

A General Equilibrium Interpretation of Damage Contingent Securities*

R. Anton Braun,[†] Richard M. Todd,[‡] and Neil Wallace[§]

October 28, 1999

Abstract

In a 1996 *Econometrica* paper, Cass, Chichilnisky, and Wu show in an endowment economy that mutual insurance and securities contingent on aggregate states support optimal risk-sharing. We extend their result to a model with production in which risk is endogenous and beliefs about the aggregate state vary across individuals. We use the model to interpret the role of new securities that are contingent on measures of total damage from natural catastrophes. Plausible special cases of the model predict the trade pattern in such securities across diverse regions and predict that such securities will not represent actuarially fair gambles.

JEL classification: D8, D61.

Keywords: Mutual insurance, risk sharing, general equilibrium, damage-contingent securities.

*The authors acknowledge the help of Brian Atwater, Karen Hovermale, Jean Rawson, and Ron Feldman, two referees and the editor. Some of the work for this paper was done while Braun and Wallace were in residence at the Research Department of The Federal Reserve Bank of Minneapolis. We are indebted to the Bank for research support. However, the views expressed are those of the authors and not necessarily those of the Bank or the Federal Reserve System.

[†]International University of Japan

[‡]The Federal Reserve Bank of Minneapolis

[§]The Pennsylvania State University and The Federal Reserve Bank of Minneapolis.

1 Introduction

Cass, Chichilnisky, and Wu (1996) show in an endowment economy that if the structure of uncertainty resembles an individual insurance situation, then a combination of mutual insurance and securities contingent on aggregate states supports optimal risk-sharing. In other words, in such settings, a complete set of Arrow-Debreu markets is not needed. We extend their result to a model with production in which the amount of risk depends on individual decisions---as when houses are built in areas prone to hurricanes.

We study a model of a static economy in which the aggregate amount of resources subject to risk is the result of individual investment decisions and in which beliefs about the aggregate state vary across individuals. For that model, we show that a Pareto efficient allocation can be supported by a competitive equilibrium with two types of securities: mutual insurance that provides for risk sharing among individuals who are ex ante similar, and a set of state-contingent securities that are indexed by the aggregate exogenous state. This extends the result in Cass, Chichilnisky, and Wu (1996) to the case of a productive economy where the level of aggregate risk is endogenous. We go on to show that the state-contingent securities can be replaced by securities contingent on aggregate losses rather than the exogenous state, despite the fact that aggregate losses are endogenous. For special cases of the model, we describe the implied pattern of trade in the contingent contracts and how they are priced. We show that there is no theoretical basis for believing that the damage-contingent securities are priced in an actuarially fair manner. If individuals have identical beliefs, then the price of a damage contingent security will in general include a risk premium that rewards individuals for taking on this risk. After presenting those results, we conclude with a brief discussion of recent policy initiatives and related discussion concerning losses from natural catastrophes.

2 A Model with Endogenous Damage

We present a static model that is designed to represent, in a simple way, regions which end up being subject to different amounts of risk. The economy consists of a finite number of islands that are indexed by h from 1 to H . Island h is inhabited by N_h people of type h . Each island is perfectly round and has a plateau in the middle of it. The land on the coast is subject to the risk of damage (from storms), while land located on the plateau is safe. Each person on island h is endowed with an ex ante identical slice of land, which includes some coastal land and some plateau land, and with a resource, y_h , which can be interpreted as labor. The only use of the resource is as an input into production of a single good, rice, on the coastal land or on the plateau land owned by that person. All type- h people are identical in terms of von Neumann-Morgenstern utility function, endowments, and technologies.

We let $u_h: R_+ \rightarrow R$ denote a type h person's von Neumann-Morgenstern utility function (for rice consumption), and we assume that u_h is strictly increasing, strictly concave, and continuously differentiable. The technologies determine the distribution of output resulting from the decision about dividing y_h between growing rice on the coast and on the plateau. If a type h person devotes y^r to coastal (risky) production and $y_h - y^r$ to plateau production, then the resulting crop is $f_h(y^r) + g_h(y_h - y^r)$ if the coastal crop is not destroyed and is $g_h(y_h - y^r)$ if it is destroyed. We assume that the functions f_h and g_h are strictly increasing, strictly concave, continuously differentiable, and satisfy $f_h(0) = g_h(0) = 0$. Finally, so as not to have to consider corner solutions, we assume that u_h , f_h , and g_h have infinite derivatives at zero. People on different islands (different types) differ in all regards, except that all grow the same good, rice.

Damage that hits island h could destroy the coastal crop of any number plots of coastal land from none of them to all of them. Let $\mathbf{N}_h \equiv \{0, 1, \dots, N_h\}$ and let $\mathbf{N} \equiv \mathbf{N}_1 \times \mathbf{N}_2 \times \dots \times \mathbf{N}_H$. A generic element $n \in \mathbf{N}$ has the form $n = (n_1, n_2, \dots, n_H)$, where we interpret n_h to be the number of coastal plots of island h destroyed. We refer to

\mathbf{N} as the set of aggregate states. We assume that each person has a subjective distribution over the set \mathbf{N} , distributions that can differ even among the inhabitants of a single island¹. As regards the underlying individual uncertainty, let (h, j) be the label for person j of type h , let $N = \sum_h N_h$, and let $\mathbf{S} \equiv \{0,1\}^N$. A generic element $s \in \mathbf{S}$ has the form $s = (s_1, s_2, \dots, s_H)$, and where $s_h = (s_{h1}, s_{h2}, \dots, s_{hN_h})$ and $s_{hj} = 0$ means that the coastal land of person (h, j) is destroyed and $s_{hj} = 1$ means that it is not destroyed. An element of \mathbf{S} is a complete description of the realization of uncertainty. We make two assumptions about the probability distribution over the elements of \mathbf{S} , assumptions which we express in terms of distributions conditional on an aggregate state $n \in \mathbf{N}$. We assume there is unanimity about the distribution over \mathbf{S} conditional on the aggregate state. And we assume that the probability that $s_{hj} = 0$ conditional on the aggregate state being n is n_h/N_h . Given the unanimity, we can let $p(s;n)$ be the probability that the state is s conditional on the aggregate state being n . Then, our assumption about the probability of individual destruction is

$$\sum_{s \in n} p(s;n) s_{hj} = 1 - \frac{n_h}{N_h}, \quad (1)$$

where the notation $s \in n$ means $s \in \mathbf{S}$ consistent with the occurrence of aggregate state n . The left-hand side of (1) is the conditional probability of no destruction for person (h, j) 's coastal land. The assumption says that that probability depends only on h and on the aggregate state through n_h , and is equal to the fraction of coastal plots on island h that are not destroyed in aggregate state n .

There is a sense in which these assumptions and our other assumptions are innocuous. Situations that lend themselves to individual insurance seem to naturally satisfy unanimity conditional on the aggregate state. As for the assumption in (1), it goes along with the notion that there is uniformity within a group for which there is to be

¹ We allow for individual-specific beliefs because we believe that there are huge inherent difficulties in predicting events like a repeat of the 1923 earthquake in Tokyo, difficulties that

mutual insurance. Put differently, one can think of our islands as being formed (by insurance companies) so that the resulting groups are ex ante identical in the senses we have assumed, including the assumption in (1).

We let $q_{hj}(n)$ be (h, j) 's subjective probability that the aggregate state is n and we let $c_{hj}(s)$ be (h, j) 's consumption in state s . Then the expected utility of (h, j) can be written as

$$U_{hj} = \sum_{n \in \mathbf{N}} q_{hj}(n) \sum_{s \in n} p(s; n) u_h[c_{hj}(s)]. \quad (2)$$

For convenience, we assume that $q_{hj}(n) > 0$, although everything we do is easily amended to deal with aggregate states that some people or all people think will never occur. We also let y_{hj}^r be (h, j) 's input into coastal production. Then the economy's resource constraints can be written as

$$\sum_{h=1}^H \sum_{j=1}^{N_h} \{c_{hj}(s) - s_{hj} f_h(y_{hj}^r) - g_h(y_h - y_{hj}^r)\} \leq 0 \quad (3)$$

for each $s \in \mathbf{S}$.

Our main goal is to show that an optimum is achieved under competition by a particular market structure: within-type mutual insurance and securities contingent on an H -element vector of total damage whose h -th component is the total damage suffered on island h . We do this in two steps. We first show that a combination of mutual insurance and securities contingent on the set of aggregate states, the elements of \mathbf{N} , achieves an optimum. Then we show that any allocation supported by that market structure is also supported by one where we replace the contracts contingent on the set of aggregate states by the contracts contingent on total damage.

very naturally give rise to a diversity of opinion.

3 Mutual insurance and contingent claims

We begin by setting out the competitive choice problem under mutual insurance on individual outcomes and aggregate-state contingent claims. Given this market structure, the individual chooses risky investment (the amount to invest in the person's coastal land), the amount of it to insure, and claims on the consumption good contingent on the aggregate states.

Let $Y_{hj}(n, s_{hj}, y^r, \theta)$ be (h, j) 's income if the aggregate-state is n , (h, j) 's individual state is $s_{hj} \in \{0,1\}$, $y_{hj}^r = y^r$ and the fraction θ of (h, j) 's risky investment is mutually insured.² The mutual insurance is such that the part of risky input insured has a net payoff in state n equal to the product of the fraction of coastal land not destroyed in aggregate-state n and the output from successful risky production of person (h, j) . In terms of our notation,

$$Y_{hj}(n, s_{hj}, y^r, \theta) = \theta \left(1 - \frac{n_h}{N_h}\right) f_h(y^r) + (1 - \theta) s_{hj} f_h(y^r) + g_h(y_h - y^r). \quad (4)$$

Later, we will see that this type of mutual insurance is self-financing within island h in any equilibrium. Notice that $Y_{hj}(n,1, y^r, \theta) \geq Y_{hj}(n,0, y^r, \theta)$ and with equality if $\theta = 1$.

We let $Q_{hj}(n)$ be the claims on consumption in aggregate state n sold by (h, j) . If $P(n)$ denotes the price of a claim on one unit of aggregate-state n consumption, then we have the usual budget constraint on sales (and purchases); namely,

$$\sum_{n \in \mathbf{N}} P(n) Q_{hj}(n) = 0. \quad (5)$$

² In some aggregate states, $Y_{hj}(n,1, y^r, \theta)$ and $Y_{hj}(n,0, y^r, \theta)$ do not both exist, because one individual state occurs with certainty. That does not create a problem provided all statements we make are interpreted to exclude the individual state that does not exist.

Finally, we let $C_{hj}(n, s_{hj})$ be (h, j) 's consumption in aggregate-state n and individual state s_{hj} . Then for $s_{hj} = 0$ and for $s_{hj} = 1$, we have,³

$$C_{hj}(n, s_{hj}) \leq Y_{hj}(n, s_{hj}, y^r, \theta) - Q_{hj}(n). \quad (6)$$

In terms of $C_{hj}(n, s_{hj})$ and using assumption (1), it follows from (2) that (h, j) 's expected utility is

$$U_{hj} = \sum_{n \in \mathbf{N}} q_{hj}(n) \left\{ \left(1 - \frac{n_h}{N_h}\right) u_h[C_{hj}(n, 1)] + \frac{n_h}{N_h} u_h[C_{hj}(n, 0)] \right\}. \quad (7)$$

Therefore, we can state the choice problem of person (h, j) as follows: choose $[C_{hj}(n, 0), C_{hj}(n, 1)]$ and $Q_{hj}(n)$ for $n \in \mathbf{N}$ and $\theta \in [0, 1]$ and $y^r \in [0, y_h]$ to maximize U_{hj} as given by (7) subject to (4)-(6).

Our first result is that $\theta = 1$ is optimal.

Lemma 1 *In the above individual choice problem, $\theta = 1$ is optimal.*

Proof. Suppose (θ, y^r, Q) is a feasible choice for (h, j) , where Q denotes a vector of contingent claims sales. Now consider instead the same choice, but with θ replaced by unity, the choice $(1, y^r, Q)$. It follows from (4) that $Y_{hj}(n, 0, y^r, 1) \geq Y_{hj}(n, 0, y^r, \theta)$.

Also, if $n_h = 0$, then $Y_{hj}(n, 1, y^r, 1) = Y_{hj}(n, 1, y^r, \theta)$. Therefore, $(1, y^r, Q)$ is a feasible choice for (h, j) . The choice $(1, y^r, Q)$ implies that $Y_{hj}(n, 1, y^r, 1) = Y_{hj}(n, 0, y^r, 1)$. That and (6) at equality, a necessary condition for an optimal choice, imply that consumption does not depend on s_{hj} under $(1, y^r, Q)$. Therefore, we let $C_{hj}(n)$ denote the consumption in aggregate state n implied by $(1, y^r, Q)$. It follows from (6) at equality and (4) that

³ Because consumption is non-negative, a consequence of this inequality is that the commitment implied by $Q_{hj}(n)$ can be met even if (h, j) 's coastal production is lost. However, in accord with the previous footnote, if $s_{hj} = 0$ is impossible in aggregate state n , then there is no such requirement.

$$C_{hj}(n) = \left(1 - \frac{n_h}{N_h}\right)C_{hj}(n,1) + \frac{n_h}{N_h}C_{hj}(n,0), \quad (8)$$

where $C_{hj}(n, s_{hj})$ on the right-hand side of (8) denotes the consumption implied by (θ, y^r, Q) . But, then, by the concavity of u_h , $U_{hj}(1, y^r, Q) \geq U_{hj}(\theta, y^r, Q)$ (and strictly if $y^r > 0$ and $n_h > 0$). ■

With $\theta = 1$, the individual choice problem simplifies because, according to (4), person (h, j) 's income in state n does not depend on s_{hj} . Therefore, we can restate the individual choice problem more simply as one of choosing y^r and $C_{hj}(n)$ for $n \in \mathbf{N}$ to maximize

$$U_{hj} = \sum_{n \in \mathbf{N}} q_{hj}(n) u_h[C_{hj}(n)] \quad (9)$$

subject to

$$\sum_{n \in \mathbf{N}} P(n) C_{hj}(n) = \sum_{n \in \mathbf{N}} P(n) \left\{ \left(1 - \frac{n_h}{N_h}\right) f_h(y^r) + g_h(y_h - y^r) \right\}. \quad (10)$$

Expression (9) is obtained from (7) by equating $C_{hj}(n,0)$ and $C_{hj}(n,1)$ which, as noted above, follows from $\theta = 1$. The budget constraint (10) follows from substituting for the Q 's in (5) from (6) at equality, while using the definition of income in (4).

Maximization of (9) subject to the budget constraint (10) is a standard competitive choice problem for a setting in which each person is endowed with a production possibility set in the space of commodities with one commodity for each $n \in \mathbf{N}$. As is well-known and is obvious from the above choice problem, person (h, j) chooses y^r to maximize income, the right-hand side of (10). Moreover, because our assumptions about f_h and g_h imply that (h, j) 's production possibility set is strictly convex, it follows that at given prices, the choice of y^r is unique and does not depend on other than (h, j) 's type---that is, on other than h . That conclusion and $\theta = 1$ imply that the mutual insurance is feasible because in aggregate state n , total type h coastal production is $(N_h - n_h) f_h(y_h^r)$, which is consistent with the per capita island h payoff that appears in (4).

Given the common type h input decision, we can define a competitive equilibrium under the market structure of this section as follows.

Definition 1 *An allocation---an input decision for each type, y_h^r for $h=1,2,\dots,H$ and $C_{hj}(n)$ for each $n \in \mathbf{N}$ and each (h, j) ---and prices, $P(n)$ for each $n \in \mathbf{N}$, is a mutual-insurance competitive equilibrium if y_h^r and $C_{hj}(n)$ for each $n \in \mathbf{N}$ solves (h, j) 's maximization problem given the prices and if the allocation is feasible; namely,*

$$\sum_{(h,j)} C_{hj}(n) \leq \sum_{h=1}^H N_h \left\{ \left(1 - \frac{n_h}{N_h}\right) f_h(y_h^r) + g_h(y_h - y_h^r) \right\} \quad (11)$$

for each $n \in \mathbf{N}$.

If we view the above economy to be one with a good for each $n \in \mathbf{N}$, then it is a standard economy. Therefore, existence of a competitive equilibrium is guaranteed. Moreover, our assumptions imply that the prices are positive, that each person consumes a positive amount in each aggregate state, and that both technologies are used on each island. We want to show that any competitive equilibrium allocation is optimal for the Pareto problem implied by our economy. To do that, we cannot simply appeal to the first welfare theorem because the market structure we are studying is not the complete-markets (Arrow-Debreu) structure for our economy. (The Arrow-Debreu market structure has markets contingent on the elements of \mathbf{S} .) Hence, we demonstrate the claim directly.

The Pareto problem is to choose $c_{hj}(s)$ and y_h^r to maximize a positive weighted average of expected utilities, as given by (2), subject to (3) for each $s \in \mathbf{S}$. Because the objective in that problem is concave and the constraint set is convex, the Kuhn-Tucker conditions are sufficient for an optimum. Let λ_{hj} denote the social planning weight on person (h, j) 's expected utility as given by (2) and let $\phi(s)$ be the non-negative multiplier associated with (3) for state s . For positive multipliers, the Kuhn-Tucker conditions are

$$\lambda_{hj} q_{hj}(n) p(s; n) u_h'[c_{hj}(s)] = \phi(s), \quad (12)$$

for all s and (h, j) , and

$$f'_h(y_{hj}^r) \sum_{s \in \mathbf{S}} \phi(s) s_{hj} = g'_h(y_h - y_{hj}^r) \sum_{s \in \mathbf{S}} \phi(s), \quad (13)$$

for all (h, j) .

We now prove that any definition 1 competitive equilibrium allocation satisfies those conditions for some social planning weights and some positive multipliers.

Proposition 1 *A definition 1 competitive equilibrium allocation is Pareto efficient. (Let \hat{y}_h^r for $h=1,2,\dots,H$ and $\hat{C}_{hj}(n)$ for each $n \in \mathbf{N}$ and each (h, j) be a competitive allocation according to definition 1. There exist positive planning weights λ_{hj} and positive multipliers $\phi(s)$ such that (12) and (13) hold.)*

Proof. We propose planning weights and multipliers and show that they and \hat{y}_h^r and $\hat{C}_{hj}(n)$ satisfy (12) and (13). Let $\hat{P}(n)$ denote the competitive equilibrium price associated with the \hat{y}_h^r and $\hat{C}_{hj}(n)$ allocation. For $s \in n$, let $\phi(s) = p(s; n) \hat{P}(n)$ and $c_{hj}(s) = \hat{C}_{hj}(n)$. Also, let $y_{hj}^r = \hat{y}_h^r$ and let λ_{hj} satisfy

$$\lambda_{hj} q_{hj}(1) u'_h[\hat{C}_{hj}(1)] = \hat{P}(1), \quad (14)$$

where 1 in (14) denotes a particular aggregate state.

By definition 1, $\hat{C}_{hj}(n)$ satisfies the familiar marginal condition,

$$\frac{q_{hj}(1) u'_h[\hat{C}_{hj}(1)]}{q_{hj}(n) u'_h[\hat{C}_{hj}(n)]} = \frac{\hat{P}(1)}{\hat{P}(n)}. \quad (15)$$

It follows, from (14), (15), and the proposal for $c_{hj}(s)$ that (12) holds.

According to the proposal,

$$\sum_{s \in \mathbf{S}} \phi(s) = \sum_{n \in \mathbf{N}} \sum_{s \in n} \phi(s) = \sum_{n \in \mathbf{N}} \sum_{s \in n} p(s; n) \hat{P}(n) = \sum_{n \in \mathbf{N}} \hat{P}(n) \quad (16)$$

and

$$\begin{aligned}
\sum_{s \in \mathbf{S}} \phi(s) s_{hj} &= \sum_{n \in \mathbf{N}} \sum_{s \in n} \phi(s) s_{hj} = \\
\sum_{n \in \mathbf{N}} \hat{P}(n) \sum_{s \in n} p(s; n) s_{hj} &= \sum_{n \in \mathbf{N}} \hat{P}(n) \left(1 - \frac{n_h}{N_h}\right),
\end{aligned} \tag{17}$$

where the last equality is the assumption in (1). By definition 1, \hat{y}_h^r satisfies the marginal condition,

$$f_h'(\hat{y}_h^r) \sum_{n \in \mathbf{N}} P(n) \left(1 - \frac{n_h}{N_h}\right) = g_n'(y_h - \hat{y}_h^r) \sum_{n \in \mathbf{N}} P(n). \tag{18}$$

It follows from (16)-(18) that $y_{hj}^r = \hat{y}_h^r$ satisfies (13). ■

That completes our analysis of mutual insurance with markets contingent on aggregate states. Our next task is to discuss the somewhat more realistic market structure in which we replace markets contingent on aggregate states with markets contingent on total damage.

4 Mutual insurance and total damage contingencies

Given common input decisions by all type- h people, total damage in aggregate state n is $(n_1 f_1(y_1^r), n_2 f_2(y_2^r), \dots, n_H f_H(y_H^r))$. Therefore, there is a one-to-one correspondence between aggregate states and such total damage vectors, given the interpretation that underlying any set of such total damage vectors is a single vector of island-specific inputs into the risky technology. Given that interpretation, we need one other assumption: each person treats parametrically, as unaffected by the person's own decision about how to allocate the resource between risky and safe production, the vector of island-specific inputs that underlies any set of total damage vectors. While that seems to be a new assumption, it is implicit in the price-taking behavior of definition 1, because that behavior assumes that each person is a small part of the trades in aggregate state contingent payoffs. That, in turn, has implicit in it that N_h is large. The new

assumption guarantees that each person acts as if he or she does not affect the probability with which a particular total damage contingency occurs.

Before formally stating a definition of equilibrium, we describe in more detail how we assume individuals behave. Imagine for a moment that a pseudo-Walrasian auctioneer announces a vector of island-specific inputs into the risky technology, \tilde{y}_h^r for each h , and a price of output contingent on the occurrence of each implied total damage vector. Given \tilde{y}_h^r for each h , the aggregate state corresponding to each total damage vector can be deduced. We assume that each person treats the price of output contingent on a given total damage vector as the price of output in the corresponding aggregate state and then behaves as described in the last section. It follows, then, that each person chooses to fully insure any risky investment; that is, chooses $\theta = 1$. That leads us to define an equilibrium under the current market structure in the following abbreviated way.

Definition 2 *An equilibrium under mutual insurance and prices contingent on total damage vectors is a vector of announced island-specific inputs into the risky technology, \tilde{y}_h^r for each h and a definition 1 equilibrium that satisfies equality between the announced inputs and the definition 1 inputs.*

Although the result is trivial, we formally state the equivalence between the two notions of equilibrium.

Proposition 2 *Any definition 1 equilibrium is a definition 2 equilibrium and vice versa.*

The result is immediate because an equilibrium under definition 2 has, in addition to the conditions for satisfaction of definition 1, H additional unknowns, \tilde{y}_h^r for each h , and a trivial condition on each additional unknown: $\tilde{y}_h^r = \hat{y}_h^r$, where \hat{y}_h^r is a definition 1 equilibrium island h input into risky production.

5 Pricing and trade

At the level of generality at which we have specified our model, it says almost nothing about the prices of claims on consumption across elements of \mathbf{N} , the set of aggregate states, nor about the pattern of trade of these claims. Depending on the structure of beliefs, the aggregate-state contingent claims may be priced at a premium or even a discount relative to the benchmark of actuarially fair prices. As a way of displaying some of the possibilities for pricing and trade patterns, we here discuss a special case.

Suppose there are two islands and that everyone has identical subjective distributions over the elements of \mathbf{N} . Suppose also that the technologies, the f_h and g_h functions, are such that in island 1 very little risky (coastal) production occurs and in island 2 very little safe production occurs. Finally, suppose that there are identical endowments of resource and identical von Neumann-Morgenstern utility functions.

In such a case, the significant variation across aggregate states is variation for island 2 outcomes. Therefore, an equilibrium has the following features. Total output varies inversely with the amount of damage suffered by island 2. And, because everyone is risk averse, the contingent prices across such aggregate states reflect not only the probabilities of those states, but also the total output in the state. In particular, if n is an aggregate state with a great deal of damage to island 2 and n' is a state with little damage to island 2, then $P(n)/q(n) > P(n')/q(n')$, where $q(n)$ is the unanimous probability that aggregate state n occurs. In addition, the residents of island 1 insure the residents of island 2 in the sense that the island 1 residents sell claims on output in state n and buy claims on output in state n' , while residents of island 2 take the reverse positions. And, because the prices satisfy the inequality just mentioned, that insurance is not actuarially fair.

Although this example is very special, it gives the flavor of the kinds of prices and trade patterns we are likely to see across disparate regions. It also illustrates the role of the two types of insurance. As is true for the model in general, the mutual insurance

provides for risk sharing within each island in the sense that it produces within-island uniformity of aggregate-state contingent income. In this example, with identical beliefs, there is no within-island diversity. Hence, all those on island 1 consume the same amount and all those on island 2 consume the same amount. The aggregate-state contingent claims provide for risk sharing across the two islands. An important feature of this example is that risk across the two islands is not completely pooled. As long as people are risk averse, consumption on the two islands will differ because residents of island 1 demand a risk premium to insure residents of island 2.

6 Comparison with complete markets

As noted in Cass, Chichilnisky, and Wu (1996), the mutual insurance market structures of sections 3 and 4 have many fewer markets than the complete markets (Arrow-Debreu) structure. The contingencies in the complete markets structure are the elements of the set \mathbf{S} . It is obvious that the set \mathbf{S} has many more elements than the set of aggregate states, \mathbf{N} , the elements of which are the contingencies in the market structures of sections 3 and 4. In particular, corresponding to each $n \in \mathbf{N}$ are many elements in \mathbf{S} .

Not only does the complete-markets set-up require more markets, for many if not most fairly natural economies, it seems to display an extreme form of market thinness. For example, for our economy consider the element in \mathbf{S} in which only the coastal production of one person in the entire economy is destroyed. Then many specifications would give rise to a complete markets competitive allocation in which that person buys claims on consumption in that state and everyone else sells such claims. But with one purchaser of such claims, price-taking behavior seems far-fetched. The market structures in sections 3 and 4 seem to suffer less from such thinness.

7 Concluding Remarks

We conclude by relating our analysis to some recent developments in the insurance industry. This discussion serves to highlight the special role played by aggregate-state contingent securities in providing efficient risk-sharing.

As measured by the damage inflicted on property, the past decade has been marked by unusually severe natural catastrophes. Insured damage from Hurricane Hugo in 1989, over \$5 billion in 1997 dollars, exceeded by more than 50% the largest insured loss from previous natural catastrophes. Three years later, insured losses from Hurricane Andrew reached almost four times those caused by Hugo, and the Northridge earthquake of 1994 caused insured damage at nearly double Hugo's level. Elsewhere, Europe has experienced unprecedented losses from wind storms, and Japan experienced property losses in the Kobe earthquake of 1995 that exceeded all previous postwar catastrophe losses.

These natural catastrophes and fears of even larger catastrophe losses have raised concerns that existing arrangements for sharing the risks of natural catastrophes are inadequate.⁴ In response to such concerns, governments have taken up or enacted initiatives involving government-sponsored insurance or reinsurance programs. Insurance commissioners in Florida, Hawaii, and California expanded their state's residential property insurance programs. At the federal level, an insurance industry proposal for a government-backed reinsurance fund was introduced into the Senate, and the Clinton administration and some Republican senators countered with a proposal that the federal government auction off reinsurance coverage for losses in the \$25 to \$50 billion range. Any such program would be in addition to existing federal crop and flood insurance programs and federal catastrophe relief programs.

⁴ Insurance industry sources claim that the US industry cannot cope with catastrophe losses that exceed \$40 billion (see Hofmann, 1996). Others have also questioned the adequacy of current arrangements for re-insurance, whereby insurance companies transfer some of their own risks and a share in the associated premium income to other companies (D'Arcy and France, 1992; Lewis and Murdock, 1996; Kunreuther, 1995; Thompson, 1995).

These events and the responses to them make one wonder what is special about risks related to natural catastrophes. Asymmetric information---which potentially gives rise to moral hazard, adverse selection, and costly ex post verification---seems, if anything, less severe in the context of natural disasters than it does in some other types of risky situations.

While insurance could induce property owners to take fewer precautions than they otherwise would, such possibilities seem to be dealt with well using deductibles. Adverse selection seems not to be a severe problem because potential insurers seem at least as knowledgeable about exposure to natural disaster risks as are individual property owners. Nor does costly ex-post verification pose special problems, although major disasters do seem to strain the insurance-adjuster resources of the insurance industry. We are, therefore, led to conclude that the special feature of natural catastrophes is that they pose risk in the aggregate, which, of course, means that at least some peoples' well-being must be contingent on outcomes.⁵

Private markets have long provided arrangements for pooling risks related to natural forces when the chances for large aggregate losses were low. The most obvious arrangement is the familiar one of property insurance, financed either as a mutual enterprise of the insured or by equity investors.

In the past, reinsurance contracts have been the principal private-sector arrangement for transferring the risks of aggregate loss. Recently, however, new arrangements involving contingent securities have begun to emerge. In December 1992 the Chicago Board of Trade (CBOT) began trade in futures and options contracts whose payoff depends on measures of aggregate claims against insurance companies for losses due to natural catastrophes. These catastrophe futures and options have since been supplemented by futures and options contingent on the average yield achieved by corn producers in certain states in the US. In addition, some insurance companies have issued so-called "act of God" bonds, whose principal and/or coupon payments may depend on

⁵ In Miranda and Glauber (1997), a similar conclusion is reached for crop insurance.

a measure of natural catastrophe losses. These securities seem to be close analogues of those in the model we have presented.

Despite that similarity, some observers have expressed skepticism about such securities. Some have suggested that markets for catastrophe risk may be one-sided in the sense that they lack “natural sellers” of the insurance they provide (Cox and Schwebach, 1992; D’Arcy and France, 1992; Niehaus and Mann, 1992; *The Economist*, 1994; Thompson, 1995). Who, in other words, will be willing to bet against large aggregate losses? The model suggests an answer, the usual answer: prices adjust to attract bets against large aggregate losses.

Relatedly, questions have also been raised about the likely rate of return on catastrophe-contingent securities. Some have suggested that the price of a CBOT catastrophe futures contract should provide a consensus expectation of the losses that go into the contract's index (Cox and Schwebach, 1992; D’Arcy, and France, 1992). Others have viewed as a defect that the new contracts seem to offer unusually high expected returns to those providing aggregate loss insurance (Schachner, 1996). Implicit in both is the standard of actuarial fairness. Our model suggests that there is no reason to expect the prices on aggregate-state contingent claims to be actuarially fair. In fact, as the example in section 5 illustrates, the expected returns may indeed be high relative to normal investment returns and to actuarially fair returns. Such discrepancies arise as an inherent part of optimal arrangements for sharing aggregate risks and do not imply that the markets for the new contracts are defective.

With islands interpreted as regions, our model says that an important component of the overall risk-sharing mechanism is a freely functioning insurance market in risk-prone areas that spreads the aggregate losses there evenly among all ex-ante identical parties. Actual markets for homeowners' insurance markets are subject to regulation, including regulation of premiums, by state governments. Furthermore, insurance contracts have traditionally bundled insurance against individual risk with insurance against aggregate risk. The equilibrium price of this bundle of insurance services appears to have risen sharply in hurricane and earthquake prone areas in recent years,

and regulators in some catastrophe-prone states may be limiting homeowners' insurance premiums to unrealistically low levels.

After Hurricane Andrew in 1992, for example, many Florida insurers revised upward their estimates of property risks in Florida and sought regulatory approval for higher premiums. Although the state's insurance commission approved substantial premium increases after 1992, many of the state's insurers still tried to reduce their Florida market share. The state government intervened to thwart this effort, passing laws that made it difficult for insurers to cancel or fail to renew coverage. A similar pattern of rate regulation disputes followed by an "availability crisis" occurred in California and Hawaii. Some observers of these availability crises view them as evidence that private markets cannot provide optimal catastrophe risk sharing (Lewis and Murdock, 1996; Thompson, 1995). This alleged market failure may instead be an example of excessive government regulation, which has kept premiums below market-clearing levels. The fact that no comparable crisis developed in the relatively unregulated market for commercial property insurance bolsters this alternative view (see Schachner, 1996).

Optimal arrangements for sharing the risks of catastrophes include both within-region mutualization and cross-region bets on the extent of aggregate loss. As noted above, these separate components of optimal insurance have traditionally been bundled in private homeowners' policies. The new state-run insurance programs in catastrophe-prone areas also promise policyholders both components. But individual state governments have no special advantages for providing cross-regional risk sharing. At best they can attempt to replicate private sector arrangements, by purchasing adequate reinsurance or using the new contingent claims markets. Worse outcomes are also possible, including vague and potentially infeasible proposals for floating and paying off bonds in the wake of a major catastrophe. If reliable arrangements are not made in advance, the actual outcomes may range from defaulting on obligations to policyholders to heavy reliance on within-state cross subsidies from less risky to more risky areas. Indeed, the emphasis on ex post borrowing in these schemes is misleading. What matters is who is liable to repay any such ex post borrowing. One likely alternative

outcome is a set of taxes that on balance subsidizes the at-risk areas in a state by placing too much of the repayment burden on other areas in the state. This will not produce cross-region risk sharing and will contribute to resource misallocation.

We end with two qualifications concerning the rosy picture that we have painted of free-market solutions to allocating risks using total-damage contingent securities. First, governments at all levels may not be able to commit to *not helping* out those who experience losses arising from natural catastrophes. Part of the solution seems simple: require that property owners be insured. Such a requirement is in place for those who have mortgages and it would seem a simple matter to extend it to everyone. Second, the kind of two-stage insurance scheme that we have used to describe optimal risk sharing is not, of course, the only possible scheme. It does, however, have one important virtue; it is transparent (see Marshall, 1974). In contrast, the bundling of the two stages that occurs under most current insurance policies is far from transparent. Holders of such bundled policies and owners and potential owners of insurance companies issuing such policies have an interest in knowing the aggregate-risk exposure of the companies. But acquiring that information is very difficult.⁶ If insurance companies focused on providing mutual policies to those in comparable risk situations, they would be engaged in activities for which they have experience; namely, the setting of criteria for placing properties into comparable ex ante risk situations. They would be out of the business of assessing the probabilities of different aggregate states, something they are not particularly well-suited to doing. It may, though, be quite reasonable for them to serve as intermediaries for their customers in the markets for bets on aggregate states.

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⁶ For an amusing instance of one insurance company's presumption that investors and policyholders did not understand its balance sheet, see the account by former Executive Life official Gary Schulte, as discussed in Todd and Wallace (1992).

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