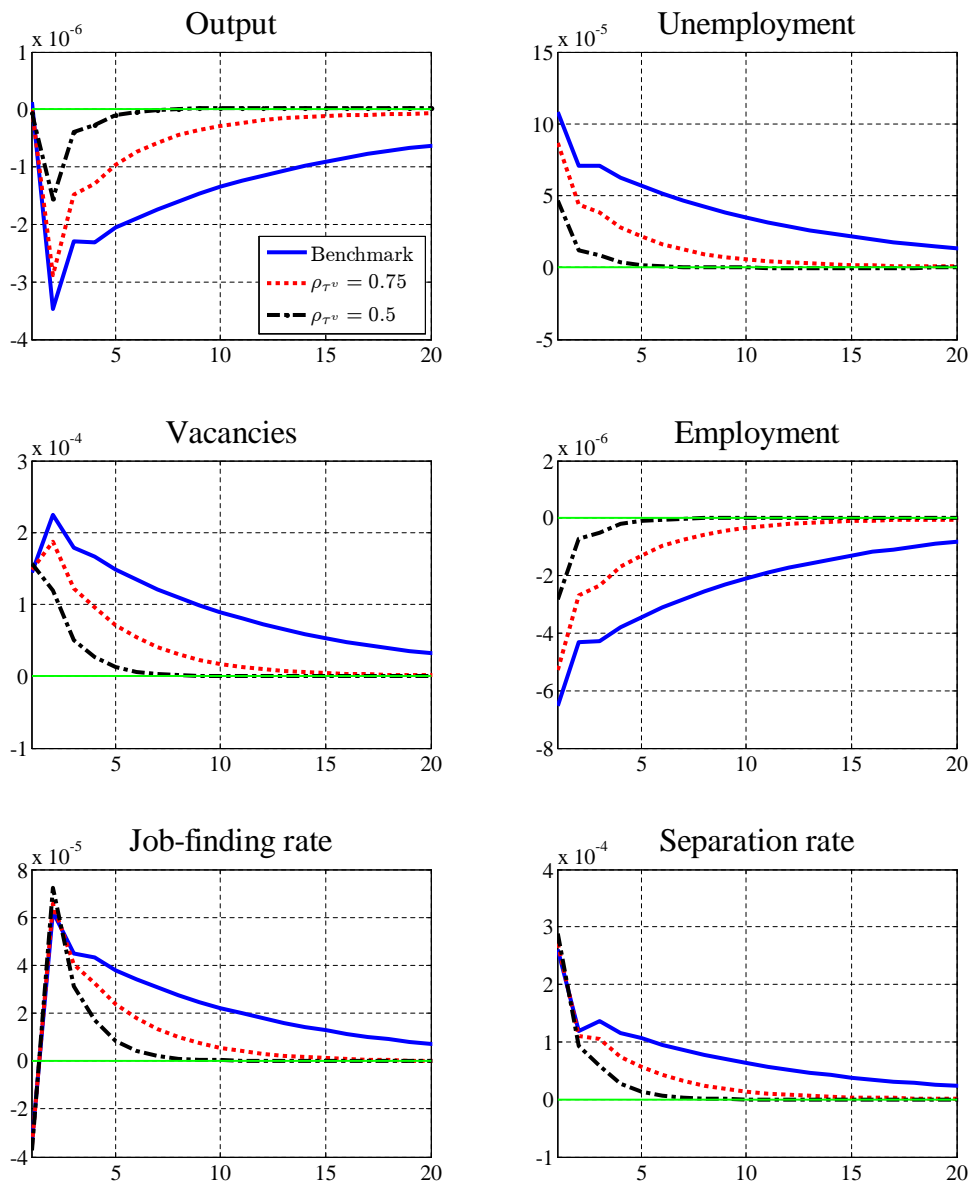
Figure 6: The role of the degree of persistence of hiring subsidy shocks

a more realistic fiscal setup with distortionary taxes. In our model, the government relies on lump-sum taxes. It is worth exploring how the economy with a realistic fiscal setup responds to a distortionary fiscal policy, in the form of not just a spending shock but also a tax shock. Also, it is worth considering the role of workers' on-the-job search when we analyze effects of fiscal stimuli on the labor market. Recent empirical studies of the U.S. labor market reveal that a large fraction of worker leavings jobs move to new jobs without intervening unemployment and this job-to-job flow is pro-cyclical (Fallick and Fleishman, 2004; Nagypál, 2004). This fact suggests that in order to study the cyclical behavior of labor market variables, it is necessary to use a model in which workers' transition between employment, unemployment, and across jobs are endogenously determined. Examining the effects of fiscal policies on labor market variables by using a model with on-the-job search is a fruitful avenue for research.

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