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Interactive Procedure for Selecting Multicriteria Inventory Policies

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I. Introduction

This paper develops an interactive procedure for assisting a manager in the selection of inventory policies. The procedure elicits a certain type of information by interaction with the manager at each step until a final inventory policy is identified.

The motivation for developing this procedure is provided by the observation that the marginal holding, ordering, and shortage costs needed to implement conventional inventory models may be difficult to measure in practice (Churchman 1961, Gardner 1980, Hanssmann 1962, and Plossl and Wight 1967). Furthermore, a majority of conventional inventory models assumes that these marginal costs are constant (for example, see Hadley and Whitin 1963 and Wagner 1975). In practice, however, these marginal costs may not be constant. This paper presents a procedure for selecting inventory policies which eliminates the need for estimating precise marginal costs and allows for the possibility of marginal cost variations.

In Section II, a conventional inventory model is briefly reviewed, and the cost measurement problems are discussed. The interactive procedure is described in Section III and an example is given. Section IV reports a computer experimental study. In these experiments the interactive procedure is applied to randomly generated inventory situations. In Section V, conclusions as well as suggestions for further research are given.

II. Conventional Inventory Model

In this section a conventional single-item, continuous-review (s, Q) inventory model (Hadley and Whitin 1963) is examined. A manager is primarily concerned with inventory control to meet three objectives or criteria:[1]

I (average annual inventory investment in dollars):

$$I = U[Q/2 + (ROP - \mu) + \int_{ROP}^{\infty} (D - ROP)\phi(D)dD], \tag{1}$$

W (workload in terms of average annual number of replenishment orders):

$$W = R/Q, \tag{2}$$

S (service level in terms of average annual number of shortages):

$$S=(R/Q)\int_{ROP}^{\infty}(D-ROP)\phi(D)dD, \tag{3}$$

where

U=unit cost in dollars,

Q=lot size in units,

ROP=reorder point in units,

μ=mean value of leadtime demand in units,

D=leadtime demand in units,

$\phi(D)$=marginal distribution of leadtime demand, and

R=average annual rate of demand in units.

Note that $Q/2$, $ROP-\mu$, and $\int_{ROP}^{\infty}(D-ROP)\phi(D)dD$ designate cycle inventory, safety stock, and lost sales, respectively. The assumptions in this formulation are that the unit cost U of the item is constant and is independent of the lot size Q, and that the average rate of demand R is constant per unit time although demand is probabilistic.

It should be recognized that the criteria, inventory investment, workload, and shortages expressed by (1), (2), and (3), respectively, are conflicting and non-commensurable. It is easily verified that these criteria are conflicting. Clearly, we cannot reduce the values on all criteria simultaneously. With a fixed workload, for example, a reduction in inventory investment leads to a lower reorder point and thereby increases shortages. Therefore, inventory decisions involve policy tradeoffs among these three criteria.

Non-commensurability of the criteria, inventory investment, workload, and shorgates, is traditionally resolved by introducing the marginal costs associated with each of these criteria. Then, the optimality condition for the problem is stated in terms of cost minimization. To see this, let

C_h=marginal inventory holding charge per unit per year,

C_p=marginal ordering cost per order, and

C_s=marginal shortage cost per unit.

In order to use cost minimization as an objective, a majority of conventional inventory models assumes that these marginal costs are constant (Hadley and

Whitin 1963 and Wagner 1975, for example). Then, we can define the general expression for total inventory cost, denoted *TIC*, as

$$TIC = C_h I + C_p W + C_s S, \tag{4}$$

where I, W, and S denote inventory investment, workload, and shortages, respectively. A decision then must be made regarding how much (Q) and when (ROP) to reorder in order to minimize *TIC*. Hence, our objective is formulated as

$$\underset{Q,ROP}{\text{minimize}}\ C_h I + C_p W + C_s S. \tag{5}$$

Since *TIC* is convex, we know that any solution to the first order condition will be an optimal solution.[2)]

While conventional inventory theory is based on the objective of cost minimization, the marginal holding, ordering, and shortage costs assumed in the theory may be difficult to measure in practice (Churchman 1961, Gardner 1980, Hanssmann 1962, and Plossl and Wight 1967). The marginal holding cost is mostly composed of the opportunity cost of the capital tied up in inventory, which is a highly subjective measure and varies from time to time to meet the changing risk environment of the firm and management goals for rates of return on investment (Brown 1967 and Plossl and Wight 1967). Most of the suggested approaches to measure the ordering cost in the accounting literature may result in average rather than marginal cost (Gardner 1980). The marginal shortage cost is particularly difficult to measure since there is little basis for its measurement in accounting methodology (Gardner 1980). In Gardner's survey (1980), he concludes that the accounting evidence shows that the marginal holding, ordering, and shortage costs assumed in the traditional theory are virtually impossible to measure in practice.

Furthermore, conventional inventory models assume a linear value scale. In other words, the marginal holding, ordering, and shortage costs are assumed to be constant and do not depend on the actual level of investment in inventory or the actual level of shortages. In practice, however, these marginal costs may not be constant. For example, the marginal cost of shortages may be small when there are only a few shortages but can be high when shortages are severe enough to cause an idle labor or an idle machine.

The cost measurement problems as well as the constant costs assumption, therefore, lead us to the examination of an approach to decision making in inventory management that avoids marginal cost measurement.

III. Interactive Procedure

In this section, we develop an interactive procedure for selecting a multicriteria inventory policy. First, we define an inventory decision making as a multicriteria optimization. Next, the interactive procedure is described. Finally, an example of the procedure is given.

3.1. *Problem Setting*

In controlling an inventory system, a manager is concerned with the following criteria; investment in inventory (I), workload (W), and service level (S). Note that the explicit forms of I, W, and S in a single-item inventory model are given by equations (1), (2), and (3) in Section II. We propose an approach that treats these criteria as independent measures of the inventory system and uses manager's tradeoffs among these criteria to make an inventory decision. This approach can be formulated as the multicriteria optimization:

$$\begin{aligned} &\text{maximize } V[I(Q, ROP), W(Q), S(Q, ROP)] \\ &\text{subject to: } 0 \leq Q \leq Q^u \\ &\qquad\qquad\quad 0 \leq ROP \leq ROP^u \end{aligned} \tag{6}$$

where V is the manager's value function (Keeney and Raiffa 1976), Q is the lot size, ROP is the reorder point, and the superscript u represents upper bounds. The value function V is a scalar-valued function such that $V(I, W, S) \geq V(I', W', S')$ if and only if an inventory policy (I, W, S) is preferred or indifferent to an inventory policy (I', W', S'). (See Debreu 1954, Fishburn 1968, 1970, and Luce and Suppes 1965 for conditions which guarantee the existence of a value function.) In (6) the manager concerns with value tradeoffs among inventory investment (I), workload (W), and service level (S) to select a preferred inventory policy (I^*, W^*, S^*). More specifically, (6) states that an inventory decision has to be made with regard to how much (Q) and when (ROP) to reorder in order to maximize the manager's preference over investment in inventory, workload, and service level.

This approach may have advantages over the conventional inventory model. First, it does not require to measure the marginal holding, ordering, and shortage costs. Secondly, the approach does not assume a linear value scale. Intuitively, the amount of investment in inventory the manager is willing to increase for a unit reduction in shortages may be small when there are only a few shortages. Whereas, the willingness to pay may be large if shortages are high enough to cause an idle labor or an idle machine. It seems that these variations in the manager's tradeoffs reflect the possible variations in the marginal holding, ordering, and shortage costs. Our approach uses explicitly the manager's tradeoffs to make an inventory decision and, thus, allows for the possibility that these marginal costs may vary. Furthermore, the approach is implemented using the policy variables which managers employ in controlling an inventory system. We often hear manager's concern about rising investment in inventory or deteriorating quality of service. Since these concerns are explicitly treated as criteria in our approach, the implementation of the solution may be facilitated.

3.2. *Description of the Interactive Procedure*

This section describes an interactive procedure for solving the multicriteria optimization problem (6). One approach to solve (6) is the direct assessment and maximization of the manager's value function which is assumed to be of a simple functional form such as an additive or multiplicative form (Nomura et al. 1985). Sometimes, however, the assumptions needed to imply the value function to be additive or multiplicative do not hold in practice (see Keeney and Raiffa 1976 for the required assumptions). The interactive procedure described in this section is developed to circumvent this problem. The procedure does not require the knowledge of the value function explicitly. Instead, it requires a minimal amount of local information about the manager's preference structure needed to solve the multicriteria problem (6).

The interactive procedure requires a manager to provide two kinds of local information concerning his preference structure over the three criteria, investment in inventory (I), workload (W), and service level (S), at each of several iterations. First, the manager assesses his local tradeoffs among investment in inventory, workload, and service level. For example, the manager is asked to answer the following type of question: "How much would you be willing to increase the investment in inventory in order to reduce shortages by a certain amount?" The tradeoff information is then used to identify several tentative inventory policies, i.e., alternative levels of (I, W, S). Second, the manager is required to make a comparison among these tentatively identified inventory policies to check for a preferred policy. These two kinds of local information are sufficient to complete one iteration of the interactive procedure. The iteration continues until a final inventory policy is identified.

It is appropriate now to describe the interactive procedure in detail. In solving the multicriteria inventory problem (6), we assume the manager's value function V to be concave and continuously differentiable on a feasibel set $\{[I(Q, ROP), W(Q), S(Q, ROP)]: 0 \leq Q \leq Q^u \text{ and } 0 \leq ROP \leq ROP^u\}$, where Q is the lot size, ROP is the reorder point, and the superscript u represents upper bounds. However, we do not assume that V is known explicitly. The interactive procedure is essentially the iterative steepest descent nonlinear mathematical programming algorithm to account for the unknown V (Boyd 1970). At each iteration k, the first step of the procedure is to find a trial solution (Q_{k+1}, ROP_{k+1}) to

$$\begin{aligned} &\text{maximize } MV_I^k I(Q, ROP) + MV_W^k W(Q) + MV_S^k S(Q, ROP) \\ &\text{subject to: } 0 \leq Q \leq Q^u \\ &\qquad\qquad\quad 0 \leq ROP \leq ROP^u \end{aligned} \tag{7}$$

where

$$MV_I^k = \frac{\partial V[I(Q_k, ROP_k), W(Q_k), S(Q_k, ROP_k)]}{\partial I(Q_k, ROP_k)},$$

$$MV_W^k = \frac{\partial V[I(Q_k, ROP_k), W(Q_k), S(Q_k, ROP_k)]}{\partial W(Q_k)},$$

$$MV_S^k = \frac{\partial V[I(Q_k, ROP_k), W(Q_k), S(Q_k, ROP_k)]}{\partial S(Q_k, ROP_k)}, \text{ and}$$

(Q_k, ROP_k) denotes the current point of (Q, ROP). The partial derivatives MV_I^k, MV_W^k, and MV_S^k represent the rate of change in the manager's total utility resulting from a unit change in I, W, and S at (Q_k, ROP_k), respectively.

We assume that the manager always prefers lower level of each criterion, or equivalently $MV_I^k<0$, $MV_W^k<0$, and $MV_S^k<0$ for any k. Since MV_I^k is a negative constant, dividing the objective function in (7) by $-MV_I^k>0$ does not affect the solution to (7). Hence, (7) is equivalent to

$$\begin{aligned} &\text{minimize } I(Q, ROP)+w_2^k W(Q, ROP)+w_3^k S(Q, ROP) \\ &\text{subject to: } 0\leq Q\leq Q^u \\ &\qquad\qquad\quad 0\leq ROP\leq ROP^u \end{aligned} \tag{8}$$

where

$$w_2^k=\frac{MV_W^k}{MV_I^k} \text{ and } w_3^k=\frac{MV_S^k}{MV_I^k}.$$

The weights w_2^k and w_3^k are the manager's tradeoffs (marginal rates of substitution) between the criterion I and the criteria W and S at the current point. The weight w_2^k measures the amount of increases in inventory investment the manager would take in order to reduce workload by one unit. Similarly, the weight w_3^k represents the amount of inventory investment the manager is willing to increase for a unit reduction in shortages. Thus we can assess these tradeoffs by asking the manager the following type of question: "How much would you be willing to increase the investment in inventory in order to reduce shortages (or workload) by a certain amount?" For example, the manager is asked to provide the change in I, ΔI, such that

$$\begin{aligned} &[I(Q_k, ROP_k), W(Q_k), S(Q_k, ROP_k)]\sim \\ &\qquad [I(Q_k, ROP_k)+\Delta I, W(Q_k), S(Q_k, ROP_k)-\Delta S] \end{aligned} \tag{9}$$

for a small fixed $\Delta S>0$, where the symbol $\sim$ reads "indifferent to." At this level, we have $w_3^k\cong\Delta I/\Delta S$.

It is noted that the minimization problem (8) can be written as

$$\text{minimize } \frac{\nabla V(I_k, W_k, S_k)}{MV_I^k}\cdot(I, W, S) \tag{10}$$

where $\nabla V(I_k, W_k, S_k)$ is the gradient of V evaluated at (Q_k, ROP_k), i.e., $\nabla V(I_k, W_k, S_k)=(MV_I^k, MV_W^k, MV_S^k)$. Therefore, the interactive procedure is the steepest descent nonlinear algorithm to account for the unknown V. The procedure requires the manager to determine the scaled gradient to the value function by evaluating the current inventory policy (I_k, W_k, S_k) and assessing his tradeoffs. The scaled gradient is then used to form a local linerar approximation to the value function. Since the criterion functions $I(Q, ROP)$, $W(Q)$, and $S(Q, ROP)$ are all convex and the tradeoffs w_2^k and w_3^k are constant, the objective function in (8) is also convex. If the upper bounds Q^u and ROP^u are assumed to be so large that we do not obtain a corner solution, any solution to the first order condition will be a solution to (8).[3] Therefore, the tradeoff information is sufficient to generate a trial solution (Q_{k+1}, ROP_{k+1}) and the manager is communicated with a new inventory policy $(I_{k+1}, W_{k+1}, S_{k+1})$.

However, the inventory policy $(I_{k+1}, W_{k+1}, S_{k+1})$ obtained in the first step of the interactive procedure may not be preferred to the previous inventory policy (I_k, W_k, S_k). This lack of improvement from iteration to iteration stems from the fact that the local linear approximation of V is sometimes poor since the solution (Q_{k+1}, ROP_{k+1}) to (8) may not be in the immediate neighborhood of the previous solution (Q_k, ROP_k). To resolve this problem, the second step of the interactive procedure is introduced so that the procedure generates a sequence of improved trial solutions converging to the optimum. At each iteration k, the second step of the procedure requires the manager to determine a weight α of the convex combination of the current tradeoffs w_i^k and the previous tradeoffs w_i^{k-1}, $\alpha w_i^k+(1-\alpha)w_i^{k-1}$ for $i=2$, 3 and $0<\alpha\leq 1$, such that these weighted tradeoffs yield an improved trial solution (Q_{k+1}, ROP_{k+1}) to (8). In other words, at each iteration k, we replace w_i^k by a weighted sum of w_i^k and w_i^{k-1}, and a unidimensional search is performed over the weight α to insure that we have an improved trial solution. Intuitively, it can be seen that αw_i^k determines the right direction for improvement and $(1-\alpha)w_i^{k-1}$ determines a penalty for moving too far. In practice, the unidimensional search might be conducted by starting $\alpha=1$ and if the new solution does not yield a preferred inventory policy over the current policy, then α is decreased.

The interactive procedure, thus, requires the manager to specify his local tradeoffs and to perform a unidimensional search at each iteration until a final inventory policy is identified. The procedure proceeds as follows:

Step 0. *Initialization*: Given an initial point (Q_0, ROP_0), compute (I_0, W_0, S_0).

Step 1. *Pre-iteration*: The manager evalutes his tradeoffs w_2^0 and w_3^0 at

(I_0, W_0, S_0). Compute a solution (Q_1, ROP_1) to (8) using these tradeoffs. Compute (I_1, W_1, S_1) at (Q_1, ROP_1). Let $k=1$.

Step 2. *Testing*: If $(Q_{k-1}, ROP_{k-1})=(Q_k, ROP_k)$ or $(I_{k-1}, W_{k-1}, S_{k-1})= =(I_k, W_k, S_k)$, then the procedure is terminated.

Step 3. *Tradeoff Assessments*: The manager evaluates his tradeoffs w_2^k and w_3^k at the current policy (I_k, W_k, S_k). Set $\xi_2^k=w_2^k$ and $\xi_3^k=w_3^k$.

Step 4. *Unidimensional Search*: The manager selects a value of α, $0<\alpha\leq 1$, such that a new solution (Q_{k+1}, ROP_{k+1}) to (8) using $w_2^k=\alpha\xi_2^k+(1-\alpha)w_2^{k-1}$ and $w_3^k=\alpha\xi_3^k+(1-\alpha)w_3^{k-1}$ yields a preferred policy $(I_{k+1}, W_{k+1}, S_{k+1})$. Let $k=k+1$ and return to Step 2.

It is quite interesting to note that a solution (Q_k, ROP_k) to (8) obtained in the first step of the interactive procedure is identical to that obtained in the conventional cost minimization model if

$$w_2^k=C_p/C_h \tag{11}$$

and

$$w_3^k=C_s/C_h, \tag{12}$$

where C_h, C_p, and C_s denote the marginal holding, ordering, and shortage cost, respectively.[4] Therefore, one way the interactive procedure can be interpreted is that the manager's tradeoffs act as surrogates for the marginal cost information. It is also noted that the procedure terminates exactly in one iteration if the manager's tradeoffs are constant. Moreover, the solution is the same that the conventional model generates if (11) and (12) hold.

3.3. *An Example of the Interactive Procedure*

This section demonstrates the interactive procedure for selecting an inventory policy. We assume that the leadtime demand is normally distributed with a mean (μ) of 750 units and a standard deviation (σ) of 300 units, that the average rate of demand (R) is 1600 units, and that the unit cost (U) is one dollar. In order to simulate the interaction process, we will assume that the form of the manager's value function is known explicitly. However, we describe the example as if the manager only provided his tradeoffs and performed a unidimensional search.

We assume the additive value function V of the form

$$V(I,\ W,\ S)=k_1V_1(I)+k_2V_2(W)+k_3V_3(S), \tag{13}$$

where V_i, $i=1, 2, 3$, are the conditional value functions and k_i, $i=1, 2, 3$, are scaling constants. We scale V and V_i such that:

$$\begin{aligned} &V(I^o,\ W^o,\ S^o)=0, && V(I^*,\ W^*,\ S^*)=1,\\ &V_1(I^o)=0, && V_1(I^*)=1,\\ &V_2(W^o)=0, && V_2(W^*)=1,\\ &V_3(S^o)=0, && V_3(S^*)=1, \text{ and}\\ &\textstyle\sum_{i=1}^{3} k_i=1 \text{ for } k_i>0, \end{aligned} \tag{14}$$

where the superscript o designates the heighest (least preferred) level of I, W, and S and the superscript $*$ denotes the lowest (most preferred) level. We also assume that the single-criterion value function V_i has the exponential form:

$$V_i(x_i)=b_i\{1-\exp[-c_i[(x_i^o-x_i)/(x_i^o-x_i^*)]]\} \tag{15}$$

for $x_1=I$, $x_2=W$, and $x_3=S$, where $b_i=1/[1-\exp(-c_i)]$. The parameter c_i can be determined if we specify a mid-value point x_i^m such that $V_i(x_i^m)=0.5$ (Keeney and Raiffa 1976, p94). The numerical tradeoffs w_2 and w_3 are given by $w_2=\dfrac{\partial V}{\partial W}\Big/\dfrac{\partial V}{\partial I}$ and $w_3=\dfrac{\partial V}{\partial S}\Big/\dfrac{\partial V}{\partial I}$, where

$$\partial V/\partial x_i=\frac{-k_ib_ic_i}{x_i^o-x_i^*}\exp\{-c_i[(x_i^o-x_i)/(x_i^o-x_i^*)]\} \tag{16}$$

for $x_1=I$, $x_2=W$, and $x_3=S$. In this example, we set $(I^*,\ I^m,\ I^o)=(0,\ 1000,\ 1600)$, $(W^*,\ W^m,\ W^o)=(1,\ 8,\ 12)$, and $(S^*,\ S^m,\ S^o)=(0,\ 600,\ 800)$, which yield $c_1=1.04$, $c_2=1.15$, and $c_3=2.44$. We also assume that the distribution of scaling constants is $k_1=0.25$, $k_2=0.25$, and $k_3=0.5$.

We shall now look at each step of the interactive procedure.

Step 0. Suppose an initial point to be $(Q_0,\ ROP_0)=(R/4,\ \mu)=(400,\ 750)$. Then, $(I_0,\ W_0,\ S_0)=(319.68,\ 4.0,\ 478.73)$.

Step 1. Tradeoffs at $(I_0,\ W_0,\ S_0)$ are $w_2^0=151.84$ and $w_3^0=5.75$. With these tradeoffs we obtain $(Q_1, ROP_1)=(833.58, 1165.34)$ which, in turn, yields $(I_1, W_1, S_1)=(843.51, 1.92, 21.85)$.

Step 2. Assume that the interactive procedure is terminated when $|(Q_k, ROP_k)-(Q_{k-1}, ROP_{k-1})|/(Q_{k-1}, ROP_{k-1})<(5\%, 5\%)$ and $|(I_k, W_k, S_k)-(I_{k-1}, W_{k-1}, S_{k-1})|/(I_{k-1}, W_{k-1}, S_{k-1})<(5\%, 5\%)$. Since $(Q_0, ROP_0)=(400, 750)$, $(Q_1, ROP_1)=(833.58, 1165.34)$ and $(I_0, W_0, S_0)=(319.68, 4.0, 478.73)$, $(I_1, W_1, S_1)=(843.51, 1.92, 21.85)$, the procedure continues.

Step 3. Tradeoffs at (I_1, W_1, S_1) are $w_2^1=86.80$ and $w_3^1=1.01$. Set $\xi_2^1=w_2^1$ and $\xi_3^1=w_3^1$.

Step 4. Set $w_2^1=\alpha\xi_2^1+(1-\alpha)w_2^0=\alpha\times 86.80+(1-\alpha)\times 115.84$ and $w_3^1=\alpha\xi_3^1+(1-\alpha)w_3^0=\alpha\times 1.01+(1-\alpha)\times 5.75$ for $\alpha\in(0, 1]$. Instead of searching for a value of α yielding the most improved solution, the unidimensional search is conducted by starting $\alpha=1$ and if the new solution is not preferred to the current solution then α is decreased by 0.1. When $\alpha=1$, $w_2^1=86.80$ and $w_3^1=1.01$ yield $(Q_2, ROP_2)=(667.24, 912.06)$ which updates the trial inventory policy to $(I_2, W_2, S_2)=(556.38, 2.36, 131.60)$. Since $V(I_1, W_1, S_1)=0.8853$ and $V(I_2, W_2, S_2)=0.8992$, (I_2, W_2, S_2) is preferred to (I_1, W_1, S_1). Consequently, we return to Step 2.

For brevity of presentation, the results of further iterations are summarized in Table 1. A preferred inventory decision is given by $(Q^*, ROP^*)=(742.61, 962.86)$, which agrees with the solutions obtained by the interactive procedure with several different initial points.

IV. Computer Experiment

A computer experiment has been undertaken to provide empirical evidence on the working of the interactive procedure. The experiment was intended to test the performance of the procedure in arriving at a preferred policy over a wide range of randomly generated inventory situations. More specifically, the objectives of the experiment were; (1) to examine the number of iterations required in identifying a preferred inventory policy; (2) to compare the solutions generated by the interactive procedure to the solutions obtained from the conventional model which assumes constant marginal costs.

Table 1

Interations of the Interactive Procedure

Iteration	Q	ROP	I	W	S	$V_1(I)$	$V_2(W)$	$V_3(S)$	$V(I,W,S)$	w_2	w_3	α
1	400.00	750.00	319.68	4.00	478.73	0.8740	0.8292	0.6840	0.7678	151.84	5.75	—
2	833.58	1165.34	843.51	1.92	21.85	0.6013	0.9532	0.9934	0.8853	86.80	1.01	1
3	677.24	912.06	556.38	2.36	131.60	0.7622	0.9290	0.9528	0.8992	109.65	1.71	1
4	741.65	988.43	645.65	2.16	78.52	0.7154	0.9404	0.9741	0.9010	101.25	1.37	1
5	719.23	955.29	608.92	2.22	97.91	0.7350	0.9367	0.9667	0.9013	104.44	1.49	1
6	727.93	967.81	622.78	2.20	90.13	0.7276	0.9381	0.9697	0.9013	103.21	1.44	1
7	724.61	962.86	617.34	2.21	93.13	0.7305	0.9376	0.9686	0.9013	103.28	1.46	—

4.1. *Experimental Design*

The experiment used the computer to simulate manager's responses. Given an explicit form of the value function, the computer evaluates the tradeoffs and checks for an improved policy at each iteration. In this experiment, we assumed the value function to be of the additive exponential form defined by equations (13), (14), and (15). Note, however, that the interactive procedure does not assume any analytic form for the value function. Its explicit form was simply assumed to be known in the experiment in order to simulate the interaction process.

The experiment was conducted by varying the marginal leadtime demand distribution, the initial point, and the range of the criteria I, W, and S.

(1) The marginal distribution of leadtime demand: The uniform, normal, and exponential leadtime demand distributions were used.[5)]

(2) The initial point: Nine different initial points, $\{(Q_0, ROP_0): Q_0 \in (R, R/2, R/4)$ and $ROP_0 \in (\mu, \mu+\sigma, \mu+2\sigma)\}$, were used, where R is the average rate of demand, and μ and σ are the mean and strandard deviation of the leadtime demand distribution.

(3) The criterion range: Three different ranges were used. Let Range 1 be $[I^* \leq I \leq I^o,\ W^* \leq W \leq W^o,\ S^* \leq S \leq S^o]$. Then, Range 2 and 3 are defined as $[I^* \leq I \leq 2I^o,\ W^* \leq W \leq 2W^o,\ S^* \leq S \leq 2S^o]$ and $[I^* \leq I \leq 3I^o,\ W^* \leq W \leq 3W^o,\ S^* \leq S \leq 3S^o]$, respectively.

The experiment was conducted using a 3×3 factorial design. Each cell of the 3×3 matrix corresponds to the marginal distribution of leadtime demand and the criterion range. In each cell, we ran thirty randomly generated problems with nine different initial points and tabulated the results. It should be noticed that this plan can be alternatively stated as a $3 \times 3 \times 9$ factorial design with repeated measures on the nine different initial points (Winer 1971).

The flow chart in Figure 1 shows the steps of the experiment. The steps are described below in detail.

1. We assume that the unit cost U is one dollar and the average rate of demand R is 1600 units.

2. We assume the range(s) of the parameter(s) of each leadtime demand distribution as follows: for uniform distribution, lower bound$=0$ and $1000 \leq$

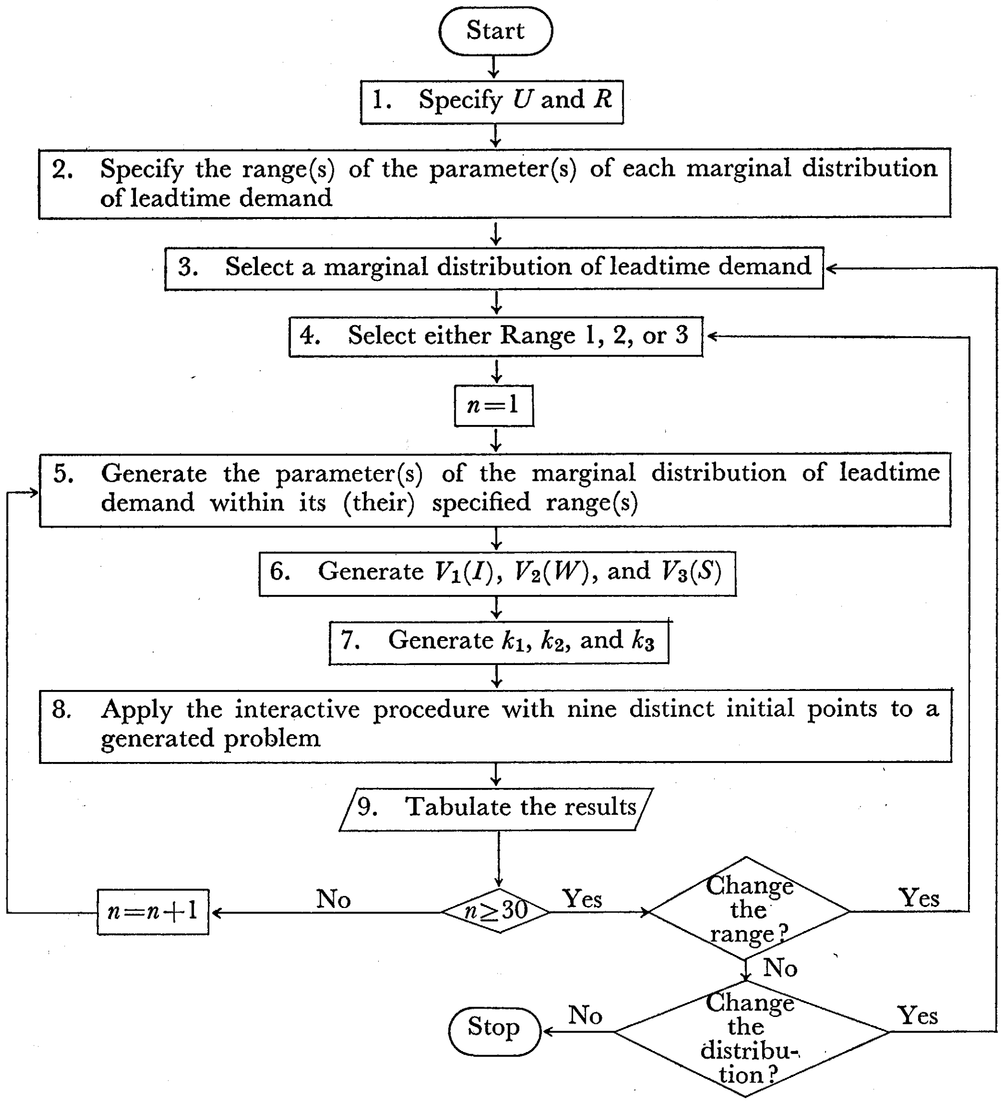

Figure 1. *Flow Chart for the Experiment*

upper bound ≤ 2000; for normal distribution, $500 \leq$ mean (μ) ≤ 1000 and $50 \leq$ standard deviation (σ) ≤ 300; for exponential distribution, $500 \leq$ mean (μ) ≤ 1000.

3. Select either a uniform, normal, or exponential leadtime demand distribution.

4. We assume that $I^*=0$, $W^*=1$, and $S^*=0$. We set I^o, W^o, and S^o as follows:[6] $I^o=2\times I(Q, ROP)$ with $Q=R$ and $ROP=\mu^u+2\sigma^u$; $W^o=2\times W(Q)$ with $Q=R/4$; $S^o=2\times S(Q, ROP)$ with $Q=R/4$ and $ROP=\mu^l$. Here, the lower bound (superscript l) and upper bound (superscript u) of the mean (μ) and standard deviation (σ) of the leadtime demand distribution are specified in 2. Select either Range 1, 2, or 3.

5. Generate randomly the parameter(s) of the leadtime demand distribution within its (their) range(s) as specified in 2.

6. Generate randomly a midvalue point within an open interval $((x_i^*+x_i^o)/2, x_i^o)$ for $x_1=I$, $x_2=W$, and $x_3=S$, which, in turn, generates $V_i(x_i)$.[7]

7. Generate randomly k_1, k_2, and k_3 such that $\sum_{i=1}^{3} k_i=1$ and $k_i>0$.

8. Apply the interactive procedure as described in Section 3.2. As a termination criterion, we select a tolerable percentage error (1%) in $|(Q_k, ROP_k)-(Q_{k-1}, ROP_{k-1})|/(Q_{k-1}, ROP_{k-1})$ and $|(I_k, W_k, S_k,)-(I_{k-1}, W_{k-1}, S_{k-1})|/(I_{k-1}, W_{k-1}, S_{k-1})$ for any iteration.

9. Analyze the performance of the interactive procedure.

4.2. *Results and Analysis*

4.2.1. *Number of Iterations with the Interactive Procedure*

Table 2 shows the number of iterations required in identifying a perferred inventory policy with the interactive procedure. The first column in this table refers to the nine different initial points from which the interactive procedure starts (see Table 3). It can be seen from this table that the number of iterations seems to depend on the type of leadtime demand distribution and the range of criteria. The overall average number of iterations was the largest (11.66) when the leadtime demand distribution is uniform and the criterion range is the narrowest (Range 1), and was the smallest (5.41) when they are exponential and the widest (Range 3). As shown in this table, the average number of iterations tends to decrease in order of uniform, normal, and exponential distribution of leadtime demand. This result could be due to the fact that the divergence of the leadtime demand distribution decreases in this order. For each of these distributions, the average number of iterations was the largest when Range 1 is employed and was the smallest when Range 3 was used. Thus, the average number of iterations tends to decrease as the criterion range spreads. This result may be because that the tradeoffs used to compute a trial solution depend not only on the levels of the criteria I, W, and S but also on the ranges of these criteria.

An ANOVA was performed to determine if there is a statistically significant difference in the number of iterations required for the identification of a preferred policy when the leadtime demand distribution, the criterion range, and the initial point are varied. We adopted a $3 \times 3 \times 9$ factorial design with repeated measures on the nine initial points (see Winer 1971). Table 4 summarizes the results of this test.[8]

The following observations can be made from these results (at the 5% significance level):

1. The marginal distribution of leadtime demand influences the number of iterations required in identifying a preferred policy.
2. The criterion range affects the number of iterations required in identifying a preferred policy.
3. The initial point also affects the number of iterations required in identifying a preferred policy.
4. The number of iterations required in identifying a preferred policy in each criterion range seems to depend on the initial point.

The Wilcoxon matched-pairs signed-ranks test (see Siegel 1956) showed that the initial point $(Q_0, ROP_0) = (R/2, \mu)$ requires less iterations than the other eight initial points. Therefore, we recommend that $(Q_0, ROP_0) = (R/2, \mu)$ be used as a starting point for the interactive procedure.

4.2.2. *The Interactive Procedure vs. the Conventional Model*

We examined the magnitude of the total inventory cost (TIC) differences between the interactive procedure and the conventional inventory model which assumes constant marginal costs.[9] The total inventory costs for each of randomly generated problems were calculated using the following solutions; (1) an optimal solution obtained by the interactive procedure, (2) a conventional

Table 2

Number of Iterations with Several Leadtime Demand Distributions

Uniform Leadtime Demand

Initial[1] Point	Range 1					Range 2					Range 3				
	Average[2] Number	Min	Max	S.D.	M/S.D.	Average[2] Number	Min	Max	S.D.	M/S.D.	Average[2] Number	Min	Max	S.D.	M/S.D.
1	11.70	6	28	5.64	2.07	8.00	4	29	4.67	1.71	6.13	3	18	3.13	1.96
2	9.90	5	24	5.25	1.89	7.80	4	27	4.17	1.87	6.53	3	19	2.92	2.24
3	12.33	6	28	5.36	2.30	7.97	4	21	3.06	2.61	6.20	4	18	2.47	2.51
4	12.43	6	31	5.96	2.09	8.63	4	31	4.93	1.75	6.40	3	17	2.82	2.27
5	10.80	6	27	5.54	1.95	7.73	3	29	4.68	1.65	6.07	3	20	3.22	1.89
6	11.60	6	23	4.63	2.51	8.17	4	27	4.06	2.01	6.47	4	20	3.06	2.11
7	12.87	6	31	5.86	2.20	8.37	4	30	4.66	1.80	6.87	4	19	2.89	2.38
8	11.27	4	31	6.00	1.88	8.00	3	30	4.62	1.73	6.47	3	20	3.16	2.05
9	12.00	6	28	5.28	2.27	8.20	4	29	4.51	1.82	6.20	4	19	2.82	2.20
Overall	11.66	4	31	5.50	2.12	8.10	3	31	4.35	1.86	6.37	3	20	2.92	2.18

1 See Table 3.

2 30 problems were generated.

Table 2 *(continued)*

Normal Leadtime Demand

Initial[1] point	Range 1					Range 2					Range 3				
	Average[2] Number	Min	Max	S.D.	M/S.D.	Average[2] Number	Min	Max	S.D.	M/S.D.	Average[2] Number	Min	Max	S.D.	M/S.D.
1	11.03	4	24	5.69	1.94	8.30	3	27	4.12	2.01	5.77	3	9	1.74	3.32
2	9.40	3	22	5.00	1.88	7.63	3	24	3.81	2.00	5.70	3	9	1.58	3.61
3	10.43	5	23	4.80	2.17	7.60	3	18	2.74	2.78	6.07	4	9	1.28	4.72
4	11.27	5	26	5.41	2.08	8.07	4	23	3.66	2.21	6.30	4	9	1.51	4.17
5	10.10	4	24	4.99	2.04	7.93	3	25	3.91	2.03	5.90	3	8	1.27	4.65
6	10.30	4	21	4.41	2.33	7.87	4	23	3.42	2.30	5.87	4	8	0.97	6.03
7	11.37	4	28	5.76	1.97	8.07	4	26	4.00	2.01	6.47	4	9	1.38	4.68
8	10.23	4	26	5.44	1.88	7.20	3	25	4.06	1.77	5.47	3	9	1.55	3.53
9	10.10	4	21	4.11	2.46	7.97	4	24	3.57	2.23	5.90	3	8	1.16	5.11
Overall	10.48	3	28	5.05	2.07	7.85	3	27	3.68	2.13	5.94	3	9	1.41	4.21

[1] See Table 3.

[2] 30 ploblems were generated.

Table 2 (*continued*)

Exponential Leadtime Demand

Initial[1] Point	Range 1 Average[2] Number	Range 1 Min	Range 1 Max	Range 1 S.D.	Range 1 M/S.D.	Range 2 Average[2] Number	Range 2 Min	Range 2 Max	Range 2 S.D.	Range 2 M/S.D.	Range 3 Average[2] Number	Range 3 Min	Range 3 Max	Range 3 S.D.	Range 3 M/S.D.
1	10.17	6	30	5.97	1.70	5.97	3	9	1.67	3.57	5.60	3	8	1.30	4.30
2	8.63	4	26	5.15	1.68	5.73	3	10	1.62	3.54	5.17	3	7	1.02	5.07
3	10.47	6	28	4.97	2.10	6.57	4	9	1.14	5.78	5.63	4	8	1.16	4.86
4	10.37	5	30	5.90	1.76	6.33	3	10	1.65	3.85	5.73	4	8	1.14	5.02
5	9.87	4	30	5.80	1.70	5.73	3	13	2.27	2.52	5.03	3	8	1.50	3.36
6	9.13	5	24	3.84	2.38	6.10	3	10	1.40	4.36	5.03	3	8	1.47	3.42
7	10.50	6	30	5.84	1.80	6.67	4	13	1.86	3.58	5.60	3	9	1.59	3.52
8	10.50	4	30	5.79	1.81	6.40	3	13	2.08	3.08	5.63	4	8	1.43	3.95
9	9.90	5	27	4.84	2.05	6.50	4	11	1.48	4.39	5.30	3	8	1.34	3.95
Overall	9.95	4	30	5.34	1.86	6.22	3	13	1.72	3.61	5.41	3	8	1.35	4.02

[1] See Table 3.

[2] 30 problems were generated.

Table 3
Nine Distinct Initial Points (Q_0, ROP_0)

ROP_0 (Initial reorder point)	Q_0 (Initial lot size)		
	R	$\frac{R}{2}$	$\frac{R}{4}$
μ	1	2	3
$\mu+\sigma$	4	5	6
$\mu+2\sigma$	7	8	9

Table 4
Analysis of Variance Test for the Number of Iterations

Source	Mean Square	d.f.*	F	F .05**
Marginal distribution of leadtime demand (A)	648.27	2	3.90	3.04
Criterion range (B)	4866.37	2	40.52	3.04
A×B	39.67	4	0.33	2.41
Error	120.09	261		
Initial point (C)	32.42	8	18.56	3.89
A×C	4.30	16	2.46	3.04
B×C	8.47	16	4.85	3.04
A×B×C	2.07	32	1.19	2.41
Error	1.75	2088		

* The degrees of freedom are modified for the conservative test (Winer 1971).
** F-values at the 5% level of significance taken from Hoel (1971, Table V).

model solution, and (3) a trial solution generated at the first iteration of the interactive procedure. More specifically, we are interested in comparing the optimal total inventory cost obtained by the interactive procedure to the *TIC* produced by the conventional model as well as to the *TIC* from the first iteration of the interactive procedure.

The constant marginal cost ratios, C_p/C_h and C_s/C_h, necessary for obtaining a conventional solution[10)] were not available in our experiment. Note that C_h, C_p, and C_s are the marginal holding, ordering, and shortage cost, respectively, and are assumed to be constant in the conventional model. We assumed, for the sake of this experiment, these marginal cost ratios to be

$$C_p/C_h = \text{average over the range of the tradeoff } w_2 \tag{17}$$

and

$$C_s/C_h = \text{average over the range of the tradeoff } w_3. \tag{18}$$

The rationale for these substitutions is given by the observation that if the assumption of constant marginal costs holds, then the interactive procedure with $w_2 = C_p/C_h$ and $w_3 = C_s/C_h$ produces the same solution as the conventional model generates. Furthermore, we used the tradeoffs w_2 and w_3 obtained at the final iteration of the interactive procedure as constant cost factors in order to calculate the total inventory costs. In this experiment, we used $(Q_0, ROP_0) = (R/2, \mu)$ as an initial point for the interactive procedure.

Table 5 summarizes the comparison results of the optimal *TIC* to the conventional *TIC* as well as to the *TIC* obtained by the first iteration of the interactive procedure. It can be seen from this table that the *TIC* obtained by the first iteration is significantly less than the *TIC* generated by the conventional model. The conventional model never produced a better total inventory cost in the 270 randomly generated problems. These results indicate that the interactive procedure with even one iteration can outperform the conventional model.

Table 5 shows that the interactive procedure with even one iteration can produce a near optimal total inventory cost. The average ratios of the *TIC* generated by the first iteration to the optimal *TIC* varied between 1.0009 and 1.0084. For the 270 generated problems, the maximum percent deviation of the *TIC* calculated by the first iteration from the optimal *TIC* was 8.38%. Therefore, one can see that even if the interactive procedure terminates quickly, the total inventory cost will close to the optimum.

Based on these results, it can be asserted that the interactive procedure is useful whenever the assumption of the conventional model (i.e., constant marginal costs) cannot be met; and that the interactive procedure with few iterations produces a less total inventory cost than the conventional model. Moreover, the total inventory cost obtained by few iterations will be close to the optimum. The observed good performance of the interactive procedure can be due to the fact that the marginal cost information is implicit in the manager's tradeoffs used in the procedure. More specifically, the interactive procedure allows for

Table 5
Mean and Standard Deviation (in parentheses) of the Ratios of Total Inventory Costs

Distribution	Ratio	Criterion Range		
		Range 1	Range 2	Range 3
Uniform	TIC^1/TIC^*	1.0055	1.0042	1.0063
		(0.0061)	(0.0111)	(0.0163)
	TIC^c/TIC^*	1.4174	1.8326	1.6953
		(0.3956)	(0.8239)	(0.7156)
Normal	TIC^1/TIC^*	1.0042	1.0048	1.0022
		(0.0048)	(0.0120)	(0.0043)
	TIC^c/TIC^*	1.2862	1.7489	1.6543
		(0.3251)	(0.7874)	(0.9553)
Exponential	TIC^1/TIC^*	1.0084	1.0025	1.0009
		(0.0124)	(0.0048)	(0.0028)
	TIC^c/TIC^*	1.3931	1.8089	1.5131
		(0.3416)	(0.7354)	(0.5138)

where:
TIC^1 =Total inventory cost calculated with a trial solution obtained at the first iteration.
TIC^* =Optimal total inventory cost.
TIC^c =Total inventory cost calculated with a conventional solution.

the possibility of marginal costs varying with the actual levels of total investment in inventory, workload, and shrtages.

It is noted that the interactive procedure will give the same result as the conventional model does if the assumption of constant marginal costs holds. Even in this case, our procedure may be attractive in some applications as the absolute values of marginal costs need not be estimated. It was argued that the marginal cost information is implicit in the manager's tradeoffs used in the interactive procedure. Therefore, the minimum contribution of our procedure will be in the consistency checking by providing an alternative way of implementing the inventory model.

V. Conclusions and Suggestions for Further Research

We have presented an interactive procedure for the selection of multicriteria inventory policies. The procedure elicits two types of information sequentially from a manager, through an interaction process, until a final inventory policy is determined. First, the manager evaluates his local tradeoffs among invenstment in inventory, workload, and service level. Second, the manager compares several tentatively identified inventory policies to check for an improved policy. These two kinds of local information are sufficient to complete one iteration of the interactive procedure. The iteration continues until a final inventory policy is identified.

The interactive procedure presented here can be coordinated on a computerized man-machine interactive system, where the computer does the mechanical portions of the procedure and allows a manager to concentrate on making tradeoff judgements and checking for an improved inventory policy. In other words, the entire procedure can be viewed by a manager as taking place in criterion space (i.e., inventory investment, workload, and service level) rather than in decision space (i.e., lot size and reorder point) and the mechanical portions of the procedure are delegated to the computer.

A computer experiment was undertaken to provide empirical evidence on the working of the interactive procedure. The results of the experiment showed that with few iterations the procedure reaches a near optimal total inventory cost. Moreover, the procedure fares well in comparison with the conventional inventory model whenever the conventional assumption of constant marginal costs cannot be validly made. These findings support our conjecture that the interactive procedure allows for the possibility of marginal cost variations.

Based on these results, the interactive procedure shows a promise for further research. The interaction with a human manager is the most important research area since many issues can only be explored if a human manager is involved. Such issues are, for example:

1. How difficult is it for a human manager to provide the required information?

2. Can a human manager understand easily the logic of the interactive procedure?

3. Is the interactive procedure easy to use for a human manager?
Another direction of further research is to develop an interactive procedure which involves a multi-item inventory system. Since the most important inventory management issues in practice involve aggregate objectives and constraints, there is a definite need for developing an interactive procedure for multi-item inventories. Such development can be made by modifying the interactive procedure addressed in this paper.

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Notes

1) We assume the unit time to be a year.
2) Assuming $Q \geq 0$ and $ROP \geq 0$ yields

$$Q=\sqrt{(2R/U)\,[C_p/C_h+(C_s/C_h)\int_{ROP}^{\infty}(D-ROP)\phi(D)\,dD]} \tag{A1}$$

and

$$\Psi(ROP)=1/[1+(C_hUQ)/(C_sR)] \tag{A2}$$

where

$$\Psi(ROP)=1-\int_{ROP}^{\infty}\phi(D)dD. \tag{A3}$$

It should be recognized that each of variables (i.e., Q and ROP) cannot be solved directly since they depend each other. The method of successive approximation can then be used to yield values for Q and ROP that simultaneously solve (A1) and (A2).

3) A solution (Q_{k+1}, ROP_{k+1}) is given

$$Q_{k+1}=\sqrt{(2R/U)\,[w_2^k+w_3^k\int_{ROP_{k+1}}^{\infty}(D-ROP_{k+1})\phi(D)dD]} \tag{A4}$$

and

$$\Psi(ROP_{k+1})=1/[1+(UQ_{k+1})/(Rw_3^k)] \tag{A5}$$

where

$$\Psi(ROP_{k+1})=1-\int_{ROP_{k+1}}^{\infty}\phi(D)dD. \tag{A6}$$

4) Compare the lot size (Q) and reorder point (ROP) expressions in (A1) and (A2) and those in (A4) and (A5).
5) Buffa and Taubert (1972, pp 96–110) indicates that the normal, Poisson, and

exponential distributions have been found to be of considerable value in representing leadtime demand distribution for inventory management. Since the Poisson distribution is discrete, we replaced it with the continuous uniform distribution.

6) The functional forms of $I(Q, ROP)$, $W(Q)$, and $S(Q, ROP)$ are given by equations (1), (2), and (3) in Section II, respectively.
7) Therefore, each single-criterion value function $V_i(x_i)$ is decreasingly concave.
8) Further statistical analyses indicated that the independence assumption required for the validity of F-test is weak in this experiment. Therefore, we employed the conservative test in which F-test can be made by reducing the degrees of freedom (Winer 1971).
9) The heuristic (s, Q) inventory model discussed in Section II is referred to as the conventional inventory model.
10) See equations (A1) and (A2) in this notes.

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