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Up(and down)-skilling and directed technical change

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Abstract

In this paper we develop a dynamic general equilibrium analog to the Roy model. Specifically, the economy is populated by heterogeneous agents who differ in ability and sort into skilled and unskilled jobs. Because skilled jobs use ability with greater intensity, high (low) ability workers sort into skilled (unskilled) jobs. As in other frameworks, this endogenous cutoff ability depends on the economy's technology and distribution of workers. In contrast to existing 'assignment' models, we incorporate endogenous skill—or, more aptly, ability—biased technical change. We use our framework to engage in a number of comparative statics. Our model's tractability allows us to contrast the short and long-run effects of changes to the economy's fundamentals. We show that, for commonly used distributions, a first order stochastic dominance (FOSD) increase in the ability distribution raises the ability requirement to work in skilled jobs in the short run. In contrast, the long-run cutoff ability may actually decline. More generally, the technological response always dampens the increase in ability requirements.

Keywords: endogenous growth, Roy, innovation, directed technical change, income inequality, overeducation

JEL Codes: O00, O40, O41

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> ...The rabbits are plentiful and stupid and even the less skilled man can ensnare a fair number in a year's hunting while the exercise of a quite appreciable degree of skill does not enable the better hunters to catch many more. The trout, on the other hand, are particularly wily and fight hard, so that many men would undoubtedly starve if they had to eat only what they themselves caught; but nevertheless the real fisherman can obtain very big catches in a year's fishing.
>
> -Roy

1 Introduction

In pioneering work, Roy (1951) proposes a framework which explains the sorting of workers into two occupations: hunting and fishing. As the opening quote indicates, the difference between both jobs is their return to ability. There are some jobs which yield greater returns to ability and jobs which yield little to no return to ability. For the lack of a better term we term the former skilled jobs and the latter unskilled jobs. Because skilled jobs yield greater returns to ability, high (low) ability workers sort into skilled (unskilled) jobs. This self-selection plays an important role in inequality (Heckman and Sedlacek, 1985). It also governs how changes to the distribution of ability, i.e education, and skill biased technical change affect inequality.

The sorting mechanics, proposed by Roy, yield insights into issues fundamental to growth economics. Consider, for instance, the findings of Hendricks and Schoellman (2014). They show that the ability gap between college and non-college bound students has increased substantially. They argue that the increase in the ability gap can explain the entire rise in the skill premium from 1910 to 1960; it can explain half of the increase of the skill premium in more recent times. Thus, sorting is playing a substantial role at the macroeconomic level. But growth economics has largely ignored Roy's insight.[1]

Our goal in this paper is to shed light on the sorting of workers through the lens of endogenous growth theory. To do so, we build an R&D-driven growth model with three building blocks. First, the economy is comprised of heterogeneous agents who differ in ability. Second, the economy's agents sort into skilled and unskilled jobs according to their comparative advantage. Third, in the spirit of Acemoglu (2002), entrepreneurs engage in R&D to develop skill specific intermediate goods.

Before proceeding further, it's important to discuss our terminology—especially because of our model's relation to directed technical change. Similar to the sorting frameworks of

[1] Jaimovich and Rebelo (2017) and Grossman and Helpman (2018) are exceptions which we discuss below.

Costinot and Vogel (2010) and Grossman and Helpman (2018), we abstract from education. When we use the terms skilled and unskilled, we do not mean college and non-college workers. We use the terms skilled and unskilled because, in equilibrium, high ability agents obtain skilled jobs and earn more than their low-ability counterparts. This is important because it is related to a key difference between our framework and directed technical change Acemoglu (2002).

Existing theories of skill biased technical change, *a la* Acemoglu (2002), assume a two factor structure. Skilled (unskilled) goods are produced using the entire skilled (unskilled) endowment and skilled (unskilled) intermediate goods. While directed technical change is a powerful framework—which yields important insight—it is silent concerning the allocation of workers across different jobs. To understand why, it's useful to reflect on the model's assumptions. An implicit assumption of directed technical change is that the marginal product of skilled (unskilled) workers using unskilled (skilled) intermediates is zero. Consequently, the economy's technology—biased or not—has no effect on the allocation of workers.

When skilled and unskilled output is sufficiently substitutable, the economy's dynamics are not globally stable; there are three potential regimes: 1) only unskilled jobs, 2) both skilled and unskilled jobs, 3) only skilled jobs. As we discuss below, if the skilled (unskilled) technology is too advanced the economy ceases the creation of new (unskilled) skilled machines and asympotitcally specializes in skilled (unskilled) production. More interestingly, however, the economy's distribution of ability can also dictate which regime emerges. We show that the first regime *must* emerge if the economy's relative supply of high ability workers is too low. In contrast, if the economy reaches a critical mass of high ability workers, the economy converges to the interior regime in which entrepreneurs develop both skilled and unskilled machines. Finally, if the economy reaches a larger threshold of high ability workers, entrepreneurs cease creating unskilled machines. In this scenario unskilled jobs disappear and low ability workers work with machines that they are relatively unproductive with.

Although we focus our attention to the allocation of workers to skilled and unskilled jobs, we see our framework as a new tool that may prove useful in a number of other applications. We briefly discuss one example. Although seemingly technical, the stability of the economy's dynamics is the driving force behind Acemoglu, Aghion, Bursztyn, and Hemous (2012). In that framework entrepreneurs invent new machines, either clean or dirty, which are used by a *homogeneous* labor force. One of the most interesting results is that when the clean and dirty production techniques are sufficiently substitutable, the leading technology always "wins". Specifically, the dynamics are globally unstable: if the dirty (clean) technology is too far ahead, innovators only create new dirty (clean) machines. Consequently, a temporary subsidy to the creation of clean machines can allow the clean technology win.

Our framework nests a special case which is similar to Acemoglu, Aghion, Bursztyn, and Hemous (2012). In our model the stability of the economy's dynamics is governed to a large extent by how 'different' the jobs are—and also how 'different' workers are. One can easily reinterpret our model as having 'clean' and 'dirty' jobs. Workers sort into occupations in which they have a comparative advantage. If clean and dirty jobs use ability with the same intensity then, as in Acemoglu, Aghion, Bursztyn, and Hemous (2012), the economy's dynamics are globally unstable. If, however, the production techniques differ enough—e.g, if clean jobs use ability with greater intensity—then the economy's dynamics are no longer globally unstable. Consequently, in contrast to Acemoglu, Aghion, Bursztyn, and Hemous (2012), a temporary subsidy to clean innovation might not be enough to make the clean technology win. The driving force behind this difference is the worker's comparative advantage. Put succinctly, if some workers are very unproductive in 'clean' jobs, they will remain in their dirty jobs which prevents the clean technology from taking over.

In recent work Jaimovich and Rebelo (2017) and Grossman and Helpman (2018) develop growth frameworks which feature worker heterogeneity and endogenous sorting mechanisms. The former generalizes Romer (1990) to include worker heterogeneity. Workers choose to work in either manufacturing—which yields a constant wage that is invariant to workers ability—or R&D which yields a positive ability premium. They use their model to discuss how changes in taxes affect economic growth. Grossman and Helpman (2018) incorporate several sources of heterogeneity. Similar to Jaimovich and Rebelo (2017), higher ability workers work in R&D. However, in contrast to Jaimovich and Rebelo (2017), Grossman and Helpman (2018) incorporate heterogeneous firms: both R&D and manufacturing. In their framework, higher productivity firms (R&D or manufacturing) hire relatively more high ability workers. This leads to a number of interesting results concerning inequality in growth.

Although our framework incorporates worker heterogeneity and endogenous growth, our structure and focus is different from Jaimovich and Rebelo (2017) and Grossman and Helpman (2018). The biggest difference is that our framework features endogenous skill biased technical change. The technological change is neutral in Jaimovich and Rebelo (2017) and Grossman and Helpman (2018). They assume that manufacturing productivity and R&D efficiency grow at the same rate and hence has no effect on the allocation of workers.[2] Instead, we focus on the sorting between different production jobs and its interrelation with skill biased technical change. While sorting between manufacturing and R&D employment is unequivocally important, only 0.6% of workers in Korea—who has the largest fraction of

[2]In Grossman and Helpman (2018) there is a positive sorting between high productivity firms and high ability workers, but firm productivity is obtained from a random draw.

workers employed in R&D—are employed in R&D. Hence in terms of up and downskilling, the primary focus of our framework, R&D employment cannot play a large role.

Our paper is organized as follows. Section 2 describes the basic setup. Section 3 solves the model. Section 4 presents the model for common functional forms and we engage in comparative statics. Section 5 develops a variant of our model which features educated and uneducated workers. Finally, section 6 concludes.

2 The model

This section presents the model. Unless necessary to avoid ambiguity, we omit time subscripts.

2.1 Production

The economy produces a homogeneous final good Y according to the technology,

$$Y = \left[Y_S^{\frac{\epsilon-1}{\epsilon}} + Y_U^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where $\epsilon > 0$ is the elasticity of substitution between skilled production, Y_S, and unskilled production, Y_U. The representative firm's profit maximization yields

$$\left(\frac{P_S}{P_U} \right)^{-\epsilon} = \frac{Y_S}{Y_U} \tag{2}$$

where P_S and P_U are the respective prices of Y_S and Y_U.

Skilled (unskilled) goods are produced according to

$$Y_S = \left(\int_A^{\infty} g_S(\alpha) \ell_S(\alpha) d\alpha \right)^{1-\beta} \int_0^{N_S} x_{Si}^{\beta} di, \tag{3}$$

and

$$Y_U = \left(\int_0^{A} g_U(\alpha) \ell_U(\alpha) d\alpha \right)^{1-\beta} \int_0^{N_U} x_{Ui}^{\beta} di, \tag{4}$$

where x_{Si} (x_{Ui}) is the quantity of a skilled (unskilled) machine i, N_S (N_U) is the endogenous measure of skilled (unskilled) machines, and $l_S(\alpha)$ ($l_U(\alpha)$) is the quantity of labor of ability α hired for skilled (unskilled) jobs. A worker with ability α has a skilled efficiency $g_S(\alpha)$ and unskilled efficiency $g_U(\alpha)$ where:

$$\frac{dg_S(\alpha) / g_U(\alpha)}{d\alpha} \geq 0; \tag{5}$$

$$\lim_{\alpha \to 0} \frac{g_S(\alpha)}{g_U(\alpha)} = 0; \tag{6}$$

$$\lim_{\alpha \to \infty} \frac{g_S(\alpha)}{g_U(\alpha)} = \infty. \tag{7}$$

In our model, a skilled job yields higher returns to ability than an unskilled job. Consequently, there's an endogenous cutoff ability, A, which segments workers into skilled and unskilled jobs. Low ability workers, endowed with ability $\alpha < A$, obtain unskilled jobs. High ability workers, endowed with ability $\alpha > A$, obtain skilled jobs. We derive the endogenous threshold ability A and discuss its properties in section 3.

The economy is comprised of L workers (which we normalize to one to ease notational burden). The distribution of workers ability is $\Phi(\alpha)$. Therefore in equilibrium

$$\int_A^\infty g_S(\alpha)\, \ell_S(\alpha) d\alpha = \int_A^\infty g_S(\alpha)\, d\Phi(\alpha); \tag{8}$$

$$\int_0^A g_U(\alpha)\, \ell_U(\alpha) d\alpha = \int_0^A g_U(\alpha)\, d\Phi(\alpha). \tag{9}$$

Equations (8) and (9) are obtained by using that the demand for workers of a given ability equals the supply: $\ell_S(\alpha) = \phi(\alpha)$ for $\alpha > A$ and $\ell_U(\alpha) = \phi(\alpha)$ for $\alpha < A$. We introduce two last assumptions

$$\lim_{A \to 0} \int_A^\infty g_S(\alpha)\, d\Phi(\alpha) < c_S; \tag{10}$$

$$\lim_{A \to \infty} \int_0^A g_U(\alpha)\, d\Phi(\alpha) < c_U. \tag{11}$$

Assumptions (10) and (11) ensure that the effective skilled and unskilled labor forces bounded. When A goes to zero, all workers obtain skilled jobs; this effective labor force is bounded by c_S. In contrast, when A goes to infinity, all workers obtain unskilled jobs; the effective unskilled labor force is bounded by c_U.

Because our structure builds upon directed technical change, it's worthwhile to discuss the differences between the two structures. In contrast to directed technical change—which has two exogenous endowments of skilled and unskilled labor—our framework incorporates heterogeneous workers who choose to work in the job, either skilled or unskilled, which yields them the highest wage. We use the term skilled because, in equilibrium, *relatively* high ability agents work in skilled jobs. One of the interesting features of our framework is that the threshold ability is endogenous and skill biased technical change will play a key role.

2.2 Factor demands

The production of skilled and unskilled goods is undertaken by perfectly competitive firms who take wages and prices as given. Consequently workers are paid their marginal revenue products and hence:

$$w_S(\alpha) = P_S(1-\beta)\,g_S(\alpha)\left(\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)\right)^{-\beta}\int_0^{N_S} x_{Sit}^\beta di; \tag{12}$$

$$w_U(\alpha) = P_U(1-\beta)\,g_U(\alpha)\left(\int_0^A g_U(\alpha)\,d\Phi(\alpha)\right)^{-\beta}\int_0^{N_U} x_{Uit}^\beta di. \tag{13}$$

Because high ability workers work in skilled jobs, we refer to $w_S(\alpha)$ as the skilled wage; analogously, we refer to $w_U(\alpha)$ as the unskilled wage. But the ratio $w_S(\alpha)/w_U(\alpha)$ is not the skill premium *a la* Acemoglu (2002). Instead it is the relative payoff of a worker, endowed with ability α, who works in a skilled job. We solve the model in section 3, but equations (12) and (13) provide insight to the allocation of workers. It's useful to look at the relative wage rate,

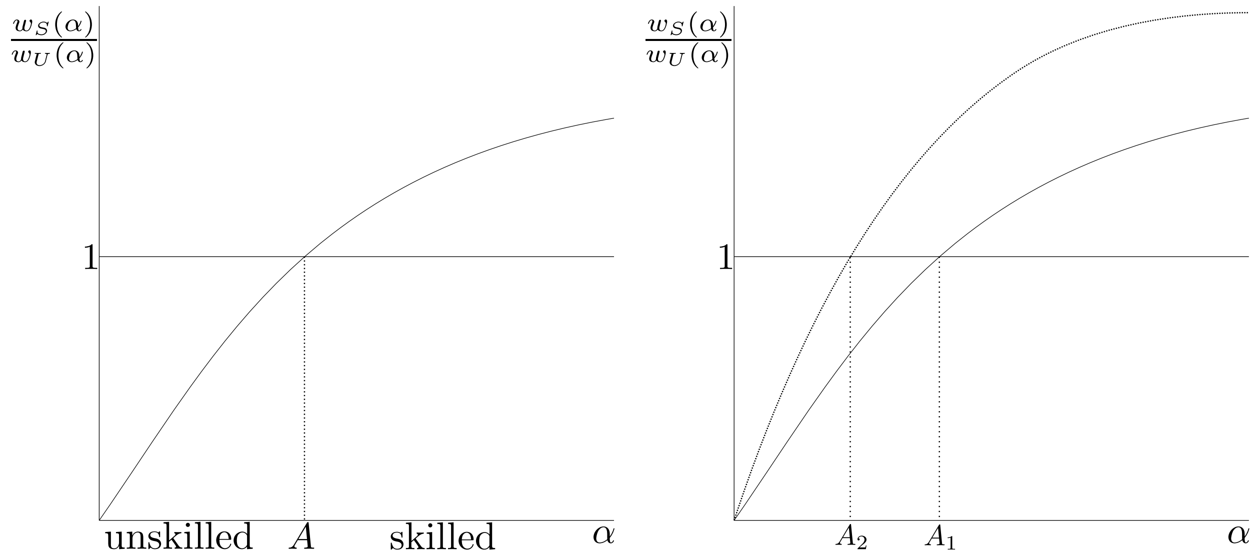
$$\frac{w_S(\alpha)}{w_U(\alpha)} = \frac{g_S(\alpha)}{g_U(\alpha)}\left(\frac{P_S}{P_U}\right)\left(\frac{\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)}{\int_0^A g_U(\alpha)\,d\Phi(\alpha)}\right)^{-\beta}\frac{\int_0^{N_S} x_{Si}^\beta di}{\int_0^{N_U} x_{Uit}^\beta di}. \tag{14}$$

We plot the relative wage rate (14) in Figure 1a.

Low (high) ability agents endowed with $\alpha < A$ ($A < \alpha$), choose to work in an unskilled (skilled) job because the relative wage $w_S(\alpha)/w_U(\alpha)$ is less (greater) than one. Assumptions (5), (6), and (7) ensure that (14) is increasing in α and that some portion of the labor force segments into both types of job. The relative payoff of working in a skilled job is increasing in the relative price of skilled output, P_S/P_U, and it is decreasing in the relative effective skilled labor force, $\int_A^\infty g_S(\alpha)\,d\Phi(\alpha) / \int_0^A g_U(\alpha)\,d\Phi(\alpha)$. It is also increasing in the relative supply of skill intensive machines, $\int_0^{N_S} x_{Si}^\beta di / \int_0^{N_U} x_{Uit}^\beta di$.

The demand for skilled machines is

$$x_{Si} = \left(\frac{P_S\beta}{p_{Si}}\right)^{\frac{1}{1-\beta}}\int_A^\infty g_S(\alpha)\,d\Phi(\alpha), \tag{15}$$



(a) Sorting pattern

(b) Skill biased technical change.

Figure 1: Partial equilibrium sorting

and the demand for unskilled machines

$$x_{Ui} = \left(\frac{P_U \beta}{p_{Ui}}\right)^{\frac{1}{1-\beta}} \int_0^A g_U(\alpha)\, d\Phi(\alpha)\,. \tag{16}$$

The demand for skilled machines is increasing in the price of the skilled output and decreasing in its own price. Symmetrically, the demand for unskilled machines is increasing in the price of unskilled goods and decreasing in its own price. Similar to directed technical change, the demand for skill specific machines depends on the amount of labor using them. In contrast to directed technical change, however, the demand for the machines and the allocation of labor is jointly determined. As Figure 1b shows, when the relative supply of skilled intermediates increases, the relative wage rate increases—this causes A to decrease and hence the relative skilled labor force increases.

2.3 Intermediate goods

Following standard practice, we normalize the unit cost of producing a machine to one. The profit of intermediate producer i in sector $j = S, U$ is thus

$$\pi_{ji} = x_{ji}\left(p_{ji} - 1\right). \tag{17}$$

Firms choose their price to maximize their profits (17) subject to the demand for machines (15) and (16). The maximization yields the pricing rule

$$p_S = p_U = \frac{1}{\beta}. \tag{18}$$

Substituting (18) and the demands (15) and (16) into (17) yields the relative profit

$$\pi \equiv \frac{\pi_S}{\pi_U} = \left(\frac{P_S}{P_U}\right)^{\frac{1}{1-\beta}} \frac{\int_A^\infty g_S(\alpha)\, d\Phi(\alpha)}{\int_0^A g_U(\alpha)\, d\Phi(\alpha)}. \tag{19}$$

The relative profit of skilled intermediate producers is increasing in the relative price, P_S/P_U, and—holding the relative price constant—is increasing in the relative effective skilled labor force, $\int_A^\infty g_S(\alpha)\, d\Phi(\alpha) \,/\, \int_0^A g_U(\alpha)\, d\Phi(\alpha)$.

2.4 New product innovation

Following Acemoglu (2002) we assume a "lab equipment" specification for R&D (Rivera-Batiz and Romer, 1991; Acemoglu and Zilibotti, 2001; Acemoglu, 2002)

$$\dot{N}_S = \eta_S R_S \text{ and } \dot{N}_U = \eta_U R_U, \tag{20}$$

where R_S is R&D spending for the skill intensive good, and R_U is R&D spending for the unskilled intensive good. Standard asset pricing conditions yield the well-known rate of return to new product creation

$$r_j = \eta_j \pi_j, \quad j = u, s. \tag{21}$$

Substituting (19) into (21) yields the relative rate of return

$$\frac{r_S}{r_U} = \frac{\eta_S}{\eta_U}\pi = \frac{\eta_S}{\eta_U}\left(\frac{P_S}{P_U}\right)^{\frac{1}{1-\beta}} \frac{\int_A^\infty g_S(\alpha)\, d\Phi(\alpha)}{\int_0^A g_U(\alpha)\, d\Phi(\alpha)}. \tag{22}$$

The relative profit of creating new skill intensive machines is increasing in the relative profit, π, and as such is increasing in the relative price and relative labor force. Changes to the relative supply of high ability affects the relative rate of return through two mechanisms. Because skilled machines use relatively high ability workers, an increase to the relative supply of skilled workers increases the marginal product of skilled machines. But a change in the relative supply of high ability workers also affects the relative price; Acemoglu (2002) refers to this as the price effect. We discuss this in detail in the next section where we close the model.

3 General equilibrium

The previous section described the model's basic setup. In this section we close the model. We first solve for the economy's instantaneous equilibrium taking the economy's technology as exogenous. We then solve for the economy's steady state and analyze its stability.

3.1 Exogenous technology

Substituting (15), (16), and (18) into (3) and (4) yields the relative supply skilled output

$$\frac{Y_S}{Y_U} = \left(\frac{N_S}{N_U}\right)\left(\frac{P_S}{P_U}\right)^{\frac{\beta}{1-\beta}}\left(\frac{\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)}{\int_0^A g_U(\alpha)\,d\Phi(\alpha)}\right) \tag{23}$$

substituting the relative demand (2) into (23) yields the relative price

$$\frac{P_S}{P_U} = \left(\frac{N_S}{N_U}\right)^{\frac{-(1-\beta)}{(\epsilon-1)(1-\beta)+1}}\left(\frac{\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)}{\int_0^A g_U(\alpha)\,d\Phi(\alpha)}\right)^{\frac{-(1-\beta)}{(\epsilon-1)(1-\beta)+1}}. \tag{24}$$

The relative price of skilled output is decreasing in the relative technology N_S/N_U and the relative labor force $\int_A^\infty g_S(\alpha)\,d\Phi(\alpha) / \int_0^A g_U(\alpha)\,d\Phi(\alpha)$. The intuition behind both is straightforward but important. When either increase, the relative production of skilled goods increases. This increase in the relative supply of skilled goods decreases the relative price. This is the well-known "price effect" (Acemoglu, 2002). The price effect plays a crucial role in directed technical change. It does in our framework as well. Equation (22) shows that the relative rate of return is increasing in the relative price. The magnitude of the price effect is governed by the elasticity of substitution, ϵ. When ϵ is large, changes to the relative supply yield small changes to the relative price. As we show below, the price effect also plays a crucial role governing the relationship between the allocation of workers and the relative technology.

In this section, we derive the relationship between the economy's technology and the allocation of workers.

Proposition 1. *Define*

$$\omega(A,n,\Phi) \equiv \frac{g_S(A)}{g_U(A)}\left(\frac{\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)}{\int_0^A g_U(\alpha)\,d\Phi(\alpha)}\right)^{\frac{-1}{(\epsilon-1)(1-\beta)+1}} n^{\frac{(\epsilon-1)(1-\beta)}{(\epsilon-1)(1-\beta)+1}}. \tag{25}$$

where

$$n \equiv \frac{N_S}{N_U}. \tag{26}$$

Under conditions (10) and (11) the short-run ability cutoff is unique and determined by the implicit equation

$$A(n,\Phi) \equiv \underset{A}{argsolve} \{\omega(A,n,\Phi) = 1\}. \tag{27}$$

All workers with ability greater (less) than $A(n,\Phi)$ work in the skilled (unskilled) jobs.
Proof: See the appendix.

We define $A(n,\Phi)$ as the short-run cutoff ability because it holds the technology constant. The following corollary characterizes the relationship between the relative technology and the allocation of workers.

Corollary 1. *When $\epsilon > 1$,*

$$\frac{dA(n,\Phi)}{d\ln n} = \frac{-(\epsilon-1)(1-\beta)}{((\epsilon-1)(1-\beta)+1)\,g'(A(n,\Phi)) + \lambda(A(n,\Phi))} < 0; \tag{28}$$

$$\frac{d\ln\left(\int_A^\infty g_S(\alpha)\,d\Phi(\alpha) \,/\, \int_0^A g_U(\alpha)\,d\Phi(\alpha)\right)}{d\ln n} = -\lambda(A(n,\Phi))\frac{dA(n,\Phi)}{d\ln n} > 0 \tag{29}$$

where

$$\lambda(A(n,\Phi)) \equiv \frac{\phi(A(n,\Phi))\,g_S(A(n,\Phi))}{\int_{A(n,\Phi)}^\infty g_S(\alpha)\,d\Phi(\alpha)} + \frac{\phi(A(n,\Phi))\,g_U(A(n,\Phi))}{\int_0^{A(n,\Phi)} g_U(\alpha)\,d\Phi(\alpha)}. \tag{30}$$

and

$$g'(A(n,\Phi)) \equiv \frac{d\ln(g_S(A)/g_U(A))}{dA} \tag{31}$$

When $\epsilon < 1$, (28) and (29) are reversed.
Proof: *See the appendix.*

Equation (28) implies that when Y_S and Y_U are substitutes (complements), an increase in n leads to a lower (higher) ability cutoff. To understand the intuition note that

$$\omega(A(n,\Phi),n,\Phi(\alpha)) = \left(\frac{g_S(A(n,\Phi))}{g_U(A(n,\Phi))}\right)\left(\frac{P_S}{P_U}\right)\left(\frac{Y_S}{L_S}\right)\left(\frac{L_U}{Y_U}\right) = 1.$$

By definition, a worker with ability $A(n,\Phi)$ must be indifferent to working in both jobs. An increase in the relative technology, n, increases the relative marginal product of working as a skilled worker. But, because the relative supply of skilled production also increases, the relative price (P_S/P_U) falls. When the goods are substitutes, the price effect is comparatively weak—consequently, the relative marginal revenue product of skilled workers increases. Because the marginal revenue product of skilled workers rises, workers reallocate towards skilled jobs—hence $A(n,\Phi)$ decreases. In contrast, when the goods are complements, the price effect is strong. In this case, the relative wage rate of skilled workers declines and hence $A(n,\Phi)$ increases. Equation (29) implies that if $A(n,\Phi)$ decreases (increases), the relative employment of skilled workers increases (decreases). Beyond the qualitative direction, however, the magnitude of the changes to the ratio of employment (L_S/L_U) is important.

Combining (28) and (29) yields

$$\frac{d\ln(\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)\,/\int_0^A g_U(\alpha)\,d\Phi(\alpha))}{d\ln n}=\frac{(\epsilon-1)(1-\beta)}{\frac{((\epsilon-1)(1-\beta)+1)g'(A(n,\Phi))}{\lambda(A(n,\Phi))}+1} \tag{32}$$

note that the term $g'(A(n,\Phi))$ captures how different skilled and unskilled jobs are with respect to their returns to ability. If the two jobs are identical (and hence $g'(A(n,\Phi))=0$), the elasticity of employment with respect to n is given by the constant $(\epsilon-1)(1-\beta)$. In contrast, when the returns to ability are much larger in the skilled sector, the denominator is large and hence skill biased technical change induces little labor reallocations. Another important determinant is the elasticity of the relative labor supply with respect to A, $\lambda(A(n,\Phi))$. When $\lambda(A)$ is large, the relative labor supply is more elastic with respect to changes in n. Ultimately, the shape of $\lambda(A)$ depends on the functional forms for $g_S(\alpha)$, $g_U(\alpha)$, and $\Phi(\alpha)$.

3.2 Endogenous technology

Having solved for the short-run cutoff ability, $A(n,\Phi)$, as an implicit function of the relative technology n and the distribution of ability Φ, we now endogenize the technology. In the steady state, the rate of return to creating skilled and unskilled machines must be equal. This leads to our next proposition.

Proposition 2. *Define*

$$\Gamma(A,\Phi)\equiv\left(\frac{\eta_S}{\eta_U}\right)\left(\frac{g_S(A)}{g_U(A)}\right)^{\frac{1}{(\epsilon-1)(1-\beta)}}\left(\frac{\int_A^\infty g_S(\alpha)\,d\Phi(\alpha)}{\int_0^A g_U(\alpha)\,d\Phi(\alpha)}\right)^{\frac{(\epsilon-1)(1-\beta)-1}{(\epsilon-1)(1-\beta)}}. \tag{33}$$

If

$$\epsilon<(1-\beta)^{-1}+1\equiv\tilde{\epsilon}, \tag{34}$$

then the economy's steady state is unique

$$A(\Phi)\equiv \underset{A}{\mathit{argsolve}}\{\Gamma(A,\Phi)=1\}; \tag{35}$$

$$n_{ss} = \left(\frac{\eta_S}{\eta_U}\right)^{\epsilon(1-\beta)+\beta} \left(\frac{\int_A^\infty g_S(\alpha)\, d\Phi(\alpha)}{\int_0^A g_U(\alpha)\, d\Phi(\alpha)}\right)^{(\epsilon-1)(1-\beta)}. \tag{36}$$

Proof: See the appendix.

Note that in the steady state, the long-run cutoff ability, and relative technology, is determined solely by the fundamentals of the economy: the R&D parameters, η_S and η_U, the ability functions, g_S and g_U, and the distribution of ability $\Phi(\alpha)$. We discuss how changes to the distribution of ability affects the economy in section 5. Before doing so, we first discuss the properties of (35). The restriction $\epsilon < \tilde{\epsilon}$, ensures that $\Gamma(A, \Phi(\alpha))$ is monotonically decreasing (increasing) for $\epsilon < 1$ ($1 < \epsilon$). When $\tilde{\epsilon} < \epsilon$, (33) might not be monotonic. We postpone a discussion of this case until section 4.

3.3 Stability of the steady state

Substituting (24) into (22)

$$\frac{r_S}{r_U} = \frac{\eta_S}{\eta_U} \left(\frac{\int_{A(n,\Phi)}^\infty g_S(\alpha)\, d\Phi(\alpha)}{\int_0^{A(n,\Phi)} g_U(\alpha)\, d\Phi(\alpha)}\right)^{\frac{(\epsilon-1)(1-\beta)}{(\epsilon-1)(1-\beta)+1}} n^{\frac{-1}{(\epsilon-1)(1-\beta)+1}}. \tag{37}$$

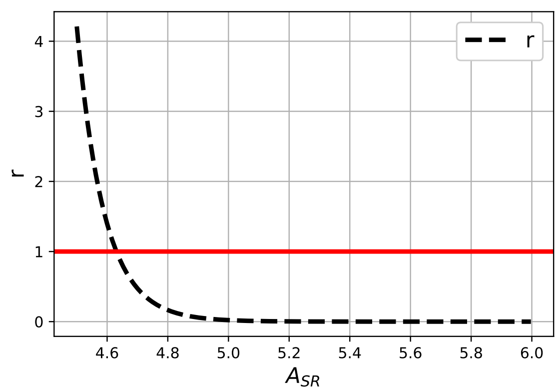
The transitional dynamics in this type of models is well-known: when the rate of return to creating skilled machines is larger (smaller) than the rate of return to creating unskilled machines, all R&D is directed to creating skilled (unskilled) machines. Therefore, on the transition path, n increases (decreases). Stability requires that the relative rate of return is decreasing in n.

Holding the allocation of workers constant, the relative rate of return is decreasing in n. As n increases, skilled workers become more productive and produce more goods. But the increased production of skilled goods decreases the relative price P_S/P_U. As the relative price decreases, so too does the profitability of creating skilled machines. This is the driving force behind the stability of directed technical change (Acemoglu, 2002). Here, however, there is an additional mechanism. Recall Lemma 1: when skilled and unskilled output are substitutes (complements), an increase in n causes the cutoff ability A to decrease and hence the relative labor supply $\int_A^\infty g_S(\alpha)\, d\Phi(\alpha) \,/ \int_0^A g_U(\alpha)\, d\Phi(\alpha)$ to increase.

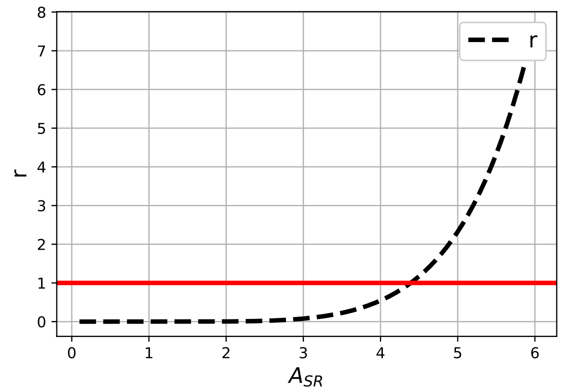
Taking the log of (37) and totally differentiating with respect to n yields

$$\frac{d\ln(r_S/r_U)}{d\ln n} = \frac{\lambda(A(n,\Phi))\left((\epsilon-1)(1-\beta)-1\right) - g'(A(n,\Phi))}{\left((\epsilon-1)(1-\beta)+1\right) g'(A(n,\Phi)) + \lambda(A(n,\Phi))}, \tag{38}$$

when $\epsilon < \tilde{\epsilon}$, the numerator is always negative—this implies that the system is globally stable.



(a) Complements: $\epsilon < 1$



(b) Weak substitutes: $1 < \epsilon < \tilde{\epsilon}$

Figure 2: Globally stable dynamics

To further understand the dynamics, it is useful to reduce the dimensionality of the problem. Combining (27) with (37) allows us to obtain the relative rate of return as a composite function of n and Φ,

$$\frac{r_S}{r_U} = \frac{\eta_S}{\eta_U} \left(\frac{\int_{A(n,\Phi)}^{\infty} g_S(\alpha)\, d\Phi(\alpha)}{\int_0^{A(n,\Phi)} g_U(\alpha)\, d\Phi(\alpha)} \right)^{\frac{(\epsilon-1)(1-\beta)-1}{(\epsilon-1)(1-\beta)}} \left(\frac{g_S(A(n,\Phi))}{g_U(A(n,\Phi))} \right)^{\frac{1}{(\epsilon-1)(1-\beta)}}. \tag{39}$$

For both cases, complements or weak substitutes, when $r > 1$, all R&D is devoted to increasing the number of skilled machines. The difference lies in how changes in n affects $A(n,\Phi)$. Equation (28) established that $A(n,\Phi)$ is increasing (decreasing) in n when the goods are complements (substitutes). Figure (2a) depicts the dynamics when the goods are complements; Figure (2b) depicts the case of weak substitutes ($\tilde{\epsilon} > \epsilon > 1$). In both cases, the system is globally stable.

4 Comparative statics

Throughout the rest of the paper, to help engage in comparative statics, we assume

$$g_S(\alpha) = \alpha^s, \text{ and } g_U(\alpha) = \alpha^u, \text{ where } 1 > s > u \geq 0. \tag{40}$$

This functional form is useful because of the simplicity of the moment generating functions; it also satisfies restrictions (5), (6), and (5).[3]

4.1 First order stochastic dominance

Here we analyze the effects of changes to the ability distribution. Recall that $\tilde{\Phi}$ first order stochastically dominates Φ iff

$$\tilde{\Phi}(\alpha) \leq \Phi(\alpha), \ \ \forall \alpha. \tag{41}$$

Stochastic domination is useful because it allows us to analyze the effects of an increase in the relative supply of high ability workers. Hence, the amount of workers beneath a given ability decreases. We now turn to the effects of a FOSD increase in the distribution from Φ to $\tilde{\Phi}$.

Lemma 1. *Suppose that $u = 0$ and that (41) holds, then*

$$\frac{\omega\left(A, n, \tilde{\Phi}\right)}{\omega\left(A, n, \Phi\right)} \leq 1 \tag{42}$$

and, consequently,

$$A(n, \Phi) < A(n, \tilde{\Phi}). \tag{43}$$

Proof: See the appendix

We will discuss the caveat $u = 0$, that unskilled jobs yield no return to ability, momentarily. We first discuss the intuition behind Lemma 1. Because ω is increasing in A, (43) follows trivially from (42). The intuition is straightforward. Holding the technology fixed, an increase in the supply of high ability workers, reduces the marginal revenue product of skilled workers. Consequently the relative payoff of working as skilled labor, ω, declines and hence the cutoff ability must increase. Our next lemma analyzes the long run effect on the cutoff ability.

Lemma 2. *Suppose that $u = 0$ and that $\tilde{\Phi}$ stochastically dominates Φ, there are two cases depending on the elasticity of substitution*

[3]The functional form implies that the wages are log-linear in ability.

$$\ln w_S(\alpha) = \ln\left(\beta^{\frac{2\beta}{1-\beta}}(1-\beta)\right) + \frac{1}{1-\beta}\ln P_S(n, A(n, \Phi)) + \ln N_S + s\ln\alpha;$$

$$\ln w_U(\alpha) = \ln\left(\beta^{\frac{2\beta}{1-\beta}}(1-\beta)\right) + \frac{1}{1-\beta}\ln P_U(n, A(n, \Phi)) + \ln N_U + u\ln\alpha$$

where the wages are log-linear in ability.

Case 1. *When* $1 < \epsilon < \tilde{\epsilon}$*,*

$$\frac{\Gamma\left(A, \tilde{\Phi}\right)}{\Gamma\left(A, \Phi\right)} \leq 1;$$

$$A(\Phi) < A(\tilde{\Phi}).$$

Case 2. *When* $\tilde{\epsilon} < \epsilon$*,*

$$\frac{\Gamma\left(A, \tilde{\Phi}\right)}{\Gamma\left(A, \Phi\right)} \leq 1;$$

provided the economy converges to the interior solution,

$$A(\tilde{\Phi}) < A(\Phi).$$

Proof: See the appendix for a discussion.

Recall that the instantaneous cutoff ability, $A(n, \Phi)$, yields the cutoff ability at an arbitrary moment in time while $A(\Phi)$ only holds in the steady state. While an increase in the relative supply of high ability workers raises the cutoff ability in the short-run, in the long run the cutoff ability can decrease. The difference lies in the technological response. When the elasticity of substitution is large enough, the technological response will be large and hence the cutoff ability actually declines.

Interestingly, when $u > 0$, an arbitrary FOSD increase in the distribution of ability yields ambiguous affects on the cutoff ability. The driving force behind changes in A lies with the changes in the effective relative labor supply of skilled workers. As we mentioned above, FOSD implies that we move (some) workers from the left of the distribution to the right. Consider a hypothetical change in the distribution where we move workers from the lowest ability to $\tilde{A} < A$, while leaving the rest of the distribution unchanged. In this scenario, the effective unskilled labor force increases relative to skilled labor.

4.2 Pareto

In this section, in order to deal with the case of $\epsilon > \tilde{\epsilon}$ and to help engage in comparative statics, we assume that ability is governed by a Pareto distribution, where

$$\Phi(\alpha) = \begin{cases} 1 - \left(\frac{b}{\alpha}\right)^{\kappa} & \alpha > b \\ 0 & b > \alpha \end{cases}, \tag{44}$$

and $\kappa > 1$.

In this section we solve for the allocation of labor in both the short and the long run. Once again, the distinction between the short and long run is the technology. In the long run, the economy's relative technology, n, is endogenous.

Following (27) the short-run cutoff ability is determined by the implicit function

$$A(n,\Phi) \equiv \underset{A}{\text{argsolve}} \left\{ A^{s-u} \left(\left(\frac{\kappa - u}{\kappa - s} \right) \frac{A^{s-\kappa}}{b^{u-\kappa} - A^{u-\kappa}} \right)^{\frac{-1}{(\epsilon-1)(1-\beta)+1}} n^{\frac{(\epsilon-1)(1-\beta)}{(\epsilon-1)(1-\beta)+1}} = 1 \right\}, \tag{45}$$

and the long-run cutoff ability, is given by

$$A(\Phi) \equiv \underset{A}{\text{argsolve}} \left\{ \left(\frac{\eta_S}{\eta_U} \right) A^{\frac{s-u}{(\epsilon-1)(1-\beta)}} \left(\left(\frac{\kappa - u}{\kappa - s} \right) \frac{A^{s-\kappa}}{b^{u-\kappa} - A^{u-\kappa}} \right)^{\frac{(\epsilon-1)(1-\beta)-1}{(\epsilon-1)(1-\beta)}} = 1 \right\}. \tag{46}$$

Because the technology (40) and distributions (44) satisfies Proposition 1's conditions, the left hand side of (45) is monotonic in A and hence there's a unique solution. The long-run cutoff ability, determined by equation (46), is not necessarily unique when $\epsilon > \tilde{\epsilon}$. We characterize the solution in our next Proposition.

Proposition 3. *There are two cases,*

Case 1. *If*

$$\frac{s-u}{\kappa - s} < \left(\left(\epsilon - 1 \right) \left(1 - \beta \right) - 1 \right), \tag{47}$$

then equation (46) yields one, unstable, root

Case 2. *If (47) is reversed, then there are either two (one stable) roots or no roots to (46). There will be two roots provided,*

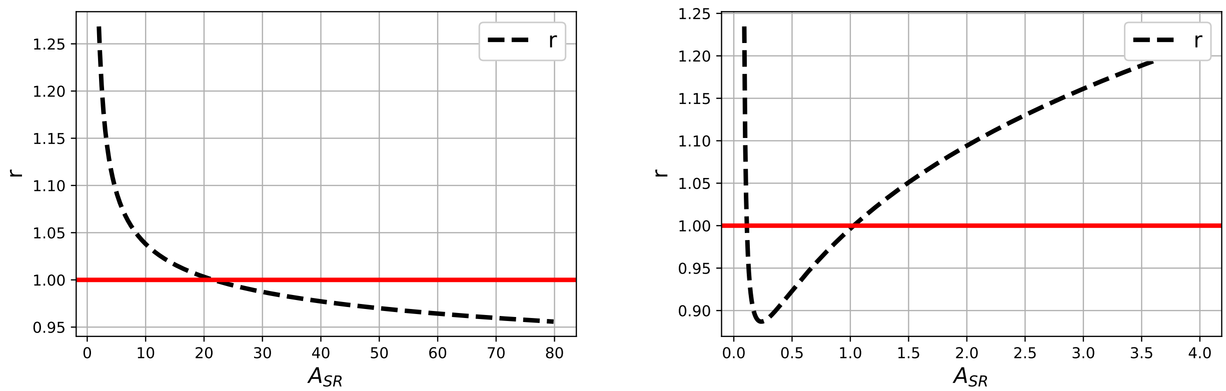
$$\left(\frac{\kappa - u}{\kappa - s} \right) \left(\frac{\eta_S}{\eta_U} \right) M^{\frac{s-u}{(\epsilon-1)(1-\beta)}} \left(\frac{M^{s-\kappa}}{1 - M^{u-\kappa}} \right)^{\frac{(\epsilon-1)(1-\beta)-1}{(\epsilon-1)(1-\beta)}} b^{(s-u)} < 1, \tag{48}$$

where

$$M = \left(\frac{(\epsilon - 1)(1-\beta)(s-u)}{(s-u) - (\kappa - s)\left((\epsilon - 1)(1-\beta) - 1 \right)} \right)^{\frac{1}{\kappa - u}}.$$

Proof: See the appendix.

Both parameter restrictions, (47) and (48), provides insight into the model. Note that equation (47) implies that when $s \approx u$, the system is necessarily unstable. Intuitively, if skilled and unskilled jobs are similar with respect to their ability requirements, an increase to the relative technology will induce a large change in the relative labor supply L_S/L_U



(a) Case 1 (b) Case 2

Figure 3: Globally dynamics—Pareto

yielding more skill biased technical change. Second, note that κ governs the concentration of ability. When κ is very large, the economy's workers are essentially identical. Therefore, similar to the case of $s \approx u$, the system is unstable. Equation (48) shows that the lower bound parameter, b, plays a crucial role. If the lower bound of ability is too high, the economy always finds it profitable to create skill-intensive goods.

Recall that the Pareto distribution $\alpha \sim 1 - \left(\frac{b}{\alpha}\right)^{\kappa}$ is comprised of two parameters, κ and b. The former governs the shape of the distribution and hence the concentration of workers at the lower end of the ability distribution. As κ increases, the percentage of workers nearby the lower support (b) increases. In contrast, a reduction in κ implies that there are fewer workers at the low end of the ability distribution. An increase in b shifts the entire distribution to the right. We focus our attention on a reduction in κ because it corresponds to a rise in education—the movement of workers from low ability to high ability.

Our next corollary, characterizes the short-run relationship between κ and the cutoff ability, $A(n, \Phi(\kappa, b))$.

Corollary 2. *In the short run:*

$$\frac{dA(n,\Phi)}{d\kappa} = \frac{A\left[\frac{u-s}{(\kappa-u)(\kappa-s)} - \frac{b^{u-\kappa}\ln\frac{A}{b}}{b^{u-\kappa}-A^{u-\kappa}}\right]}{(s-u)\left((\epsilon-1)(1-\beta)+1\right)+(s-\kappa)-\frac{(u-\kappa)A^{u-\kappa}}{b^{u-\kappa}-A^{u-\kappa}}} < 0$$

Proof: Apply the implicit function theorem to equation (45).

This corollary is not particularly surprising. When the concentration parameter increases, there are relatively few high ability workers and hence the cutoff ability is low. We now turn to the effect of an increase in κ on the long-run cutoff ability, $A(\Phi)$.

Corollary 3. *In the long run:*

$$\frac{dA(\Phi(\kappa,b))}{d\kappa} = \frac{-A\left(\left(\epsilon-1\right)\left(1-\beta\right)-1\right)\left[\frac{u-s}{(\kappa-u)(\kappa-s)} - b^{u-\kappa}\frac{\ln\frac{A}{b}}{b^{u-\kappa}-A^{u-\kappa}}\right]}{s-u+\left(\left(\epsilon-1\right)\left(1-\beta\right)-1\right)\left[\left(s-\kappa\right)-\frac{(u-\kappa)A^{u-\kappa}}{b^{u-\kappa}-A^{u-\kappa}}\right]} \tag{49}$$

There are two cases:

Case 1. *If*

$$\epsilon < \tilde{\epsilon},$$

then

$$\frac{dA(\Phi(\kappa,b))}{d\kappa} > 0.$$

Case 2. *If*

$$\epsilon > \tilde{\epsilon},$$

then

$$\frac{dA(\Phi)}{d\kappa} < 0.$$

Proof: Apply the implicit function theorem to equation (45).

Corollary 3 establishes two things. Our first result is that if skilled and unskilled goods are sufficiently substitutable, a decrease in κ (which increases the right tail of the ability distribution) can actually decrease the cutoff ability to be a skilled worker. Second it yields the magnitude of the change. This, by itself, is not particularly interesting. But when it is combined with corollary 2 we obtain our next Proposition

Proposition 4. *Suppose that the concentration parameter falls from κ_1 to κ_2. When $\epsilon < \tilde{\epsilon}$,*

$$A(n_{ss1}, \Phi(\kappa_1)) < A(n_{ss2}, \Phi(\kappa_2)) < A(n_{ss1}, \Phi(\kappa_2)) \tag{50}$$

where

$$n_{ss1} = \left(\frac{\eta_S}{\eta_U}\right)^{\epsilon(1-\beta)+\beta}\left(\frac{L_S(A(\Phi(\kappa_1))}{L_U(A(\Phi(\kappa_1))}\right)^{(\epsilon-1)(1-\beta)};$$

$$n_{ss2} = \left(\frac{\eta_S}{\eta_U}\right)^{\epsilon(1-\beta)+\beta}\left(\frac{L_S(A(\Phi(\kappa_2))}{L_U(A(\Phi(\kappa_2))}\right)^{(\epsilon-1)(1-\beta)}.$$

When $\epsilon > \tilde{\epsilon}$,

$$A(n_{ss2}, \Phi(\kappa_2)) < A(n_{ss1}, \Phi(\kappa_1)) < A(n_{ss1}, \Phi(\kappa_2)) \tag{51}$$

Proof: See the appendix.

Proposition 4 sheds light on how education affects the difficulty of obtaining skilled jobs. When the relative supply of high ability workers increases, the ability requirement to obtain a skilled job increases. But this also induces biased technical change. Consequently, as (50) shows, after the economy's technology responds the cutoff ability is lower than the immediate increase. Interestingly, when ϵ is large enough, the cutoff ability actually declines. In other words, in the long-run an increase in the relative supply of high ability workers reduces the ability requirement to obtain skilled jobs.

5 Conclusion

Whether or not increases to the relative supply of skilled workers leads to more overeducated workers is obviously important. Recent empirical work has found that increases to the relative supply of skilled workers leads to deskilling (and hence more overeducated workers), at least at local levels (Modestino, Shoag, and Ballance, 2015). However, our results suggests the need for longer time horizons to analyze the effects. Our model predicts that increases to the relative supply of skilled workers leads to deskilling—in the short run. But in the long-run, following the induced technical change, the threshold ability to work in the skill-intensive sector may increase or decrease.

Our model also yields new insight into directed technical change. As in Acemoglu (2002) and Acemoglu (1998), our model is able to generate strong-bias. But the parameter restrictions necessary for strong-bias are incompatible with global stability. Intuitively, if skilled and unskilled tasks are strong enough substitutes, the price effect is weak. We show that the price effect is crucial for dynamic stability. Finally, we think our model might prove useful in a wide variety of applications that directed technical change, in its current form, is unsuitable for. For instance, the existing literature has no labor reallocations. Consequently directed technical change is silent concerning, substantial, labor misallocations (Hsieh and Klenow, 2009). In contrast, a key driving force of our models results is the reallocation of labor across jobs/tasks.

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